Composite Semi-Rigid Construction

ROBERTO T. LEON

INTRODUCTION

Over the past twenty years several hundred articles on analysis and behavior of semi-rigid steel frames have appeared in the technical literature.[1] This body of knowledge indicates that semi-rigid or partially restrained (PR) frames possess many economical, constructional, and technical advantages over rigid frames and frame-shear wall systems. However, it is safe to say that only a small percentage of the literature on this subject addresses design issues directly.[2–5] The result is that even though semi-rigid or partial-restraint (PR) connection behavior is recognized and allowed by most specifications, very few structural engineers have made explicit use of them in design.[6] In general these few designers possess what are considered advanced analysis and design tools, consisting mostly of computer programs developed in-house. This situation is beginning to change, however, because advanced analysis techniques[7,8] are beginning to be widely discussed and disseminated and public-domain and commercial programs incorporating them will soon be available.

This paper describes the development of a particular type of semi-rigid construction which the author and his co-workers have developed over the past 10 years. The paper is divided into three main parts. The first part presents some important considerations on semi-rigid behavior which apply irrespective of the connection type being used. They are included here to illustrate the differences between simple, fully rigid, and partially rigid structures and to highlight their impact on limit states. The second part of the paper deals with the design of semi-rigid structures utilizing the composite action of the floor system. Traditionally, the additional strength and stiffness provided by the floor system is ignored in the analysis of steel buildings, except to idealize it as a rigid diaphragm for lateral loads. This part of the paper intends to show why it is economic, structurally efficient, and safe to utilize the additional strength and stiffness of the floor slab in design. The last part of the paper deals with detailing issues related mainly to the seismic performance of semi-rigid composite systems.

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SEMI-RIGID BEHAVIOR AND LIMIT STATES

Traditional approaches to frame design overlook the actual stiffness response of joints and adopt ideal behavioral models, i.e., the "simple" model in simple frames and the "rigid" model in continuous construction. In reality, most connections exhibit semi-rigid or partially restrained (PR) behavior. This behavior is recognized in current specifications either explicitly by PR (LRFD) and Type 3 (ASD) framing, or implicitly by Type 2 (ASD) framing. In the latter, the connection restraint is only considered for the lateral loads.

The behavior of structural connections can be visualized for design purposes with the aid of moment-rotation (M-θ) curves (Figure 1). These curves are generally taken directly from individual tests or by using best-fit equations derived from many tests.[9,10] From a M-θ curve the connection can be characterized by its stiffness, strength, and ductility. The stiffness corresponds to the slope of the M-θ curve and changes continuously as the moment increases. For design purposes it is customary to assume a linear approximation for the service range ($\theta < \theta_{ser}$), generally in the form of a secant stiffness ($K_{conn} = M_{ser} / \theta_{ser}$).

It is important to recognize at the outset that for design purposes an exact, non-linear moment-rotation curve such as that shown in Figure 1 may not be necessary. In fact, only two important points need to be known for design. The first corresponds to the serviceability level where the stiffness, which can be approximated as a straight line between the origin and (M_{ser}, θ_{ser}), must be known for deflection calculations. The second point is the ultimate strength (M_{ult}) and

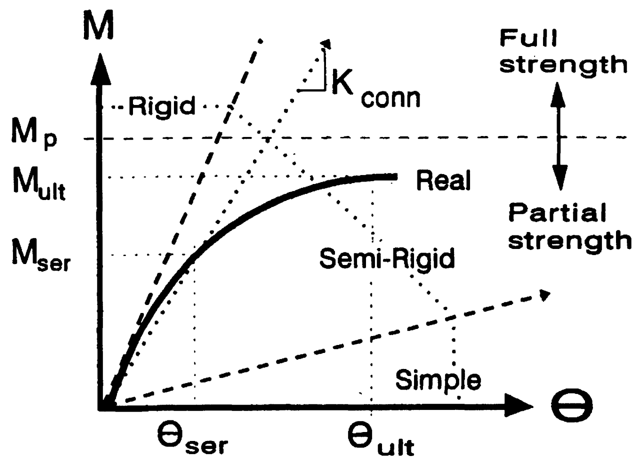

Fig. 1. M-θ curve for a typical connection.

rotation (θ_{ult}) achievable by the connection to insure that adequate plastic redistribution of stresses can occur. For the type of connections discussed in this paper (primarily semi-rigid composite Type 1, see Figure 10), research has shown that either a bilinear or trilinear approximation is sufficient for design.

To clarify the range of values for stiffness associated with the regions labelled rigid, semi-rigid, and simple in Figure 1, the case of a floor girder under distributed loads in a braced frame will be examined first. It should be recognized that the terms rigid, semi-rigid, and simple are only meaningful as long as the slope of the M-θ curve is linear or close to it, and thus are applicable primarily to the serviceability limit state. As the slope begins to change above M_{ser}, and particularly as it approaches its capacity (M_{ult}), a distinction between connection types based on stiffness becomes meaningless. At this level a more useful differentiation stems from whether the connection is full strength (i.e., capable of transferring the full M_p of the connected beam) or partial strength. For semi-rigid frames at the ultimate-strength limit state, only plastic analysis or advanced methods that incorporate the non-linear characteristics of the spring should be used. Because the rotations necessary to develop the ultimate strength of a connection may be very large, the issue of connection ductility becomes important and should be checked.

The well-known slope-deflection equations can be modified to account for the presence of linear springs (Figure 2) by incorporating the concentrated rotation of the springs at the end of the element and correcting for the fixed-end moments.

For the common case of equal semi-rigid connections at either end of the beam ($K_A = K_B = K_{conn}$), the new slope-deflection equations are:[11]

$$M_{AB} = \frac{2EI}{L}\frac{1}{1+3\alpha}\left[\left(\frac{2+3\alpha}{1+\alpha}\right)\theta_A + \left(\frac{1}{1+\alpha}\right)\theta_B - 3R\right] - \frac{M_{RAB}}{1+\alpha} \qquad (1)$$

$$M_{BA} = \frac{2EI}{L}\frac{1}{1+3\alpha}\left[\left(\frac{2+3\alpha}{1+\alpha}\right)\theta_B + \left(\frac{1}{1+\alpha}\right)\theta_A - 3R\right] - \frac{M_{RBA}}{1+\alpha} \qquad (2)$$

$$\alpha = \frac{2EI}{LK_{conn}} \qquad (3)$$

In Equations 1 and 2, θ_A and θ_B are the beam-end rotations, $R = \Delta / L$ where Δ is the relative lateral displacements of ends A and B, and the fixed-end moments are M_{RAB} and M_{RBA}.

As can be seen, Equations 1 and 2 are very similar to the usual slope-deflection ones with the introduction of the modifier containing the term α. As the K_{conn} goes to infinity, α goes to zero and Equations 1 and 2 revert to the original slope-deflection equations. The readers are referred to Reference 11 for a complete description of the implementation of both the slope-deflection and moment distribution methods for structures with semi-rigid connections. Similar treatment, utilizing the stiffness matrix approach and therefore more suitable to computer applications, is available in many textbooks and references.[12,13]

Figure 3 shows a plot of both the centerline deflection and end moments for a beam of length L and rigidity EI with semi-rigid end connections subjected to a uniformly distributed load w. The horizontal axis is logarithmic and is given by $\alpha / 2$, so a rigid connection corresponds to the left end and a simple connection to the right end of this graph. The logarithmic nature of this axis is important, because the variability of connection stiffnesses even among supposedly identical specimens is large.[14] This large variability, however, has only a limited effect on the overall behavior of a highly redundant structure. The left vertical axis, normalized by the deflection corresponding to the fixed-fixed case, $\delta = wL^4 / 384EI$, gives the centerline deflection. The right vertical

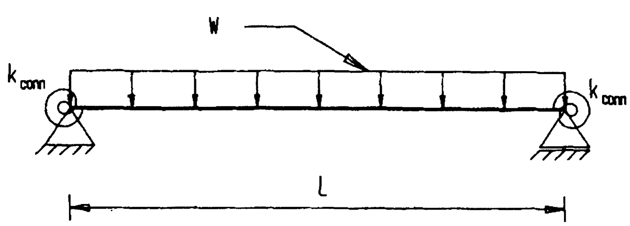

Fig. 2. Simple beam with semi-rigid (PR) connections.

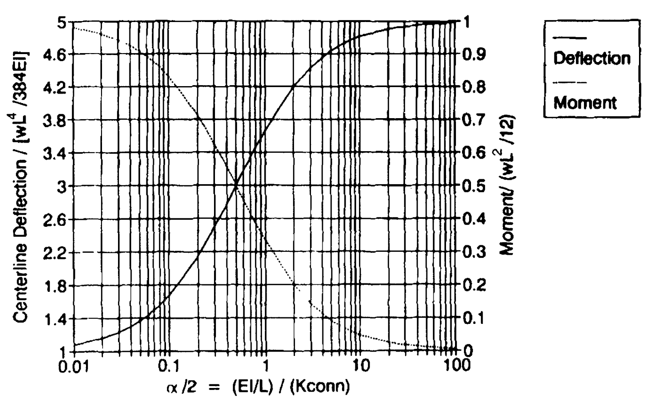

Fig. 3. Deflections and end moments as a function of α/2.

axis, normalized by the end moment for the fixed-fixed case ($M = wL^2/12$), gives the moment.

Plots such as Figure 3 allow us to make some arbitrary decisions as to what values of $\alpha/2$ correspond to the rigid, semi-rigid, and simple regions. Since it is common to assume that at least 10 percent of the negative moments can be redistributed, one may assume that connections that achieve 90 percent of fixity can be considered rigid. From Figure 3, this corresponds to an $\alpha/2$ less than 0.05. At the other extreme, one can consider connections not capable of transferring more than 20 percent of the beam moment capacity to be pinned. This corresponds to an $\alpha/2$ greater than 2.

Insofar as deflection is concerned, Figure 3 shows that the $\alpha/2$ limits discussed above correspond to 1.4 and 4.2 times the fixed end deflections. Based on deflection considerations, it could be argued that setting the limit for the rigid case at $\alpha/2 = 0.05$ is somewhat generous. In fact, this illustrates that true fixity for displacements is very hard to achieve even for the stiffest connections available today and that most connection types should be modeled as semi-rigid for deflection calculations.

The term α, defined by Equation 3, is important because it indicates that a given connection per se cannot be regarded as rigid or semi-rigid based on its stiffness (K_{conn}); it can only be classified as rigid or semi-rigid with respect to the framing members (thus the EI/L for the beam). Therefore Figure 1 should be non-dimensionalized as shown in Figure 4. The vertical axis should be related to a reference moment capacity, generally the plastic moment capacity (M_p) of the framing beam. The horizontal axis should be non-dimensionalized by dividing the rotation by a reference rotation, such as that of the simply supported beam when it reaches its plastic moment ($\phi_p = wL^3/24EI$ for the case of a distributed load, or $\phi_p = M_pL/3EI$ in general). This approach is taken by the new Eurocodes,[15,16] which define the behavior for beam and girder connections in braced and unbraced structures as shown in Figure 4. The Eurocode limits for rigid connections are very strict, particularly for unbraced frames, and in fact few if any connections in use today will meet this criteria.

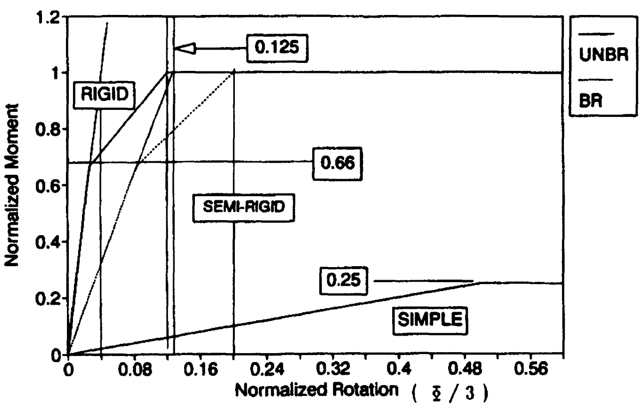

Fig. 4. Eurocode criteria for connection behavior.

Another important serviceability criterion is vibration control. Figure 5 shows the change in natural frequency (f_n) for a beam with semi-rigid connections (Figure 2). The plot shows the value of K_n as a function of $\alpha/2$, where:

$$f_n = K_n \pi \sqrt{\frac{EI}{mL^4}} \tag{4}$$

The variables are as before, with m being a distributed mass. The limits on this graph are 3.1416 for the simple support case and 4.7300 for the fixed support case.* In this case the criteria of $\alpha/2$ less than 0.05 gives a frequency very close to that of the fixed-fixed case. Figure 5 also indicates that it takes only a little restraint ($\alpha/2 < 10$) before the effect of the semi-rigid connection on the vibration characteristics is significant. This is why typical measurements on vibrations of continuous composite floors indicate that the first natural frequency is somewhat higher than calculated from a pinned-pinned analysis.

For the case of lateral drift under service loads, a simple example of a one-story, one-bay frame will be discussed. From the slope-deflection equations for a pinned-based rigid frame ($K_{base} = 0$, $K_{conn} = \infty$), this drift is:

$$\Delta = \frac{PH^2}{6E}\left(\frac{H}{I_{col}} + \frac{L}{2I_{beam}} + \frac{3E}{K_{conn}}\right) \tag{5}$$

Figure 6 shows the results of an analysis for the general case of a one-story, one-bay frame with springs both at the connections to the beam (K_{conn}) and at the base of the structure (K_{base}). A simple formula for the drift, such as Equation 5, cannot be written for this general case. The figure shows five cases of $K_{base}/(EI_{col}/H)$ (K = 0, 1, 2.5, 5, 10, and ∞) versus a varying K_{conn} ($\alpha/2 = (EI/L)_g/K_{conn}$). The calculations are for a frame with an I_g = 2,000 in.[4], L_b = 288 in., I_{col} = 500 in.[4], L_c = 144 in., P = 2.4 kips, and w = 0.08333 kip/in. The vertical axis gives the deflection as a multiplier (τ) of the case where $K_{conn} = K_{base} = \infty$. The value for the latter is 0.0253 in. For the case of $K_{base} = \infty$, as the connection stiffness decreases, the deflection reduces to that of a cantilever subjected to $P/2$ ($\tau = 3.25$). For the other extreme ($K_{base} = 0$), the deflections increase rapidly from $\tau = 4.0625$ as the stiffness of the connection is decreased since we are approaching the unstable case of a frame with pins at all connections.

Figure 6 indicates that there are infinite combinations of K_{base} and K_{conn} for a given deflection multiplier. Consider the case of a one-story, one-bay frame with the properties given for Figure 6. Most footings are not perfectly rigid or pinned,

*Some solutions for this case available in manuals such as *Formulas for Natural Frequency and Mode Shapes* by Blevins are wrong.

with the practical range probably being $1 < K_{base} < 10$. Thus for a target deflection multiplier of, say, 3, one can design the frame with a pinned base and a K_{conn} approaching infinity ($\alpha = 0$), or one can design a rigid footing with a connection having an $\alpha = 2$. This flexibility in design is what makes semi-rigid behavior both attractive and a bit disconcerting. It is attractive in that it provides the designer with an ability to dictate the way the structure will behave. It is a bit disconcerting because most designers are not familiar with semi-rigid analysis and thus do not have an "engineering feel" for how these structures behave. It is precisely the need to choose an arbitrary α that distinguishes semi-rigid design from rigid or simple frame design.

Insofar as strength is concerned, joints can be classified as full-strength when they are capable of transferring the full moment capacity of the beams, or as partial-strength connections. The moment-rotation curve shown in Figure 1 does not reach the full M_p capacity, and thus is a partial strength connection. In fact, the most advantageous designs may occur with connection capacities in the range of $M_p / 2$ to $3M_p / 4$ as will be shown with the design examples in the next section.

For the ultimate-strength limit state, semi-rigid, partial-strength connections offer many advantages in the design of composite floors. The most obvious one is that the known ultimate strength of the connection allows for a simple plastic analysis solution while providing excellent serviceability. Figure 7 shows the plastic capacity of a beam with three equally spaced concentrated loads of the same magnitude. It assumes that hinges form over the supports (negative moment) and under one of the point loads (positive moment). The figure shows the results for varying values of M_p^+ and M_p^- for both interior and exterior spans. This load case corresponds to the one to be used for Design Example 1 in the next section, but similar diagrams can be quickly derived for any loading condition.

Most importantly, the advantages of using plastic design do not disappear when we deal with unbraced multistory frames. In general, plastic design for multistory frames is complicated because many collapse mechanisms need to be checked even when the designer has a good idea of which one governs. Because the connections can be designed as fuses and carry only live and lateral loads, the plastic collapse mechanism for a semi-rigid composite frame will almost invariably be a beam sidesway one (Figure 8). The hinges will be located at the supports because the strength of the composite section under positive moment will generally be 3–5 times higher than the capacity of the connections. Thus it is not necessary to check mechanisms where hinges occur inside the beam span. The advantages of using plastic design will be more fully described in the examples in the following section.

The issue of partial strength needs some explanation from the capacity design standpoint.[17] Capacity design, which explicitly or implicitly is built into most modern seismic design codes, implies that under large lateral loads the frame will deform by concentrating the deformations and energy dissi-

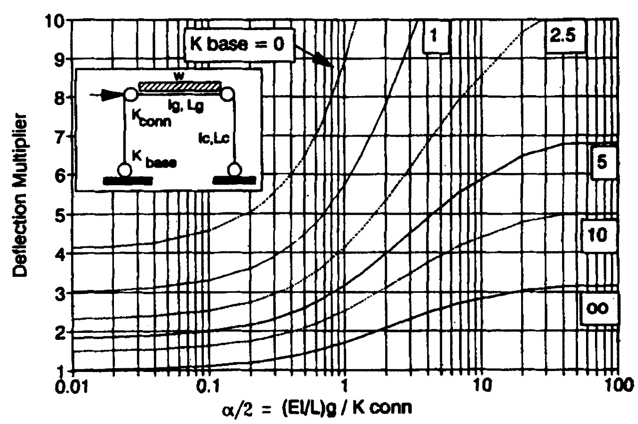

Fig. 6. One-story portal fame (PR base and connections).

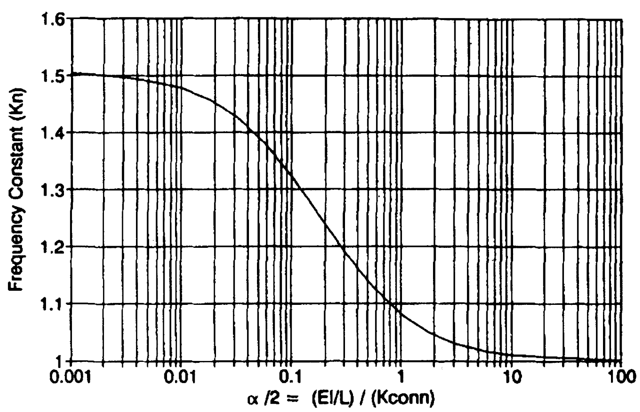

Fig. 5. First natural frequency for beams with PR connections.

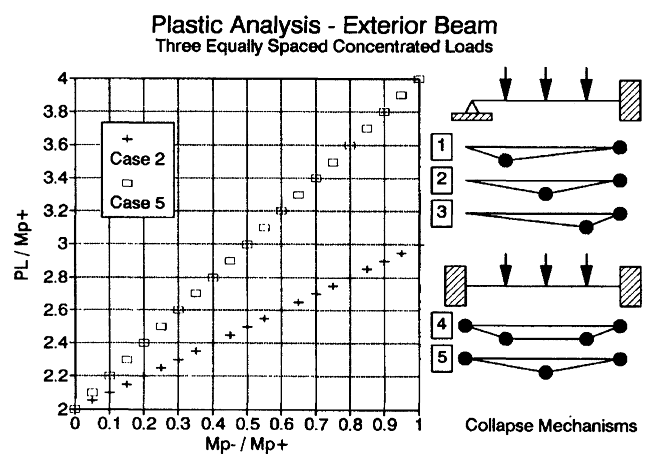

Fig. 7. Collapse load of composite beam sections.

pation in a few, well-detailed areas. All other parts of the structure must remain elastic and are detailed such that yielding will not occur there. For seismic design it is also essential to recognize that sway mechanisms associated with the yielding or buckling of the columns in unbraced frames are undesirable for capacity design. The formation of story mechanisms and the subsequent stability problems require that a strong-column–weak-beam approach be followed. The best way to prevent column problems is to limit the forces, and particularly the moments, to the columns. Semi-rigid connections can provide (1) a "fuse" that permits the deformations to be concentrated at the beam ends and (2) a well-defined strength such that unintentional overstressing of the columns is unlikely. To satisfy the assumptions of plastic design the connections must possess large ductility and exhibit slightly hardening behavior, limiting the plastic deformations and the structure's sway.

Because semi-rigid behavior is usually linked to larger sways than for rigid frames, stability is an important consideration. Figure 9 shows the multiplier of the buckling load ($\pi^2 EI_c / L_c^2$) from a first-order analysis for a portal frame with pinned bases.

The figure shows the results for a number of combinations of I_g / I_c. For the case of a very rigid girder with rigid connections, the solution approaches the theoretical value of 2.407. A sharp transition zone in behavior occurs as the stiffness $\alpha / 2$ increases above 0.2. It should be noted that at ultimate conditions exact analyses show that the difference between rigid and semi-rigid drifts for multistory frames subjected to dynamic lateral loads is small.[21] However, a linear elastic analysis with linear springs and loads applied statically will show substantial differences at the service level if the $\alpha / 2$ is large.

SEMI-RIGID COMPOSITE CONNECTIONS

Over the past 10 years the author and his co-workers at the University of Minnesota have developed the concept of semi-rigid composite (SRC) connections. These connections utilize the additional strength and stiffness provided by the floor slab which is activated by adding shear studs and slab reinforcement in the negative moment regions adjacent to the columns. Four different types of connections have been in-

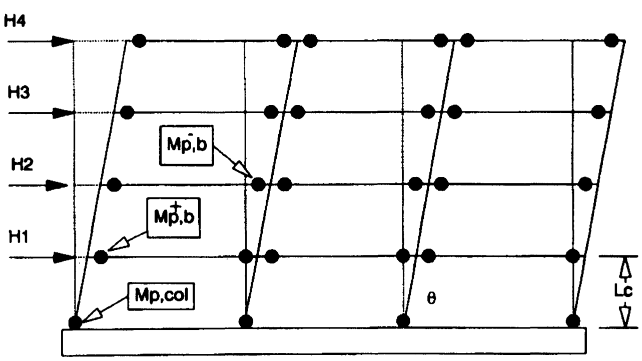

Fig. 8. Plastic mechanism for a semi-rigid composite frame.

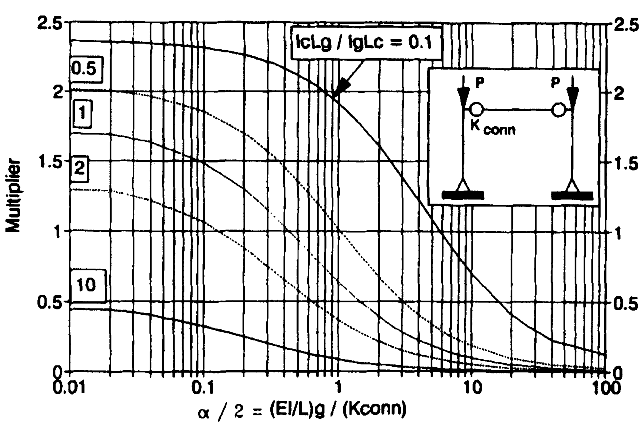

Fig. 9. Buckling loads for a portal frame.

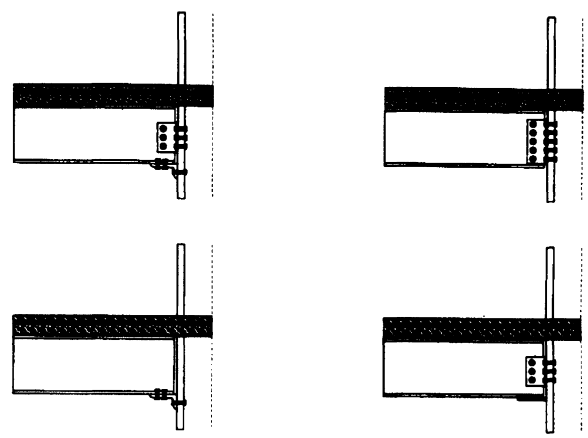

Fig. 10. Typical semi-rigid composite connections.

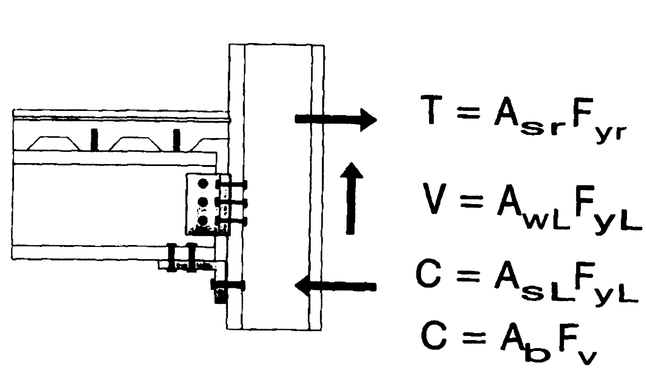

Fig. 11. Moment resistance mechanism.

vestigated (Figure 10), and their performance described elsewhere.[2,5,17]

The main advantage of semi-rigid connections is that they can be easily detailed to limit their strength. The moment resistance (Figure 11) is provided by a couple between the slab steel and the bottom connector (either an angle or a plate). Thus by varying the amount of slab steel the strength of the connection can be controlled. Because the slab steel yields primarily in tension and because it is located well above the top flange of the framing beam, it offers superior stiffness characteristics over, for example, a top angle which will yield in a combination of axial and bending modes. The use of double angles as a web connection for shear helps maintain a low but nearly constant stiffness well into the inelastic range. When semi-rigid composite connections are used, problems with local buckling of the beam flanges near the connections, shear yielding of the column panel zone, and formation of weak-column–strong-beam mechanisms can be avoided.

The equations that are of interest for frame analysis are the ones that express moment (M) as a function of the relative rotation between a connected beam and column (θ). Two of these equations have been developed, one for positive bending and one for negative bending[18] for SRC Type 1 connections. These equations supersede the original ones proposed in Leon and Ammerman.[2]

For positive bending:

$$M = C1\,(1 - e^{(-C2 \times \theta)}) + (C3 + C4)\,\theta$$

where:

$$C1 = 0.2400\,[(0.48 \times A_{wL}) + A_{sL}]\,(d + y_3)\,F_{yL}$$
$$C2 = 0.0210\,(d + y_3 / 2)$$
$$C3 = 0.0100\,(A_{wL} + A_{sL})\,(d + y_3)\,F_{yL}$$
$$C4 = 0.0065\,A_{wL}\,(d + y_3)\,F_{yL}$$

For negative bending:

$$M = C1\,(1 - e^{(-C2 \times \theta)}) + C3 \times \theta$$

where:

$$C1 = 0.1800\,[(4 \times A_{rb} \times F_{yrb}) + (0.857 \times A_{sL} \times F_{yL})](d + y_3)$$
$$C2 = 0.7750$$
$$C3 = 0.0070\,(A_{sL} + A_{wL})\,(d + y_3)\,F_{yL}$$

θ = relative rotation milliradians
A_{wL} = area of web angle, in.^{2}
A_{sL} = area of seat angle, in.^{2}
A_{rb} = effective area of slab reinforcement, in.^{2}
d = depth of steel shape, in.
y_3 = distance from top of steel shape to center of gravity of slab reinforcement, in.
F_{yL} = yield stress of angles, ksi
F_{yrb} = yield stress of rebar, ksi

To have a reasonable overall behavior, the connections that give moment resistance to the frames should have adequate strength and stiffness in both positive and negative bending. In negative bending the semi-rigid composite connections naturally have a high initial stiffness, and a well-defined yielding plateau. The attached beam rotates relative to the column by pushing the seat angle against the column and by stretching and yielding the slab reinforcement. The contribution of the web angles at service levels is minimal, but becomes important at ultimate since they provide most of the hardening in the M-θ curves as the rotations increase above 0.005 radians.

For positive bending the behavior is less favorable. The seat angle has a tendency to be opened (unfolded) by the rotating beam, and the contribution of the web angles is much more significant. It is for this reason that to have satisfactory stiffness and strength in positive bending the seat and web angles must be chosen much larger than what is required for negative bending.

The most efficient type of connection is one that behaves as a full-restraint one for service loads, yields at low drifts (0.5 percent to 0.75 percent), and has sufficient ductility and toughness to insure good hysteretic behavior up to 2 percent drift (0.02 radians assuming the mechanism of Figure 8). Semi-rigid composite, partial-strength connections meet this criteria, and thus are a most efficient structural solution. This will be illustrated with two examples.

Design Example 1

The first design example will be that of a continuous floor system in a braced frame. A three-span girder with a total length of 96 ft will be designed for dead loads of 80 psf and live loads of 100 psf. This girder supports floor beams spanning 28 ft in the perpendicular direction every 8 ft, for a total of three point loads per span. The connections to the exterior columns will be assumed as pinned since an overhang would be required to anchor the slab reinforcement. The steel will be A572 Grade 50 and a 3-in. concrete slab (f_c' = 4 ksi) on 3-in. metal deck ($Y2$ = 4.5 in) will be assumed.

The construction dead loads are assumed as 60 psf and the construction live loads are taken as 15 psf. Based on these, the most economical steel section would be a W21×44. If we assume typical current construction practice and design these girders as simply supported composite beams, for the ultimate-load condition the composite section required with a W21×44 would have a $Y2$ = 6.5 in. and a ΣQ_n = 650 kips. In order to satisfy the $Y2$ requirement, either the slab would have to be thickened to 4.5 in. increasing the dead loads significantly, or the height of the deck changed.

Probably a more practical design would be to increase the section to a W21×50 with $Y2$ = 4.5 in. and ΣQ_n = 560 kips. The service-load deflection in this case would be about $L_b / 460$ for the full live load, well within the allowable limits.

If one still wanted to utilize the W21×44 section, one could provide rigid connections at the beam ends to obtain a moment diagram similar to the one labelled "Rigid" in Figure 12.

This design would be very inefficient because the negative moments over the support are large, and the connections will have to develop the full M_p of the girder. The strongest part of the section, the positive bending capacity at midspan, would be underutilized in this case. The deflection for this case would be much lower, about $L_b / 2{,}300$, and this degree of restraint cannot be justified either by strength or serviceability considerations.

If we were to provide a semi-rigid connection such as the one shown in Figure 11, with eight #4 bars in the slab, a L8×4×½ for the seat angle, and 2L4×4×⅜ ten inches long for the web angles, one would obtain a "softer," semi-rigid connection. Figure 13 shows the entire M-θ curve for this connection based on Equations 5–8 as well as a trilinear approximation (TriL) as suggested by References 2 and 3. The latter will be used in Design Example 2.

For this type of connection, Figure 12 shows two analyses based on a secant stiffness approach for both the service (SR SER) and ultimate (SR ULT) design levels. Because the connection has a large initial stiffness up to the service level but low stiffness from there to the ultimate load level, the negative moment capacity required changes very little from service to ultimate. All the extra capacity required is in the positive moment region, where the composite action is utilized to its fullest. In fact a W21×44 with $Y2$ = 4.5 in. and ΣQ_n = 650 kips would satisfy the strength requirements in this case. The deflection for this case would be about $L / 720$, with a period of vibration about 20 percent higher than the simply-supported W21×50.

If we were to investigate this section for a plastic mechanism, Figure 7 indicates that the exterior span will control. For the given section properties (M_p = 3,000 kip-in. for the connection and M_p^+ = 8,200 kip-in for the composite beam), the M_p^- / M_p^+ = 0.36 for a PL / M_p^+ = 2.36 or an ultimate P = 50.4 kips. This compares to the required P = 45.1 kips. Thus the semi-rigid beam represents a much more efficient structural solution, and curiously, one where the elastic and plastic envelopes match very closely.

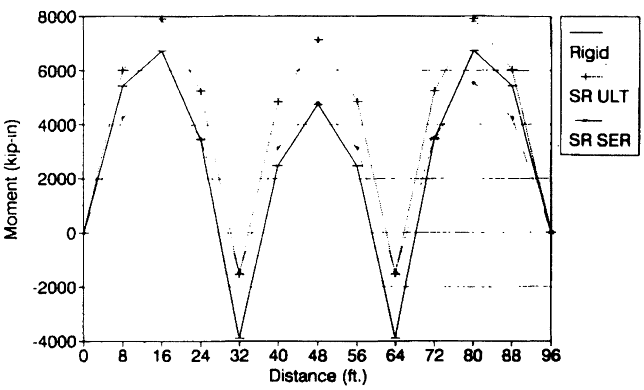

Fig. 12. Moment diagram for Design Example 1.

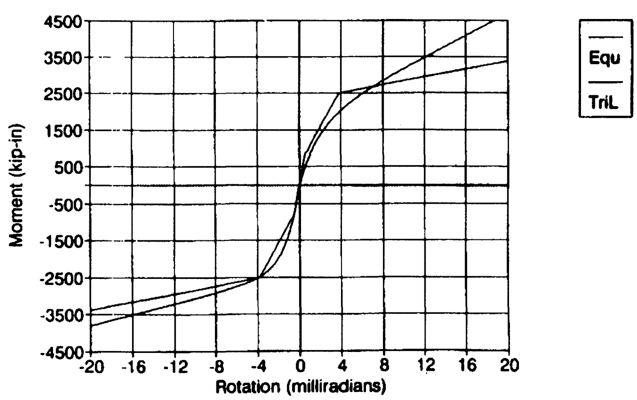

Fig. 13. M-θ for the connection used in Design Example 1.

Design Example 2

The second design example deals with an unbraced four-story, three-bay frame with Type 1 connections (Figure 14). The bays will be assumed on a 32 ft by 28 ft grid, and the design loads will be assumed similar to those of Design Example 1. For the lateral load design, wind loads corresponding to exposure B and 90 mph zones and earthquake loads for UBC Zone 2 with poor soil conditions were used. The wind loads governed the design, but the detailing is controlled by seismic concerns.

The design procedure for this frame can be summarized as follows:[20]

a. Determine the size of the steel beams based on construction loads.
b. Size composite sections, as in Design Example 1, for the factored gravity loads assuming that the connection can take 50 percent to 75 percent of the M_p of the steel beam alone.
c. From a rigid frame analysis for lateral loads, determine the preliminary size of the columns. For this analysis utilize beam properties (I_{cb}) as given by:

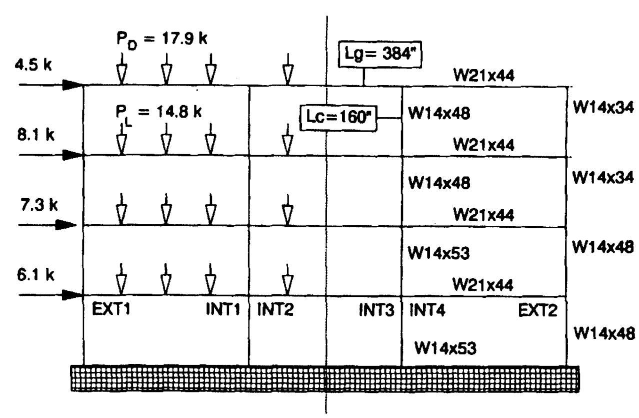

Fig. 14. Frame for Design Example 2.

$$I_{cb} = 0.6 I_{LBp} + 0.4 I_{LBn} \quad (6)$$

where I_{LBp} and I_{LBn} are the positive and negative moments of inertia of the composite beam.

d. Conduct a final analysis for the lateral-load case utilizing either a linear elastic analysis program that incorporates linear springs or one that accounts for the non-linear spring behavior. If only linear springs are used, drift under the full lateral load should be checked by assuming that the connections reach the following moment at a rotation of 0.002 radians:

$$M_{\theta_s} = 0.17\ [4A_{ry}F_{yr} + A_{sl}F_{ysl}](d + Y3) \quad (7)$$

At ultimate, check that connection moments at ultimate do not exceed:

$$M_{\theta_u} = 0.245\ [4A_rF_{yr} + A_{sl}F_{ysl}](d + Y3) \quad (8)$$

and that the connection rotation does not exceed 0.02 radians. Care should be taken to properly model the sequence in which the loads are applied. Typically most of the dead loads are applied to the simply supported beams, while the live and lateral loads are applied to the composite ones.

e. Detail the connections as described in the next section.

The right side of Figure 14 shows one of the possible structural designs that will satisfy all the requirements of the current LRFD provisions. Figure 15 shows the lateral drift versus deflection behavior for four variations of this system. The normalized values for deflection correspond to $H / 400$ and the lateral load is the full design wind load (approximately 26 kips per frame in this case) with $1.0DL$ and $0.5L$ acting simultaneously. The curves are for the case where all dead loads were applied to the beams as simply supported elements first, half of the live loads were then applied to the composite elements and connections, and only the wind load was increased ($1.0D + 0.5L + NW$, where N is the normalized factor plotted) until failure. The analysis program accounts for all non-linear geometric, material, and stability effects directly.

In the case labelled EXTOO, the exterior connections were assumed as pinned, and only the four interior connections at each floor were active. For the case labelled EXTSS, the exterior connections were made up of a top and seat angle, with M-θ curves equal to one-half of those of the interior connections. For the case labelled EXTSR, it was assumed that the slab overhangs the exterior columns by four feet, allowing the use of semi-rigid composite connections over the exterior columns. Plotted also in this graph, for comparison, is the behavior of a rigid frame, RIGID, using I_{cb} for the beams and full moment connections everywhere.

For the case of no exterior connections (EXTOO), the frame does not quite meet the $H / 400$ criteria (i.e., the normalized deflection at the normalized load of 1 is greater than 1). The evolution of the connection moments for this case is shown in Figure 16, with the connections reaching rotations of about 10 milliradians in both the positive (–2,700 kip-in) and negative (3,000 kip-in) directions at drifts of about 1.25 percent It should be noted that this drift is not comparable to

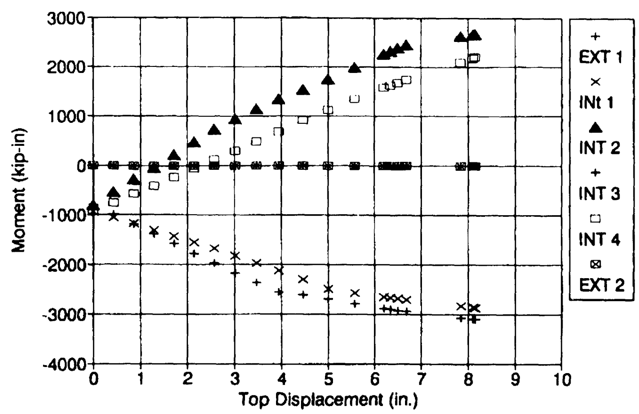

Fig. 16. Evolution of moments for Case EXTOO.

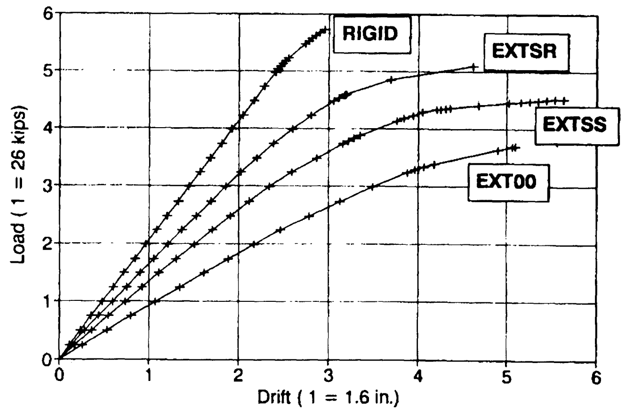

Fig. 15. Load-deflection curces for Design Example 2.

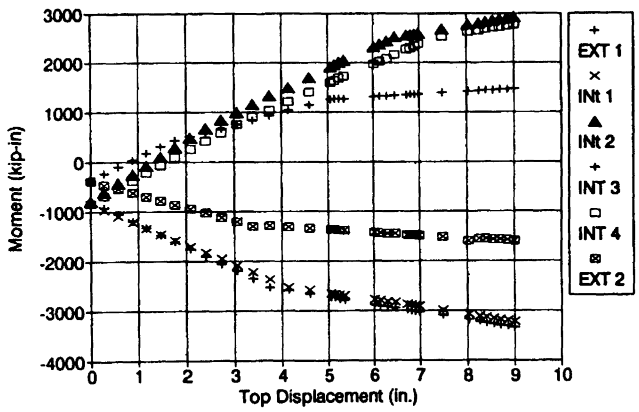

Fig. 17. Evolution of moments for Case EXTSS.

that obtained under seismic loads because it is achieved statically.

For the case of EXTSS, the evolution of moments is shown in Figure 17. In this case the exterior connections reach their ultimate capacity at drifts between 4 and 5 in., but the added strength increases the overall normalized strength to about 4.5. For the case of EXTSR (Figure 18), the evolution of moments shows clearly that all the connections were participating about equally in carrying the lateral loads, but with the exterior connections carrying slightly more than the interior ones. This shows that it is important, from the structural standpoint, to provide some type of connection at the exterior columns. In comparing the values of moment attained in the connections with the actual and idealized connection curves (Figure 13), it is clear that the connections were near 0.002 radians at the service level and near 0.010 radians at ultimate. The equations given above were derived intending to match the values at these two rotations and should not be extrapolated. In particular, the values for positive moments can substantially overestimate the connection capacity at rotations greater than 0.010.

As noted before, the analysis is much simpler if a plastic analysis approach is followed. For the failure mechanism shown in Figure 8 and using nominal values for moment capacities (no ϕ factors), the multiplier at collapse for the EXTSR case is 10.90. This implies that the frame is much stronger than needed. This value does not compare well with the exact analysis, which resulted in a load factor of 5.12. The large difference stems from the appreciable effect that the change in geometry has on the plastic capacity. Following the simplified approach by Horne and Morris,[21] the plastic capacity is given as 4.83. Figure 19 shows the trend in behavior. As the lateral deflection increases, the plastic second-order solution diverges from the rigid plastic one very quickly. The analysis reached a maximum at a normalized deflection of 4.6; the dotted line after that point represents an assumed failure path. The latter failure path becomes asymptotic to the plastic second-order analysis at normalized deflections of about 6.

Similar analysis for the other frames showed similar trends, with the softer frames showing greater strength losses when compared to the rigid plastic case. Because the deflections at ultimate are very large compared to those for a rigid case (Figure 15), the changes in geometry cannot be ignored. Simplified rules are available which allow the designer to both determine when this needs to be calculated and how to compute this reduction.

Detailing

The design of the connections used in Design Examples 1 and 2 will now be discussed, and the reader is referred to References 2 and 3 for more details on the connection design. For Design Example 1, a connection with a service load capacity around 2,000 kip-in was required, while for Design Example 2, a connection with an ultimate capacity of around 3,000 kip-in was needed.

Assuming eight #4 bars, Grade 60 bars in the slab, with their centroid around 4.5 in. above the top of the beam, the moment capacity at yield assuming the mechanism shown in Figure 11 is about 2,500 kip-in. Assuming ϕ = 0.85, the resistance will be around 2,125 kip-in for service loads. Because the bars do not yield uniformly, i.e., the bars near the column yield first, the M-θ softens somewhat below this limit but the service load level of 2,000 kip-in can be achieved with a high rotational stiffness. This connection has a stiffness around 1×10^6 kip-in/rad or an α = 0.27 for the case of Example 1.

When the ultimate state is reached the bars will be strain-hardening, and a conservative assumption for this overstrength ($1.4F_y = F_u$ was used) results in a capacity just below 3,000 kip-in. Both these initial calculations ignore the contribution of the web angle which will be significant at rotations above 0.005 radians.

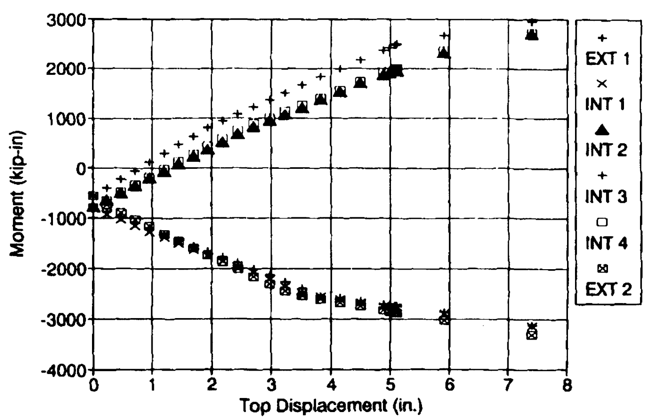

Fig. 18. Evolution of moments for Case EXTSR.

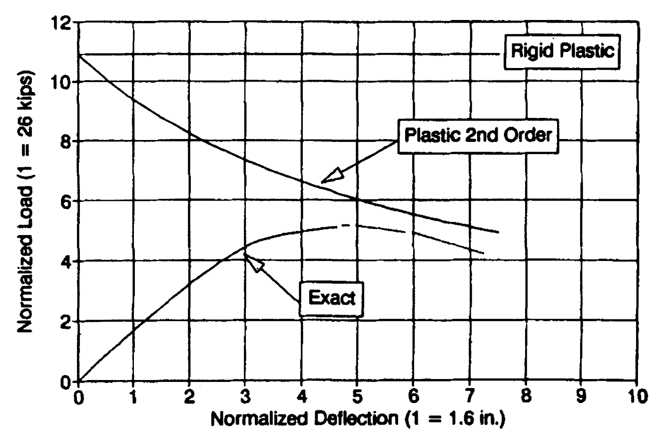

Fig. 19. Second-order effects on plastic capacity.

The bottom angle is designed so that positive and negative connection behavior is approximately symmetrical both in strength and stiffness. Because a seat angle pulls out under positive moments, it yields near the toe of the angle under a combination of axial and bending forces. For angles with thicknesses between 0.5 and 0.75 in. this occurs at about 60 percent to 75 percent of axial yield capacity of the leg connected to the beam, and a value of 66 percent is recommended for this type of design. If we want approximately the same strength in both directions, the total angle area should be the area of the rebar times (60 / 36) to account for the different materials times (1 / 0.66) to account for the early yield of the angle. This results in an angle with an area of 4.00 in.,^{2} which could be provided by a 8-in. wide angle 0.5-in. thick.

The critical design parameter for the bottom angles is the bolt capacity. Given the design values of 2,000 and 3,000 kip-in, the total shear in the bottom bolts at service and ultimate are 79 and 118 kips respectively. Assuming that there will be four bolts in the connection to the beam, four ⅞-in. A325X bolts (28.1 kips each) will be sufficient for ultimate strength. It should be recalled that the ϕ factor for these bolts is 0.65, and thus a bolt failure is unlikely even if its tabulated capacity is exceeded slightly. At service loads these bolts will provide only about 41 kips of frictional resistance, which is about half of what is needed to prevent slip under the full service loads. Because the design equations (M-θ) have the frictional capacity of ¾-in. and ⅞-in. A325 bolts built in, this is not a serious matter. Of course, all other requirements such as shear connection (⅜-in. angles, ten inches long will be sufficient), bearing strength and minimum spacing have to be satisfied.

The detailing provisions for both the slab steel and the connection elements are the key to good performance. The longitudinal slab steel should be kept with a column strip less than or equal to seven column flange widths, and should extend at least 12 inches past the point of inflection. The bar size should be kept small (less than a #5), and at least three bars on either side of the column should be used. Transverse steel must be provided at each column line, and must extend at least 12 inches into the slab strip. To reduce serviceability problems a minimum of 0.1 in.^{2} of steel per linear foot must be provided over the girders, with this reinforcement extending at least 24 inches on either side of the girder. Care must be taken that fully tightened bolts are used everywhere, that local buckling of the beam flange or web in negative moment regions does not occur, and that yielding of the column panel zone be avoided. Full shear connection should be provided since the effect of partial connection has not been investigated.

CONCLUSIONS

Semi-rigid composite connections are traditional steel frame connections in which the additional strength and stiffness provided by the floor slab has been incorporated by adding shear studs and slab reinforcement in the negative moment regions adjacent to the column. The work reported here and other work by the author (References 2, 3, 16, 17, 19, 22, 23) indicates that:

1. Significant increases in both the strength and stiffness of simple connections can be achieved by providing some continuous slab reinforcement over the column lines. The added cost is only that of a few supplementary slab bars, since no additional shear studs or bolts are required.
2. Semi-rigid composite connections provide a large amount of ductility and extra reserve capacity. A structure designed as semi-rigid composite would provide a very high degree of protection against progressive collapse failures.
3. The failure mechanisms for these connections are well understood and design procedures to prevent them have already been developed.

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