Inelastic Design of Steel Girder Bridges

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ABSTRACT

This paper presents existing inelastic design procedures for steel girder bridges. Comparisons of the inelastic methods and the conventional methods are presented. This paper also discusses past and current analytical and experimental research for developing behavior models to extend inelastic design techniques for bridges comprising non-compact sections.

INTRODUCTION

Inelastic design procedures offer the potential for significant cost savings by accounting for a better estimate of the true strength and behavior of the bridge. Inelastic limits can show a significant increase of capacity over that predicted by conventional elastic or pseudo-plastic procedures used in present bridge specifications.[1] Furthermore, inelastic techniques permit greater design flexibility such as optimizing material use as is done in plastic design procedures, eliminating cover plates and flange transitions, and quantifying the redistribution characteristics for more consistent safety considerations.

In 1986, AASHTO adopted the "Guide Specifications for Alternate Load Factor Design (ALFD) Procedures for Steel Beam Bridges Using Braced Compact Sections."[2] The inelastic procedures in the guide specs are based on the autostress (shakedown) design method. Using this design flexibility to reduce costs, more economical designs have been achieved with the ALFD procedures.[6] The provisions are also applicable to evaluating existing bridges.

One objective of this paper is to remove the confusion and perceived complexity of ALFD design and shakedown procedures. Simply stated, ALFD procedures theoretically determine the redistributed negative pier moments for continuous, compact beams instead of using the arbitrary 10 percent in the current load factor design (LFD) provisions.[1] ALFD can be viewed as a better analysis applied to nearly the same performance requirements as LFD procedures. The ALFD provisions are currently limited to bridges comprising braced, compact sections as is LFD when using the 10 percent redistribution.

This paper presents existing inelastic design procedures for steel girder bridges. Comparisons of the inelastic methods and the conventional methods are presented. This paper also discusses current analytical and experimental research for developing behavior models to extend inelastic design techniques for bridges comprising non-compact sections.

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INELASTIC DESIGN PROCEDURES

For bridges comprising braced, compact sections, Table 1 illustrates the structural performance requirements for the LFD and ALFD procedures for the service, overload, and maximum load levels used in AASHTO specifications.[1] Service limits are imposed to provide adequate fatigue life and to limit service load deflections. Overload limits ensure satisfactory riding quality by limiting permanent deformations produced by occasional heavy (overload) vehicles. Maximum load limits provide adequate structural capacity for infrequent very heavy (maximum) vehicles.

The ALFD procedures require limit-state checks to assure satisfactory structural performance at the same three load levels as the LFD procedures. At service load, exactly the same checks are required. At the other two load levels, the required checks are intended to satisfy the same performance requirements as those in the LFD procedures. The ALFD procedures, however, utilize more refined inelastic analytical procedures to calculate the actual beneficial effects of inelastic redistribution of moments rather than relying on the arbitrary 10 percent shift specified for the LFD procedures. The design limits for the various load levels are discussed below.

Service Load Limits

For LFD and ALFD, the nominal dead load and live load plus impact $[D + L(1 + I)]$ must meet fatigue and deflection limits. The requirements are identical for the two methods.

Overload Limits

To assure satisfactory riding quality at overload, the LFD method limits the maximum positive and negative moments (calculated elastically) to a fraction of the nominal yield moment: 0.95 for composite sections and 0.80 for noncomposite sections. These limits were developed from experimental data from the AASHO road tests conducted in the late 1950s and early 1960s.[3]

For compact sections, up to 10 percent of the peak negative-bending moment at a pier (interior support) in a redundant continuous beam may be shifted to the positive-bending region before checking the sections. This provision is intended to account, in an approximate way, for the redistribu-

Table 1. LFD and ALFD Design Limit States		
Load Level	**LFD**	**ALFD**
Service $[D + L(1 + I)]$	Fatigue, Deflections	Fatigue, Deflections
Overload $[D + 5/3L(1 + I)]$	10% Redistribution of Neg. Pier Moments. Pos. and Neg. Elastic Moments $\leq 0.95M_y$ (Composite) and $\leq 0.80M_y$ (Noncomposite)	Redistributed Neg. Pier Moments calculated. Positive Elastic Moments $\leq 0.95M_y$ (Composite) and $\leq 0.80M_y$ (Noncomposite)
Maximum Load $1.3[D + 5/3L(1 + I)]$	10% Redistribution of Neg. Pier Moments. Pos. and Neg. Elastic Moments $\leq M_p$	Redistributed Neg. Pier Moments calculated through Mechanism Analysis using M_{pe} at Neg. Pier

tion of moment that can occur in continuous girders due to yielding at piers. Such yielding reduces the peak negative-bending moments and increases the positive-bending moments. Thus, it amounts to a shift of moment from negative to positive regions. However, by limiting the negative pier moments to 0.95 or $0.80M_y$ after the 10 percent redistribution, little or no theoretical yielding is actually allowed over the supports.

At overload, the ALFD procedures explicitly permit controlled local yielding over the piers, but limit the maximum positive moments to the same fractions of the yield moment as the LFD procedures. This is more consistent with the intent of the overload design philosophy. Furthermore, the actual amount of the moment shift is calculated directly in the ALFD procedures. The permitted local yielding at piers causes positive automoments that assure elastic behavior under subsequent overloads. The automoment stresses counteract the stresses produced by the applied elastic moments over interior supports and add to the stresses produced by the applied elastic moments in positive-bending regions. Because the beam eventually behaves elastically (shakes down), there is no need to limit stresses in negative bending. Small permanent deformations due to the local yielding in negative bending are computed directly and may be included in the dead-load camber.

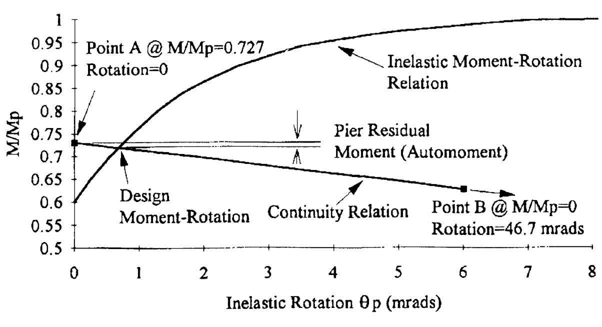

Fig. 1. Graphical determination of moment and automoment at pier..

In the past, the beam-line method[4,5] was generally used to calculate the automoments at overload.[2] In this method, two relationships are used to define the moment at the pier: a continuity relationship for the structure and an inelastic moment-rotation curve for the cross section at the pier.[6] Figure 1 illustrates the beam-line method in graphical form. For continuous beams with more than two spans, however, an iterative procedure is required to get the correct pier moments and plastic rotations. The effects of yielding in positive-bending regions can be accounted for in the beam-line method, but the procedure to do so is somewhat cumbersome, especially for a continuous beam with more than two spans.

Schilling[7,8,9] extended the beam-line method with the unified autostress method (UAM) which has the advantage that it can be applied to all three limit-state checks for multiple spans. Previously, different methods of analysis were used in these different checks. Dishongh[10,11] also developed a residual damage analysis (RDA) method, similar to UAM, which can also analyze girders comprising non-compact sections that exhibit an unloading behavior with increasing inelastic rotation. Both methods have the ability to account for inelastic moment-rotation behavior in the positive moment regions.

Maximum Load Limit

For compact girders, the LFD method limits the elastic moments at maximum load to the plastic moments for all sections and 10 percent of the pier moments may be shifted to the positive-bending region. Thus, the LFD procedures account for the ability of compact sections to sustain moments above the yield moment and account in an approximate way for the redistribution of moments due to inelastic behavior in continuous spans with compact girders. In fact, for compact girders, the LFD maximum load limit is actually an incremental collapse (shakedown) limit, a limit that meets with resistance from practicing engineers when stated as a "shakedown" limit.

The shakedown limit states that for the negative moment region,

$$M_e^- + m_r^- \geq -M_p^-$$

and for the positive moment region,

$$M_e^+ + m_r^+ \leq M_p^+$$

where

M_e^- = elastic moment at the neg. pier region,
m_r^- = residual moment at the neg. pier region,
M_p^- = plastic moment capacity at the neg. pier region,
M_e^+ = elastic moment at the pos. region,
m_r^+ = residual moment at the pos. region, and
M_p^+ = plastic moment capacity at the pos. region.

For LFD, m_r^+ and m_r^- are represented by the 10 percent redistribution of negative pier moments. Thus, the LFD maximum load limit is an approximate measure of the incremental collapse capacity. A drawback to the LFD method is that it ignores the possible moment unloading behavior which may occur in pier sections that must significantly rotate inelastically to provide the redistribution. The section may not be able to maintain M_p at the required rotation.

In contrast to the LFD procedures that limit the elastic moments to the plastic moment capacities for compact sections, the ALFD procedures permit the strength of the bridge to be determined by appropriate inelastic analytical methods such as the plastic-design mechanism method. These methods reflect the inherent strength of continuous spans. However, tests have shown that certain compact shapes may not possess adequate inelastic rotation capacity at the full plastic moment during hinge rotation.[15-18] The ALFD provisions ensure adequate rotation capacity by using an effective plastic moment, M_{pe}, in the mechanism analysis at the first hinge. The level of M_{pe} depends on the plastic-rotation curve and the amount of plastic rotation required at the first hinge up to incipient mechanism formation.

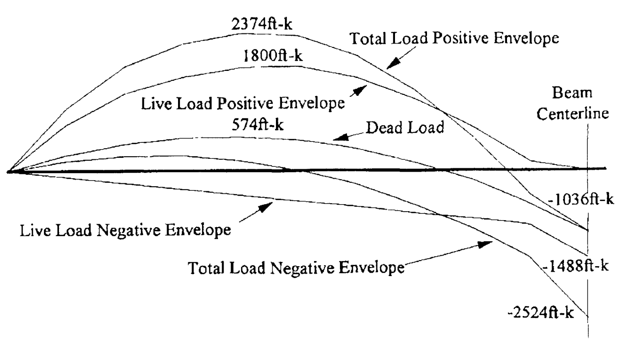

Fig. 2. Overload level elastic moments for first span.

The required plastic rotation depends primarily on the characteristics of the bridge and the plastic-rotation curve depends primarily on the web and compression-flange slenderness ratios for the sections at the piers. Semi-empirical formulas defining M_{pe} in terms of these slenderness ratios were developed in earlier autostress studies[4,12] and are given in the ALFD guide specifications.[2] The formulas, which apply only to compact sections, assure that the sections will provide a plastic-rotation capacity exceeding 0.063 radians; that is, the actual moment will remain above M_{pe} over a plastic rotation up to 0.063 radians.

This amount of plastic rotation was determined in tests[12] to correspond to that provided by sections proportioned for plastic design by Part 2 of the 1978 AISC building specifications,[13] which were in effect at the time the formulas were developed. This amount of plastic rotation was expected to be sufficient to allow continuous bridges to reach their maximum strength.

The effective plastic moment from the semi-empirical formulas equals the actual plastic moment when both the web and compression-flange slenderness ratios meet the corresponding limiting ratios for plastic design given in Part 2 of the 1978 AISC specifications, but is less than the actual plastic moment when either slenderness ratio exceeds its limiting value. For steel with a nominal yield strength of 50 ksi, these limiting ratios are 7.0 for compression flanges (projecting width/thickness) and 55.3 for webs (depth/thickness). Sections with slenderness ratios not exceeding these limiting values are called ultra-compact sections.[9]

DESIGN EXAMPLE

To illustrate the ALFD procedures, a two-span bridge design is examined and compared to the LFD design provisions. The bridge has two 75 ft spans with an 11 ft girder spacing. Design loads are the HS20 truck and lane loads.[1] A trial compact and braced W36×210 (F_y = 50 ksi) noncomposite section will be examined. The yield moment M_y = 2996 ft-k and the plastic moment M_p = 3471 ft-k. The dead load is assumed at 1.46 kips/ft, the lateral distribution factor is 2.0, and the impact factor is taken as 1.25. Figure 2 illustrates the overload level elastic moment diagrams for a typical girder.

For LFD, the elastic moments (M_e) are limited to 80 percent of the yield moment (0. 80 M_y). For the negative moment region,

$$M_e = 2524 \text{ ft-k} \geq 0.8M_y = 2397 \text{ ft-k}$$

which does not meet the criteria. However, since the girder is braced and compact, up to a 10 percent redistribution of the negative pier moment can be applied. The required amount to be redistributed is 127 ft-k (M_e-0.8M_y) which is 5 percent of the elastic moment. The pier moment becomes 2397 ft-k which is the limiting value of 0.8M_y = 2397 ft-k. Redistributing 127 ft-k of the pier moment increases the positive region moment by 40 percent of the redistributed moment. (i.e. 127 $\times$ 0.4 = 51 ft-k). Thus the positive moment becomes 2374 + 51 = 2425 ft-k which exceeds $0.8M_y$. Therefore, the W30×210 section does not meet the LFD criteria.

ALFD provisions calculate the actual (theoretical) redistribution of negative pier moment. This is accomplished by determining the specific moment and inelastic rotation of the pier section by matching the continuity relationship for the elastic structure (M-θ) and the inelastic moment-rotation relationship for the section (M-θp). The beam-line method and assumed relationships located in the ALFD guide spec are used here. Figure 1 shows the graphical method for determining the specific M-θp for the pier section. Two points are determined for the continuity relation. Point A is the elastic moment normalized by M_p with no inelastic rotation. Point B is the rotation at the support associated with no elastic moment (pin ended). The intersection of the line defined by the two points and the section moment-inelastic rotation behavior determines the theoretical M-θp for the pier section. The automoment at the pier is the difference between the elastic moment and the calculated moment.

Determining Point B requires a structural analysis of a simple beam loaded with the loads that cause the largest moment over the pier for the girder.

For this design case,

Point A

$$\theta_A = 0, M_A / M_p = 2524 / 3471 = 0.727$$

Point B

$$\theta_B = 2\left[\frac{WL^3}{24EI} \times \frac{Pab(L+a)}{6LEI}\right] \times 1000$$

$$= 46.7 \text{ mrads } \& \ M_B / M_p = 0$$

where

W = Factored Uniform Load = 1.46 + (5⁄3)0.32(2)(1.25) = 2.79 k/ft, and

P = Factored Applied Load = 5⁄3 (9)(2)(1.25) = 37.5 kips

From Figure 1, the moment at the pier is $M / M_p = 0.72$, or M = 2499 ft-k. Thus the automoment at the pier becomes $0.01M_p$ = 25 ft-k (automoment = $M - M_e$ = −25 ft-k: 25 ft-k opposite of M_e). Figure 3 shows the applied elastic overload level moments, the automoments present due to inelastic rotation of the pier section, and the total overload level moments after the structure has shaken down.

To complete the ALFD overload level design check, the elastic moments in the positive region and the redistributed moment must be less than $0.80M_y$:

$$M^+ = 2374 + 25(0.4) = 2384 \text{ ft-k} \leq 0.8M_y = 2397 \text{ ft-k}$$

Therefore, the W30×210 meets the ALFD overload level criteria.

The LFD checks fail because the arbitrary redistribution of the negative moment required to meet the moment limit at the pier exceeds the expected redistribution and therefore increases the positive region moment beyond the $0.8M_y$ limit. The ALFD procedure calculates a much lower redistributed moment (1 percent) and meets the positive moment region criteria.

For the LFD maximum load design check, the elastic moments are 1.3 times the overload moments. These elastic moments themselves are less than their respective moment capacities so no redistribution is necessary:

$$M_e^+ = 3086 \text{ ft-k} \leq 3471 \text{ ft-k}$$

$$M_e^- = 3281 \text{ ft-k} \leq 3471 \text{ ft-k}$$

Therefore, the maximum load criteria are met. if some of the pier moment is redistributed, the maximum load criterion is actually a maximum shakedown limit (incremental collapse) with an assumed 10 percent pier moment residual moment field. However, the LFD criteria assume that the pier section can inelastically rotate a significant amount and maintain the plastic moment capacity. For compact sections this may not be true. Thus, the LFD provisions may actually be overestimating the girder capacity.

The ALFD procedures use a mechanism limit for maximum load and a reduced moment capacity at the pier (effective plastic moment M_{pe}) to account for local buckling during plastic rotation.

The pier effective moment capacity is

$$M_{pe} = R_f M_{pf} + R_w M_{pw}$$

where

R_f = flange local buckling reduction factor = $\dfrac{0.0845E(t/b')^2}{F_{yf}} \leq 1.0$

R_w = Web local buckling reduction factor = $\dfrac{1.32E(t_w/D_{cp})^2}{F_{yw}} \leq 1.0$

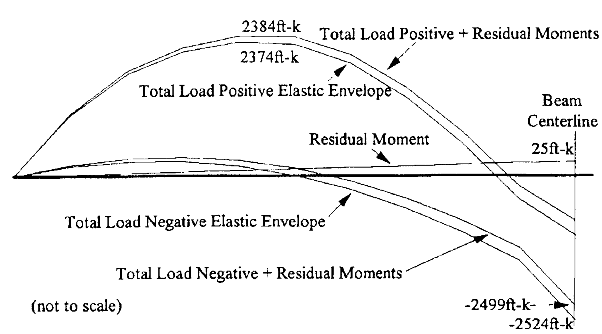

Fig. 3. Overload level moments after shakedown for first span.

M_{pf} = Flange plastic moment capacity component, and
M_{pw} = Web plastic moment capacity component

The reductions (R_f and R_w) are transitions between ultra-compact (large rotation at M_p) and compact (limited rotation at M_p) sections. M_{pe} is the moment capacity at a specified and usually sufficient rotation.

For the W30×210 section,

$$R_f = \frac{0.0845(29000)(0.222)^2}{50} = 2.42 \geq 1.0$$

$$R_w = \frac{1.32(29000)(0.0452)^2}{50} = 1.56 \geq 1.0$$

Therefore, since R_f and R_w are greater than 1.0, the section is ultra-compact and there is no reduction in capacity ($M_{pe} = M_p$). The section has adequate inelastic rotation capacity at M_p. It is then clear that the ALFD maximum load mechanism check for the various load cases are met since the LFD shakedown limit capacity is adequate.

If the section is ultra-compact, the ALFD maximum load capacity will always be greater than the corresponding LFD capacity. However, if M_{pe} is less than M_p, the LFD capacity may exceed the ALFD capacity since the LFD method ignores possible moment capacity loss over the pier when subjected to inelastic rotation.

At the overload check, the W36×210 section does not meet the LFD design criteria. The redistributed pier moment required to reduce the negative moment to $0.8M_y$ increased the positive moment beyond $0.8M_y$. For ALFD, there is no restriction limit for the pier moment after the redistribution. The moment rotation and continuity relations dictate the negative design moment. The girder shakes down and, at least for this girder, redistributes less moment to the positive moment region than the LFD method.

The maximum load check does not often control the girder design. The LFD method uses a shakedown limit with an assumed residual moment field. It also assumes that the negative pier section can maintain the plastic moment up to this limit. The ALFD method uses the collapse mechanism limit with an effective moment capacity M_{pe}. If $M_{pe} = M_p$, the maximum load capacity will be nearly the same as the shakedown capacity for a single girder with typical truck loads.[20]

FUTURE OF INELASTIC DESIGN PROCEDURES

Proposed load and resistance factor (LRFD) bridge specifications (NCHRP Project 12-33) have been developed recently.[14] These proposed specifications include both elastic and inelastic design procedures. The elastic procedures account for inelastic effects in the same approximate way as the present LFD provisions. Specifically, they permit the moments calculated by elastic methods to reach the plastic moment in compact sections and permit a 10 percent longitudinal redistribution of peak negative moments in continuous spans.

Inelastic procedures, which are encouraged as an alternative to the elastic procedures, are similar to those in the ALFD guide specifications and apply only to compact sections. However, with widespread use of built-up members or plate girders, there is a definite need to extend the applicability of inelastic design methods to consider noncompact sections. This need is confirmed in Section C6.10.11.1 of the third draft of the proposed AASHTO LRFD Bridge Design Specifications:

> "[Inelastic design] methods are applicable to both compact and noncompact sections if the plastic rotation characteristics of such sections are known. These characteristics, however, have not yet been adequately established for the full range of noncompact section geometries."

The proposed specifications do not permit inelastic procedures for non-compact sections explicitly. There is a definite need for comprehensive inelastic design provisions for steel girder bridges comprising compact as well as noncompact girders for composite and noncomposite construction.

RECENT RESEARCH ON NONCOMPACT GIRDERS

The ALFD guide specifications[2] cover only compact sections that have a compression-flange slenderness not exceeding 9.2 and a web slenderness not exceeding 86 for 50-ksi steel. Therefore, additional research has been sponsored by AISI[6,15-18] to extend the guide specifications to noncompact girders, which have higher web and/or flange slenderness ratios. The main concern with such girders is the plastic-rotation behavior in negative-bending regions and the shear capacity through the full range of plastic rotation. The higher slenderness ratios change the unloading portion (after reaching the maximum moment) of the plastic-rotation curves and thereby affect the strength of the girder calculated in the maximum load check.

Because the loading portion of the negative-bending plastic-rotation curve depends primarily on residual stresses and concrete behavior rather than buckling, it is not expected to be significantly different for compact and noncompact sections. In the overload check, therefore, the same procedures are expected to apply to both compact and noncompact sections.

In the first phase of the sponsored work by AISI,[15,16] three noncomposite, noncompact girders of 50-ksi steel were tested to develop negative-bending plastic-rotation curves, especially the unloading portion of such curves. The web and compression-flange slenderness ratios for all girders were near the maximum permitted by AASHTO[1] for noncompact girders with transverse stiffeners. The three tests included a variety of conditions that affect rotation characteristics: symmetric and unsymmetric cross sections, long and short spans, relatively large and small amounts of initial out-of-flatness, and relatively large shear stresses. From the results, a lower-bound plastic-rotation curve was developed for plate girders with web and compression-flange slenderness ratios at the maximum values permitted by AASHTO for noncompact girders with transverse stiffeners.

In the next phase,[6] 50 exploratory designs were made to show whether the plastic-rotation characteristics defined by the lower-bound curve would permit economical autostress designs for noncompact plate girders. The results showed that economical autostress designs are possible with the lower-bound curve and suggested that even more economical autostress designs could be achieved by improving the plastic-rotation characteristics of negative-bending sections.

Next,[17,18] three noncomposite girders with improved rotation characteristics were tested. All had ultra-compact compression flanges and a close spacing of the first transverse stiffener on both sides of the midspan load (yield location) to provide the improved rotation characteristics. Each specimen had a different web slenderness; one had a slenderness ratio about the same as that of the three girders tested previously, and the other two specimens had lower ratios to provide an additional improvement in the rotation characteristics.

The transverse stiffener on each side of the midspan load was one-sided but was welded to the compression flange to provide more restraint for this flange. Additional transverse stiffeners were spaced so that the shear stress was less than 0.6 of the maximum shear capacity as limited by tension-field action; the permissible moment-capacity of the girder may be reduced if this stiffener spacing is exceeded.[1]

The results showed that the rotation characteristics were considerably better than those of the girders tested previously and should be suitable for practical autostress designs. The results were also used to develop a family of simplified plastic-rotation curves that depend on the web slenderness and can be used in the unified autostress method and other inelastic analysis methods.[7,8]

CURRENT RESEARCH ON INELASTIC DESIGN PROCEDURES

The author is the principal investigator on a recently initiated project, "Development and Experimental Verification of Inelastic Design Procedures for Steel Bridges Comprising Noncompact Sections,"[19] supported by NSF, AISI, AISC, and the Missouri Highway and Transportation Department. The purpose of this work is to fill in the knowledge gaps pertaining to inelastic behavior of steel girder bridges by establishing the inelastic load characteristics of noncompact steel sections typically used in the bridge industry and developing analytical and structural behavior models that accurately predict the performance of inelastically loaded structures. The main objectives of the project are:

1. confirm ALFD (autostress) procedures by performing continuous-span tests
2. develop ALFD (autostress) design provisions for noncompact, composite, and noncomposite girders
3. study progressive yielding (shakedown) effects in composite, compact, and noncompact girders
4. study nonlinear concrete effects (cracking, slippage, crushing) in composite girders and their influence on inelastic and elastic design and evaluation procedures
5. obtain data on the magnitude of permanent deflections at loads corresponding to the overload check in the LFD and ALFD procedures
6. obtain data on the shape of the plastic rotation curves for composite, noncompact, and compact sections
7. confirm that the concrete slab provides adequate support to resist local and lateral buckling of the compression flange in positive-bending regions of continuous composite girders.
8. obtain data on the length of the plastic hinges that develop at ultimate load
9. study the effect of dead load applied to the steel section in unshored composite girders

To accomplish these objectives, sequential-load (shakedown) and ultimate-load tests will be performed on three 50 percent-scale composite, continuous-span specimens: a three-span compact rolled beam, a two-span noncompact welded girder, and a two-span welded girder with ultra-compact flanges and a noncompact web.

The three types of sections chosen cover the practical range of flange and web slenderness ratios expected to be used with inelastic procedures. A three-span configuration was chosen for the compact-section test to maximize progressive-yielding (shakedown) effects, which tend to be more important in three-span bridges than in two-span bridges.

The experimental results in conjunction with other relevant information, will be used to develop comprehensive, practical ALFD (autostress) design procedures for noncompact, composite and noncomposite girders, which are not presently covered by the ALFD guide specifications.

SUMMARY

Currently, ALFD procedures are limited to bridges comprising compact sections. The provisions are more or less an improvement of the LFD procedures in that they have incorporated recent knowledge to better analyze the redistribution characteristics of continuous girders. The ALFD provisions also allow explicit yielding over the pier section and estimate the permanent set at the overload level. At the maximum load level, the ALFD procedures account for possible capacity loss in the pier sections due to significant inelastic rotation. LFD procedures use an arbitrary redistribution of pier moments and assume the pier section can inelastically rotate at full plastic moment capacity up to incremental collapse.

Advances in the state-of-the-art of bridge design clearly indicate the need for generalized inelastic steel girder bridge design procedures which include all possible girder cross-sectional shapes, both compact and noncompact. This need is confirmed by the proposed AASHTO LRFD Bridge Design Specifications'[14] encouragement to use inelastic design procedures. To meet this need, past research coupled with the current research described herein will be used to develop comprehensive, practical inelastic design procedures for noncompact, composite, and noncomposite girders, which are not presently covered by current ALFD design provisions.

REFERENCES

1. *Standard Specifications for Highway Bridges*, 14th edition, AASHTO, 1989.
2. *Guide Specifications for Alternate Load Factor Design Procedures for Steel Beam Bridges Using Braced Compact Sections*, AASHTO, 1986, and *Interim Specifications* through 1989.
3. Vincent, G. S., "Tentative Criteria for Load Factor Design of Steel Highway Bridges," *Bulletin No. 15*, AISI, New York, NY, March 1969.
4. Carskaddan, P. S., Haaijer, G., and Grubb, M. A., "Computing the Effective Plastic Moment," *Engineering Journal*, AISC, Vol. 19, No. 1, 1982.
5. Haaijer, G., Carskaddan, P. S., and Grubb, M. A., "Autostress Design of Steel Bridges," *Journal of Structural Engineering*, ASCE, Vol. 109, No. 1, January 1983.
6. Schilling, C. G., "Exploratory Autostress Girder Designs," *Report on Project 188*, AISI, July 1986.
7. Schilling, C. G., "Unified Autostress Method," *Preprint 920041*, 71st Annual Meeting of the Transportation Research Board held in Washington, DC, January 12–16, 1992.
8. Schilling, C. G., "Unified Autostress Method," *Engineering Journal*, AISC, Vol. 28, No. 4, 1991.
9. Schilling, C. G., "A Unified Autostress Method," *Report on Project 51*, AISI, November 1989.
10. Dishongh, B. E., "Residual Damage Analysis: A Method for the Inelastic Rating of Steel Girder Bridges," Ph.D. Thesis, University of Minnesota, Minneapolis, MN, January 1990.
11. Dishongh, B. E., Galambos, T. V., "Residual Deformation Analysis for Inelastic Bridge Rating," *Journal of Structural Engineering*, ASCE, Vol. 118, No. 6, June 1992.
12. Grubb, M. A., and Carskaddan, P. S., "Autostress Design of Highway Bridges, Phase 3: Moment-Rotation Requirements," *Research Laboratory Report*, United States Steel Corporation, Monroeville, PA, July 1981.
13. *Manual of Steel Construction*, 7th edition, AISC, Chicago, 1978.
14. "Development of Comprehensive Bridge Specifications and Commentary—Third Draft LRFD Specifications and Commentary," *NCHRP Project 12-33*, Modjeski and Masters, Harrisburg, PA, April 1992.
15. Schilling, C. G., "Moment-Rotation Tests of Steel Bridge Girders," *Journal of Structural Engineering*, ASCE, Vol. 114, No. 1, January 1988.
16. Schilling, C. G., "Moment-Rotation Tests of Steel Bridge Girders," *Report on Project 188*, AISI, April 1985.
17. Schilling, C. G., "Moment-Rotation Tests of Steel Girders with Ultracompact Flanges," *Proceedings of the April 9–11 1990, Conference of the Structural Stability Research Council.*
18. Schilling, C. G., and Morcos, S. S., "Moment-Rotation Tests of Steel Girders with Ultracompact Flanges," *Report on Project 188*, AISI, July 1988.
19. Barker, M. G., "Development and Experimental Verification of Inelastic Design Procedures for Steel Bridges Comprising Noncompact Sections," research under contract with the National Science Foundation, Washington, D.C., 1993–1996.
20. Barker, M. G., "The Shakedown Limit State of Slab-On-Girder Bridges," *Ph.D. Thesis*, University of Minnesota, Minneapolis, MN, June 1990.