Optimal Design of Steel Frames to Enhance Overall Stability

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Civil engineering structures are generally proportioned by the method of trial and error, wherein provisional designs are analyzed and checked for compliance with specified design criteria. If the performance of the structure does not meet the specified criteria, then the provisional design is modified. Design improvements are generally inspired by the desire to save weight in materials and are often achieved through *ad hoc* procedures suggested by the violated criteria (*e.g.* if a stress limit in a certain member is exceeded, then the size of that member is increased). A design that satisfies all specifications is adequate, but is probably not the best design available.

Computational mechanics and optimization can be valuable allies of the structural design engineer, particularly in the final design stages of complex facilities or simpler facilities governed by complex behavioral criteria. Both of these sources of complexity often push the limits of experience and intuition. The design of building structures to provide adequate overall stability is an instance in which design improvements are not always intuitively obvious, and hence might benefit from a more systematic computational approach.

In this paper we present the simplest case of the general stability-based design procedures developed in our earlier work.[3,4,8,9] Specifically we examine the problem of the design of a planar steel framed structure subjected to a single load case, using a single design variable for each cross section. The value of examining this special case is its power of illustration. The formulas that drive the design procedure, reduced to their simplest forms, yield insight into the physical interpretation of the design process. The design procedure is applied to a four-story frame, initially designed according to UBC and AISC specifications. The optimized design is then checked for conformance with these same specifications. Through this example we hope to clarify the connection between the more traditional design and the enhanced design, and to examine the fate of design constraints not formally included in the procedure.

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FORMULATION

Pezeshk and Hjelmstad[9] showed that if one maximizes the fundamental linearized buckling eigenvalue of a framed structure, holding the volume of material constant, the limit capacity of the structure increases without compromising the rate of post-limit degradation of load carrying capacity. The advantage of such an observation is that it gives one an optimization objective that involves only the elastic properties of the structure. The main structural analysis task required to drive the design optimization process is the linearized buckling eigenvalue problem.

Suppose that we have a framed structure subjected to a loading μf, where f is a fixed pattern of loads and μ is a scalar multiplier with which we can proportionally adjust the magnitude of the loading. The following eigenvalue problem governs the elastic buckling of the structure:

$$K\phi = \mu G\phi \tag{1}$$

where K represents the elastic stiffness matrix, and G is the ordinary geometric stiffness matrix that depends on the axial loads in the members of the structure when it is subjected to the forces f (i.e. $\mu = 1$). The fundamental eigenvalue μ is the value of the load factor at which the structure would buckle elastically. The fundamental eigenvector ϕ is the buckled shape that the structure would adopt at this critical load level. The linearized buckling eigenvalue problem is fictitious because it ignores the possibility that inelastic action might commence well before the elastic buckling load level. However, we do not intend to use this problem to predict carrying capacity of the structure, but rather we intend to use it to assist us in proportioning the members in a way that will enhance both the strength and the overall *inelastic* structural stability.

Let our structure be characterized by n groups of elements, the elements in each group having identical geometric and material properties. Further, let each group have a single design variable x_i that describes the cross sectional geometry of the member. For the example presented in this paper we shall take the design variable to be the moment of inertia of the cross section. Another application might suggest taking some other parameter as the design variable, such as the cross sectional area or the depth of the section.

Following Pezeshk and Hjelmstad[9] we pose the structural design problem as a standard constrained optimization problem as follows

$$\text{Maximize} \qquad \mu(x) \tag{2}$$

$$\text{Such that} \qquad w^t a(x) - W = 0 \tag{3}$$

$$\text{and} \qquad \underline{x}_i \le x_i \le \overline{x}_i \quad i = 1,\ldots,n \tag{4}$$

where W is the fixed weight of the structure, n is the total number of groups, and $a(x) = \{a_1(x_1), a_2(x_2), \ldots, a_n(x_n)\}$ is a vector containing the cross sectional areas of the n groups. A superscript t indicates the transpose of the matrix or vector that it follows. The vector w contains the specific weights of the individual groups with each component w_i being computed as the sum of the density times the length of all of the members in group i. The process of allocating the structural elements to groups tends to be subjective, and will generally lead to somewhat different optimal designs.

We bound the design variables between the limits $\underline{x}_i$ and $\overline{x}_i$ in order to reflect the importance of other design criteria that we have chosen not to treat explicitly in the optimization phase of the design. For example, floor deflection limits might tend to control beam sizes while the stability criterion may be relatively insensitive to the properties of the beams. One could recognize this hidden constraint through placing a lower bound on the beam design variables. In optimization the bounding inequality constraints on the design variables are best dealt with through an *active set* strategy. At each stage of the iteration one checks the values of the design variables against the bounds and accordingly determines whether they are binding constraints or not. One then dubs each design variable as either active or passive in accord with a predefined strategy. Usually it is not wise to alter the status of each variable at every iteration because of the tendency of the variables to cycle from active to passive and back to active repeatedly, thereby inhibiting convergence of the design. The most stable (but most laborious) active set strategy maintains a fixed active set until the iteration converges. At the converged state one evaluates the bounding constraints, removing only the most violated constraint to the passive set. The process is repeated until no change in the active and passive set needs to be made. Many other strategies are available and tend to be variations on this theme.

To make the ensuing discussion simpler we make the following notational reinterpretation of our original optimization problem. We assume that an active set exists and that W is the difference between the total fixed weight of the structure and the weight of the members in the passive set. The number of groups n is then reinterpreted to be the number of active groups and the vectors $a(x)$ and w contain only the areas and specific weights of the active groups. With this simplification the optimization problem described by Equations 2, 3, and 4 reduces to one with a single equality constraint within each design iteration.

The contribution of group i to the structural stiffness, $K(x)$, can be expressed as a summation, over the elements in that group, of axial and bending stiffnesses as follows

$$K_i(x_i) = a_i(x_i)\sum_{m\in i} k_m^A + j_i(x_i)\sum_{m\in i} k_m^B \equiv K_i^A(x_i) + K_i^B(x_i) \tag{5}$$

where k_m^A is the axial part of the member stiffness matrix of element m, a member of group i, with the cross-sectional area $a_i(x_i)$ factored out and k_m^B is the bending part of the member stiffness kernel of element m with the moment of inertia $j_i(x_i)$ factored out. The matrices K_i^A and K_i^B are the axial and bending contributions to the group stiffness matrix; we have defined them for subsequent use.

Using the method of Lagrange multipliers, one can derive the conditions for optimality of the design variables x, often referred to as the *optimality criteria*

$$Q_i(x) \equiv \frac{W}{E(x)}\left[\frac{E_i^A(x_i) + c_i(x_i)E_i^B(x_i)}{w_i a_i(x_i)}\right] = 1 \quad i = 1,\ldots n \tag{6}$$

where $E_i^A(x_i)$ and $E_i^B(x_i)$ are the normalized axial and bending strain energies possessed by group i while the structure is bent in the fundamental mode, given respectively by

$$E_i^A(x_i) = \frac{\phi^t K_i^A \phi}{\phi^t G \phi} \quad E_i^B(x_i) = \frac{\phi^t K_i^B \phi}{\phi^t G \phi} \tag{7}$$

and the dimensionless scalar $c_i(x_i)$ is an intrinsic geometric property of the cross section that depends upon area, moment of inertia, and their first derivatives (indicated by a prime after the argument) as

$$c_i(x_i) \equiv \frac{j_i'(x_i)a_i(x_i)}{j_i(x_i)a_i'(x_i)} \tag{8}$$

As a simple illustration of $c_i(x_i)$, consider a rectangular cross section with width b and depth d. Let the design variable be the depth. Since the area is $a(d) = bd$ and the moment of inertia is $j(d) = bd^3 / 12$, the parameter c takes the value $c = 3$. For an ideal wide-flange section, one could choose the depth of the section as the design variable and hold the ratio of flange area to web area, $\xi \equiv 2bt_f / dt_w$, constant. One can approximate the area as $a(d) = (1 + \xi)dt_w$ and the moment of inertia as $j(d) = (1 + 3\xi)d^3t_w / 12$. The parameter c then takes the value

$$c(d) = 3\left[\frac{(1+\xi)(1+2\xi)}{(1+3\xi)}\right] \tag{9}$$

For other sections, notably available W-sections that do not adhere to the special assumption of constancy of the ratio of flange area to web area, the parameter c will depend upon the design variable selected. However, the two simple examples should help convey the physical character of this parameter.

The total strain energy in the system is designated as $E(x)$ and can be computed as the sum of group strain energies as follows

$$E(x) = \sum_{i=1}^{n} [E_i^A(x_i) + c_i(x_i)E_i^B(x_i)] \tag{10}$$

A physical interpretation of the optimality criteria expressed by Equation 6 is that ratio of strain energy to weight should be equal for all groups when the structure is bent in the shape of the fundamental buckling mode. Furthermore, the ratio of strain energy in the ith group to the total strain energy in the structure is equal to the ratio of the weight of that group to the total weight of the structure. This interpretation is typical for this class of problems. If we let each group consist of a single member and neglect the energy due to bending (i.e. consider a truss), we recover the classical result that the strain energy per unit volume is constant over all members for the structure with the largest buckling eigenvalue.

Choice of Design Variables

We confine our attention to selection of members from wide flange steel sections. Ideally, these members might be selected from the collection of available rolled steel shapes, a set which is generally discrete. Efforts have been made to formulate algorithms to optimize on the discrete set (e.g. Liebman, Khachaturian and Chanaratna[6]). It has also been common to attempt to cover the discrete set with a set of continuous functions (e.g. Brown and Ang,[2] Walker[11]) because optimization on a continuous domain is far more efficient than optimization on a discrete domain. The best method of representing the design space is an issue that is far from being settled. For simplicity, we adopt the continuous functional dependence between cross-sectional area and moment of inertia described by Pezeshk and Hjelmstad[9] which is a slight modification of the empirical relations proposed by Walker[11] for economy wide flange steel sections.

SOLUTION PROCEDURE

The optimum design must satisfy the optimality criteria given by Equation 6 and the fixed total weight constraint. Since these equations are nonlinear, they must be solved by an iterative procedure. The algorithm used here is based on the optimality criterion approach. The optimality criterion approach, used in conjunction with a scaling procedure, will move the initial design toward a configuration that satisfies the optimality criteria and the constraints.

The algorithm steps are as follows: (1) Select an initial design, e.g. a preliminary design, based on the AISC specification, determined by traditional means. Initialize the active set to include all groups and the passive set to be empty. (2) Scale the active design variables to meet the total weight constraint, e.g using Equation 12 of the following section. (3) Perform a linearized buckling analysis of the structure for the current design to determine the fundamental buckling load and buckled shape. (4) Update the design variables, *e.g.* using the recursion relationship given in Equation 13 of the subsequent section. (5) Check optimality criteria. If

$$\sum_{i=1}^{n} |Q_i(x_i) - 1| \geq \varepsilon \tag{11}$$

where ε is a convergence tolerance appropriate to the application, then go to Step 2. (6) Evaluate the bounding constraints given by Equation 4. If no bounding constraints are violated, then stop; the optimum has been achieved. Otherwise, place the group with the most violated constraint into the passive set and go to Step 2.

Scaling Procedure

Since the weight constraint is not exactly enforced at each iteration, it is necessary to scale the design variables to keep the design feasible. To satisfy the weight constraint after each iteration, each *active* design variable is scaled by an amount ζ, where ζ is a scaling factor determined from the following equation:

$$w^t a(\zeta x) = W \tag{12}$$

Design Variable Update Procedure

One of the simplest algorithms for updating the design variables is based on the exponential recursion formula

$$x_i^{\chi+1} = x_i^{\chi}[Q_i(x_i^{\chi})]^{\gamma} \quad i = 1,\ldots,n \tag{13}$$

where χ denotes the iteration counter and the exponent γ is a parameter which determines the magnitude of the adjustment in the design variable. The central idea of the recurrence relationship is that the term in the bracket tends to unity at the optimum and any deviation from unity indicates a need for adjustment in the design variables. If the value is less than one, the associated design variable is dominating and needs to be reduced. If the value is greater than one, the associated design variable needs to be increased. The parameter $\gamma > 1$ keeps the adjustment factors closer to unity to stabilize the iteration. One typically uses $\gamma = 2$ or 3 unless the progress toward a solution is erratic, in which case a larger value is warranted. At the optimum the design variables will be unchanged by Equation 13.

EXAMPLE

In the remainder of the paper we present an application of the stability design method to a particular framed structure. The example demonstrates the performance of the design procedure and its effect on the proportions of the subject structure. The presentation includes a discussion of the initial design

Table 1. Design of Interior Columns						
Story	K_x	P_{D+L} (kips)	M_{D+L} (ft-k)	$P_{3/4(D+L+Q)}$ (kips)	$M_{3/4(D+L+Q)}$ (ft-k)	Section
1	1.42	163.2	12.9	120.0	23.2	W10×39
2	1.52	120.6	18.7	89.2	32.0	W10×39
3	1.52	78.1	13.8	57.8	24.7	W10×33
4	1.42	35.7	24.1	24.0	40.0	W10×33

Table 2. Design of Exterior Columns						
Story	K_x	P_{D+L} (kips)	M_{D+L} (ft-k)	$P_{3/4(D+L+Q)}$ (kips)	$M_{3/4(D+L+Q)}$ (ft-k)	Section
1	1.65	89.8	28.4	72.0	27.7	W10×39
2	2.02	66.4	41.2	52.5	39.0	W10×39
3	2.08	42.9	31.0	33.0	31.5	W10×33
4	1.89	19.2	52.5	15.0	44.2	W10×33

Table 3. Design of Girders			
Story	Max $M_{(D+L)}$ (ft-k)	Max $M_{3/4(D+L+Q)}$ (ft-k)	Section
1	80.60	75.0	W16×31
2	80.60	76.0	W16×31
3	80.60	70.7	W16×31
4	67.33	56.0	W16×26

method, a description of the optimization process, and the subsequent analyses of the initial and the optimized design.

The example problem chosen is a four-story frame with the topology given in Figure 1. This frame was initially designed by Pique and Rosset[7] to meet the Uniform Building Code (UBC).[10] The design loading consisted of: (1) 80 psf for dead load; (2) 40 psf for a typical floor live load; (3) and 20 psf for the roof load. Lateral loads and their distribution were computed for Zone 4 following the UBC recommendations.

Pique and Rosset[7] designed the frame using full dead load and live load to determine maximum moments in the girders. Following UBC recommendations the base shear was determined as $V = ZIKCSW = (1)(1)(0.67)(0.0679)(367) = 16.7$ kips, in which $Z = 1$ (high seismic zone), $I = 1$ (importance factor), and $K = 0.67$ (moment resisting frame). The weight of the structure, W, was 367 kips. The base shear was established from the UBC response spectrum and has the value of $CS = 0.05 / T^{1/3} = 0.0679$. The lateral loads were distributed according to the UBC and are shown in Figure 2. For the combination of dead plus live plus earthquake loads $(D + L + Q)$, the allowable increase in working stress of 33% was followed (Section 2303 of UBC[10]). Analysis of the preliminary design was done for load combinations of $(D + L)$ and $0.75(D + L + Q)$.

The American Institute of Steel Construction (AISC[1]) specification was used for the design requirements on steel sections. The yield stress was taken to be 36 ksi for all sections. The sections were assumed fully braced against lateral buckling, giving an allowable bending stress of $F_b = 24$ ksi for the girders. The columns were assumed to be braced in the out-of-plane directions; therefore, K_y for all columns was taken as one. Table 1 shows the results of the analyses of the structure and design profiles. After checking all design requirements, W10×33 sections were chosen for the top two story columns, W10×39 for the bottom two story columns, and W16×31 for the bottom three story girders and W16×26 for the top story girders.

The UBC design was used as the initial design for the optimization. The initial design was optimized by maximizing the first buckling eigenvalue of the structure under combined lateral, dead, and live loads. Elements were grouped in six categories: (1) exterior columns in the bottom two stories; (2) exterior columns in the top two stories; (3) interior col-

Table 4. Properties of Optimized Design								
	Stiffness Properties from Optimization		Stiffness Properties of the Selected Rolled Members			Yield Properties of the Selected Members		
Member	A (in.^{2})	I (in.^{4})	Section Chosen	A (in.^{2})	I (in.^{4})	N_O (k)	V_O (k)	M_O (in.-k)
Bottom two-story exterior columns	13.6	480	W14×48	14.1	485	506	93.2	2800
Top two-story exterior columns	6.8	135	W10×26	7.61	156	275	53.4	1130
Bottom two-story interior columns	13.3	471	W14×48	14.1	485	506	93.2	2800
Top two-story interior columns	11.1	282	W12×35	10.3	285	375	74.7	1860
Bottom three-story girders	10.3	400	W12×35	10.3	285	375	74.7	1860
Top story girders	5.5	105	W10×19	5.62	96.3	203	51.2	781

umns in the bottom two stories; (4) interior columns in the top two stories (5) girders in the bottom three stories; and (6) girders in the top story. The optimization converged in 13 iterations resulting in an optimized design with the properties given in Table 4, which lists the cross-sectional area (A) and the bending moment of inertia (I) obtained from the optimization, the selected wide flange members along with their cross-sectional area (A), and moment of inertia (I), axial capacity (N_O), shear capacity (V_O), and flexural capacity (M_o).

The volume of the initial design is 44,988 in.^{3} and the volume of the optimized design is 46,236 in.^{3} The difference in the volume is because final members were picked from a discrete set, rather than directly, using the results of the optimization (see Columns 2 and 5 of Table 4). The optimization algorithm increased the fundamental buckling eigenvalue of the structure from 22.1 to 28.0.

The compliance of the optimized design with AISC-ASD[1] specification was checked and is summarized in Tables 5 and 6. The member loads shown in Tables 5 and 6 were obtained from a linear, elastic finite element analysis. Each column was checked against the interaction Equations H1-1 and H1-2 of AISC-ASD[1] with the summary of the results given in Tables 5 and 6. The columns are assumed to be braced in the out-of-plane directions with K_y for all columns taken as one and K_x factors based on the alignment charts given in the AISC-ASD[1] specification. All of the columns of the optimized design performed satisfactorily. The performance check for the girders is summarized in Table 7 wherein it is shown that all the girders performed satisfactorily.

Nonlinear Static Performance

While inelastic second-order analysis is not required as part of the design procedure, it is useful for assessing the outcome of the design. Knowing that the initial design and the optimized design satisfy the AISC specification, both designs

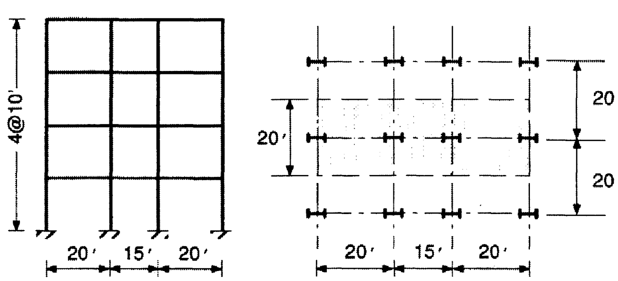

Fig. 1. Topology of Pique-Rosset frame.

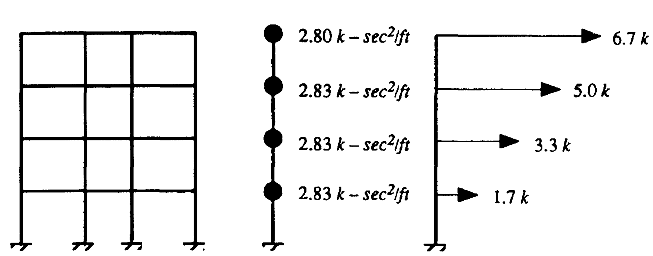

Fig. 2. Pique-Rosset frame topology, mass distribution, and code lateral force distribution.

were analyzed under a combination of statically applied lateral loads obtained from the UBC. Nonlinear analyses were performed to evaluate the overall stability of the initial and the optimized designs. There are commercially available finite element packages that can perform this level of nonlinear analysis. The results of the nonlinear analyses are given in Figure 3.

One can observe that the load-carrying capacity of the optimized structure increased substantially (30%) with an improvement in post-limit behavior. The analysis therefore reveals that the optimized design is better than the initial design in terms of overall strength and stability.

During nonlinear analysis, the hinge formation for both the initial and the optimized was monitored. The hinge pattern at collapse, referred to as the collapse mechanism, for both designs is given in Figure 4. It is interesting to note that the collapse mechanism of the optimized design is more in line with the strong-column, weak-girder design philosophy than is the initial design. The optimization procedure does not enforce a specific type of collapse mechanism, and yet the algorithm automatically picked a design with a desirable collapse mechanism.

CONCLUSIONS

We have shown in this paper that the overall stability of a planar framed structure can be enhanced by maximizing the fundamental linear elastic buckling eigenvalue of the structure, holding the total weight fixed. In its simplest form, the optimality criteria for this objective is that the combined axial and flexural strain energy of the structure, bent in the fundamental buckling mode, be distributed in accord with the weight distribution of the structure. An algorithm to achieve this objective is simple to implement and involves the solution of the linearized buckling eigenvalue problem as the main computational task. The resulting structure generally has a limit load considerably higher than the initial design, particularly if the preliminary design has been carried out by traditional means. In addition to increased strength, the structure will also generally exhibit improved post-limit behavior. These tendencies were demonstrated through an example design of a four-story frame building, wherein it was shown that by maximizing the fundamental buckling eigenvalue of the frame, the overall performance of the frame improved.

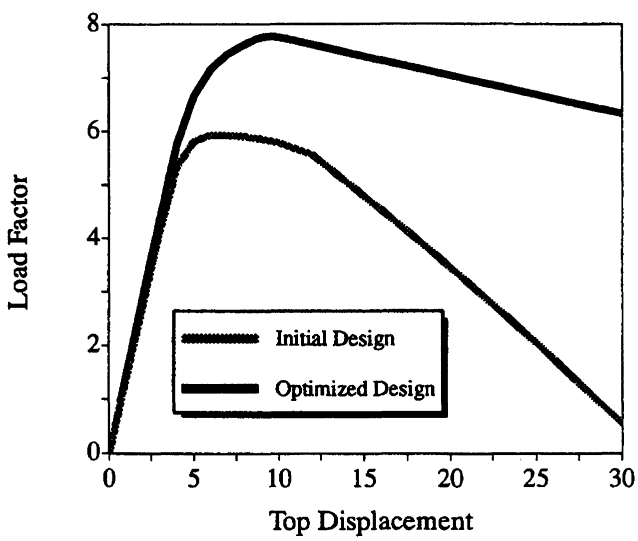

Fig. 3. Load deformation response.

ACKNOWLEDGMENTS

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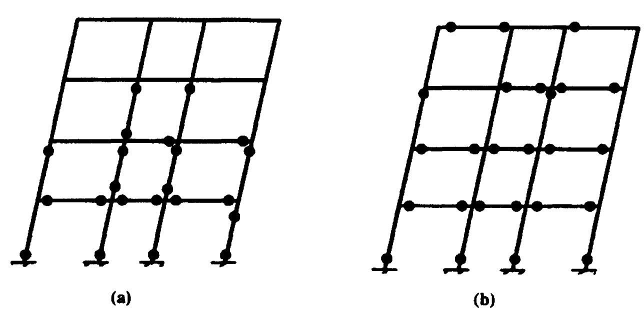

Fig. 4. Collapse mechanisms, (a) initial design; (b) optimized design.

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Table 5.
Performance Check of Interior Columns

Story	**1**	**1**	**2**	**2**	**3**	**3**	**4**	**4**
Load	$D+L$	$0.75(D+L+Q)$	$D+L$	$0.75(D+L+Q)$	$D+L$	$0.75(D+L+Q)$	$D+L$	$0.75(D+L+Q)$
Axial (kips)	163.3	128.18	121.1	92.7	78.70	60.00	35.88	27.17
Moment (k-in.)	124.50	298.20	186.48	324.00	118.92	251.64	283.68	299.52
Member	W14×48		W14×48		W12×35		W12×35	
Length (in.)	120		120		120		120	
L_c, L_u (in.)	86.4,98.4		86.4,98.4		70.8,109.2		70.8,109.2	
A (in.^{2})	14.10		14.10		10.30		10.30	
I_X (in.^{4})	485		485		285		285	
r_X (in.)	5.85		5.85		5.25		5.25	
r_y (in.)	1.91		1.91		1.54		1.54	
K_X (1)	1.53		1.67		1.42		1.32	
$K_X L_X / r_x$	31.38		34.26		32.46		30.17	
L_y / r_y	62.83		62.83		77.92		77.92	
F_a (2)	17.16		17.16		14.41		14.41	
F'_{ex}	151.78		127.29		142.15		164.13	
F_{bx}	22		22		22		22	
C_m	0.85		0.85		0.85		0.85	
f_a	11.58	8.88	8.59	6.57	7.64	5.83	3.48	2.64
f_b	1.77	4.24	2.63	4.58	2.61	5.52	6.22	6.57
f_a / F_a	0.68	0.52	0.50	0.38	0.53	0.40	0.24	0.18
$\frac{C_m f_{bx}}{(1 - f_a / F'_{ex}) F_{bx}}$	0.07	0.17	0.11	0.19	0.11	0.22	0.25	0.26
Total (3)	**0.75**	**0.69**	**0.61**	**0.57**	**0.64**	**0.63**	**0.49**	**0.44**
$f_a / 0.6F_y$	0.54	0.41	0.40	0.30	0.35	0.27	0.16	0.12
f_{bx} / F_{bx}	0.08	0.19	0.12	0.21	0.12	0.25	0.28	0.30
Total (4)	**0.62**	**0.60**	**0.52**	**0.51**	**0.47**	**0.53**	**0.44**	**0.42**

(1) Equation E2-1 or E2-2 of AISC-ASD
(2) Equation E2-1 or E2-2 of AISC-ASD
(3) Interaction Equation H1-1 or H1-3 of AISC-ASD
(4) Interaction Equation H1-2 or H1-3 of AISC-ASD

Table 6.
Performance Check of Exterior Columns

Story	1	1	2	2	3	3	4	4
Load	$D+L$	$0.75(D+L+Q)$	$D+L$	$0.75(D+L+Q)$	$D+L$	$0.75(D+L+Q)$	$D+L$	$0.75(D+L+Q)$
Axial (kips)	90	72	66	53	42	33	19	15
Moment (k-in.)	313	330	474	448	256	267	528	433
Member	W14×48		W14×48		W10×26		W10×26	
Length (in.)	120		120		120		120	
L_c, L_u (in.)	86.4,98.4		86.4,98.4		62.4,98.4		63.4,98.4	
A (in.^{2})	14.10		14.10		7.61		7.61	
I_x (in.^{4})	485		485		144		144	
r_x (in.)	5.85		5.85		4.35		4.35	
r_y (in.)	1.91		1.91		1.36		1.36	
K_x (1)	1.77		1.77		1.80		1.70	
$K_x L_x / r_x$	36.31		36.31		49.66		46.90	
L_y / r_y	62.83		62.83		88.24		88.24	
F_a (2)	17.16		17.16		14.41		14.41	
F'_{ex}	113.28		113.28		60.57		67.90	
F_{bx}	22		22		22		22	
C_m	0.85		0.85		0.85		0.85	
f_a	6.38	5.10	4.68	3.73	5.57	4.38	2.53	1.95
f_b	4.46	4.69	6.69	6.33	9.18	9.58	18.91	15.52
f_a / F_a	0.37	0.30	0.27	0.22	0.39	0.30	0.18	0.14
$\frac{C_m f_{bx}}{(1 - f_a / F'_{ex}) F_{bx}}$	0.19	0.19	0.28	0.26	0.40	0.41	0.77	0.63
Total (3)	**0.56**	**0.49**	**0.56**	**0.48**	**0.78**	**0.71**	**0.95**	**0.76**
$f_a / 0.6F_y$	0.29	0.23	0.21	0.17	0.25	0.20	0.11	0.09
f_{bx} / F_{bx}	0.21	0.22	0.31	0.29	0.43	0.44	0.88	0.72
Total (4)	**0.50**	**0.45**	**0.52**	**0.46**	**0.68**	**0.64**	**0.99**	**0.81**

(1) Equation E2-1 or E2-2 of AISC-ASD
(2) Equation E2-1 or E2-2 of AISC-ASD
(3) Interaction Equation H1-1 or H1-3 of AISC-ASD
(4) Interaction Equation H1-2 or H1-3 of AISC-ASD

Table 7.
Performance Check of Girders

Story	M_{D+L} (ft-k)	$M_{3/4(D+L+Q)}$ (ft-k)	Section	Allowable Stress (ksi)	Calculated Stress (ksi)
1	47.2	47.4	W12×35	24	12.47
2	47.4	48.5	W12×35	24	12.76
3	49.1	46.7	W12×35	24	12.91
4	36.9	35.2	W10×19	24	23.55