Geometric Properties of Hot-Rolled Steel Angles Including the Effects of Toe Radii and Fillet

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ABSTRACT

Hot-rolled steel angles are an important class of structural members. Currently, the values listed in the AISC handbook for the properties of these sections do not include the effect of corner radii except for torsion constant. In the interest of standardization and consistency, it is suggested that these effects be included in the derivation of all properties. The paper presents a complete list of formulas for the geometric properties of angles including the effect of corner radii, as well as the effect of corner radii on the properties. In addition, relevant aspects of metric conversion are discussed.

INTRODUCTION

Steel angles are widely used in several kinds of structures such as trusses, towers, and many bracing systems. They are some of the most popular steel shapes because of the basic simplicity and adaptability of their shape which facilitates easy fabrication and erection. They could be either single or built-up angles. Single angles include equal-leg and unequal-leg hot-rolled and cold-formed angle shapes (which are primarily 90° angles), and different types of 60° angles. Strength of steel angles, as in the case of other structural shapes can be estimated from the geometric and material properties of the members and their boundary conditions. The geometric properties depend upon the shape and size of the section. For over a century, the basic section parameters such as thicknesses and widths of various components of the section have been defined or controlled by industry standards. The remaining properties of the section such as the area of cross section, moments of inertia, torsion, and warping constants can be calculated from these basic section parameters. Methods for the derivation of some of the more complicated of these properties are discussed in several leading texts and manuals on the subject (Timoshenko, 1961).

STRUCTURAL STEEL HANDBOOKS

It has been customary for various national bodies to produce design guides, manuals, and handbooks for use by the design community. These (*e.g.*, Bethlehem Steel, 1978) and several texts (*e.g.*, Madugula and Kennedy, 1985) list various geometric properties of standard sections. However, there is in general, a lack of published information on the formulas to compute geometric properties of rolled steel sections taking into account the effect of fillet radii.

For structural shapes such as wide flange sections, the general properties can be derived if the values of section depth, flange width, flange and web thicknesses, and fillet and toe radii are known. Although the actual dimensions differ slightly within allowable tolerances, the section properties are derived based on nominal dimensions. The properties for all structural shapes except steel angles are derived by including the effects of corner radii. For steel angles however, the effects of fillet and toe radii are ignored by the industry practice in North America (AISC-ASD, 1989; AISC-LRFD, 1986; CISC, 1990; Bethlehem Steel, 1978). In most countries, however, the effect of toe and fillet radii are considered in the derivation of section properties for all sections including steel angles (BS:4848-1972; IS:808-1984). The basis for such a difference appears to be mainly tradition which might have originally been founded on certain rolling practices of the last century. The chief explanation for the current practice in North America is that the rolling practices of various mills tend to give different toe and fillet radii for steel angles, and hence cannot be standardized. However, notwithstanding this objection to standardizing the nominal values of toe and fillet radii of steel angles, recently the AISC-ASD (1989) handbook listed the torsion constant of steel angles by including the effect of fillet radius. The current practice can be summarized as:

"In calculating the theoretical weights, properties and dimensions of the rolled shapes listed in (North American) manuals, fillets and roundings have been included for all shapes except angles. The properties of these rolled shapes are based on the smallest theoretical size fillets produced; dimensions for detailing are based on the largest theoretical size fillets produced. These properties and dimensions are either exact or slightly conservative for all producers who offer them. Equal leg and unequal leg angle (L) shapes of the same nominal size available from different producers have profiles which are essentially the same, except for the size of fillet between the legs and the shape of the ends of the legs." (AISC-LRFD, 1986) As an exception, the torsion constant of

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steel angles is calculated by taking into account the fillet radius. In the interest of standardization and uniformity in design practices, it is important to include the effect of corner radii on all the other geometric properties of steel angles also.

OBJECTIVES

The present paper has the following objectives:

1. To present formulas for the computation of all the important geometric properties of 90° hot-rolled steel angles by including the effects of corner radii.
2. To find the effect of fillet and toe radii on the various geometric properties of steel angles and to arrive at a recommendation regarding their inclusion in the standard lists of properties in structural steel handbooks.
3. To examine recent statutory requirement in the U.S. regarding the use of metric properties and dimensions as applicable to steel angles.

Properties of Steel Angles

This section gives a listing of several important properties of hot-rolled steel angles. A typical cross section is shown in Figure 1. The properties have been derived by including the effects of corner properties. The warping constant is an exception to this rule and does not include the effect of corner radii in hot-rolled angles. The omission of the effects of corner properties for the calculation of warping constant of angle sections can be justified since the effect of warping constant on the column strength is very small and is limited to very low slenderness ratios (< 40).

$$A = (h + b - t)t + 0.2146(r_1^2 - 2r_2^2) \tag{1}$$

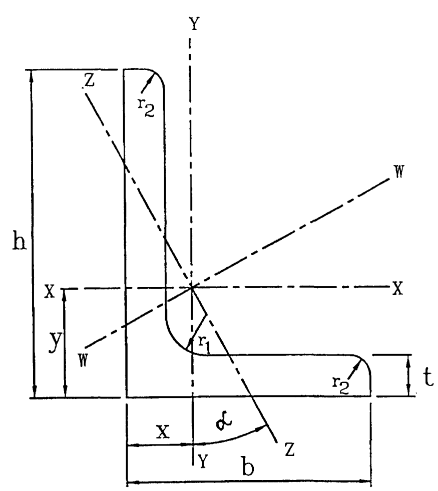

Fig. 1. Typical cross section of hot-rolled steel angle.

$$x = c_y = \frac{1}{A}[(ht + b^2 - t^2)\frac{t}{2} + 0.2146\ \{t(r_1^2 - r_2^2) - br_2^2 + 0.223(r_1^3 + 2r_2^3)\}] \tag{2}$$

$$y = c_x = \frac{1}{A}[(bt + h^2 - t^2)\frac{t}{2} + 0.2146\ \{t(r_1^2 - r_2^2) - hr_2^2 + 0.223(r_1^3 + 2r_2^3)\}] \tag{3}$$

$$e_y = b - x = b - c_y \tag{4}$$

$$e_x = h - y = h - c_x \tag{5}$$

$$I_y = \frac{ht^3}{3} + \frac{(b-t)t}{12}[(b-t)^2 + 3(b+t)^2] + 0.0076(r_1^4 - 2r_2^4) + 0.2146[r_1^2(t + 0.223r_1)^2 - r_2^2(t - 0.223r_2)^2 - r_2^2(b - 0.223r_2)^2] - Ac_y^2 \tag{6a}$$

$$S_y = \frac{I_y}{e_y} \tag{6b}$$

$$I_x = \frac{bt^3}{3} + \frac{(h-t)t}{12}[(h-t)^2 + 3(h+t)^2] + 0.0076(r_1^4 - 2r_2^4) + 0.2146[r_1^2(t + 0.223r_1)^2 - r_2^2(t - 0.223r_2)^2 - r_2^2(h - 0.223r_2)^2] - Ac_x^2 \tag{7a}$$

$$S_x = \frac{I_x}{e_x} \tag{7b}$$

$$I_{xy} = -\frac{hb(h-t)(b-t)t}{4(h+b-t)} + 0.2146r_1^2(t - c_x + 0.2234r_1)(t - c_y + 0.2234r_1) - 0.2146r_2^2(e_x - 0.2234r_2)(t - c_y - 0.2234r_2) - 0.2146r_2^2(e_y - 0.2234r_2)(t - c_x - 0.2234r_2) - 4.439 \times 10^{-3}(r_1^4 - 2r_2^4) \tag{8}$$

$$\tan 2\alpha = \frac{2I_{xy}}{(I_y - I_x)} \tag{9}$$

$$e_z = \max\,[\,|e_x\sin\alpha - x\cos\alpha|,\ |-y\sin\alpha - x\cos\alpha|,\ |e_y\cos\alpha - y\sin\alpha|\,] \tag{10}$$

$$e_w = \max\,[\,|e_x\cos\alpha + x\sin\alpha|,\ |x\sin\alpha - y\cos\alpha|,\ |-y\cos\alpha - e_y\sin\alpha|\,] \tag{11}$$

$$I_z = I_x\sin^2\alpha + I_y\cos^2\alpha + I_{xy}\sin 2\alpha \tag{12a}$$

$$S_z = \frac{I_z}{e_z} \tag{12b}$$

$$I_w = I_y\sin^2\alpha + I_x\cos^2\alpha - I_{xy}\sin 2\alpha \tag{13a}$$

$$S_w = \frac{I_w}{e_w} \tag{13b}$$

$$r_w = \sqrt{\frac{I_w}{A}} \tag{14}$$

$$r_z = \sqrt{\frac{I_z}{A}} \tag{15}$$

The distances of plastic neutral axes from the outer faces of legs (x_p and y_p) and the plastic section moduli (Z_x and Z_y) can be calculated by using the formulas listed below. If the section is assumed to be made of rectangles, the calculation of plastic section properties is relatively simple. It needs a one-stage iteration. Assume that the plastic neutral axis falls in one of the two legs and estimate its location. If the assumption is incorrect, use the formulas for the other leg. However, if the effect of corner radii are included, the plastic neutral axis can pass through the fillet or toe. In such a case, a second iteration is necessary to determine the location of plastic neutral axis within the fillet or toe. The following formulas reflect the two-stage iteration. A total of four cases (Figure 2) are considered for each axis as described below:

Y-axis

Case (a): $x_p < (t - r_2)$,

$$x_p = \frac{1}{2h}[(b - t + h)t + 0.2146(r_1^2 - 2r_2^2)] \tag{16a}$$

$$\begin{aligned} Z_y = {} & \frac{hx_p^2}{2} + \frac{h(t - x_p)^2}{2} + t(b - t)\left(\frac{b+t}{2} - x_p\right) - \\ & 0.2146r_2^2(t - x_p - 0.2234r_2) + \\ & 0.2146r_1^2(t - x_p + 0.2234r_1) - \\ & 0.2146r_2^2(b - x_p - 0.2234r_2) \end{aligned} \tag{17a}$$

Case (b): $(t - r_2) < x_p < t$,

$$c_1 = t - x_p;\ c_2 = \sqrt{c_1(2r_2 - c_1)};\ \sin\alpha_c = \frac{c_2}{r_2}$$

$$A_3 = \frac{\alpha_c r_2^2}{2} - \frac{c_2(r_2 - c_1)}{2};\ A_2 = c_1 r_2 - A_3;\ A_1 = 0.2146r_2^2 - A_2$$

$$x_{A3} = \frac{r_2^3}{12A_3}(6\alpha_c - 3\sin 2\alpha_c - 4\sin^3\alpha_c)$$

$$x_{A2} = \frac{1}{2A_2}(c_1^2 r_2 - 2A_3 x_{A3})$$

$$x_{A1} = \frac{1}{A1}(0.04794r_2^3 - A_2 x_{A2}) - c_1$$

$$x_p = \frac{1}{2h}[(h + b - t)t + 0.2146(r_1^2 - r_2^2) + A_1 - A_2] \tag{16b}$$

$$\begin{aligned} Z_y = {} & \frac{hx_p^2}{2} + \frac{h(t - x_p)^2}{2} + t(b - t)\left(\frac{b+t}{2} - x_p\right) + \\ & 0.2146r_2^2(t - x_p + 0.2234r_1) - \\ & A_1 x_{A1} - A_2(t - x_p - x_{A2}) - \\ & 0.2146r_2^2(b - x_p - 0.2234r_2) \end{aligned} \tag{17b}$$

Case (c): $t < x_p < (t + r_1)$,

$$c_1 = x_p - t;\ c_2 = \sqrt{c_1(2r_1 - c_1)};\ \sin\alpha_c = \frac{c_2}{r_1}$$

$$A_3 = \frac{\alpha_c r_1^2}{2} - \frac{c_2(r_1 - c_1)}{2};\ A_2 = c_1 r_1 - A_3;\ A_1 = 0.2146r_1^2 - A_2$$

$$x_{A3} = \frac{r_1^3}{12A_3}(6\alpha_c - 3\sin 2\alpha_c - 4\sin^3\alpha_c)$$

$$x_{A2} = \frac{1}{2A_2}(c_1^2 r_1 - 2A_3 x_{A3})$$

$$x_{A1} = \frac{1}{A_1}(0.04794r_1^3 - A_2 x_{A2}) - c_1$$

$$x_p = \frac{1}{2t}[(b - h + t)t + A_1 - A_2] \tag{16c}$$

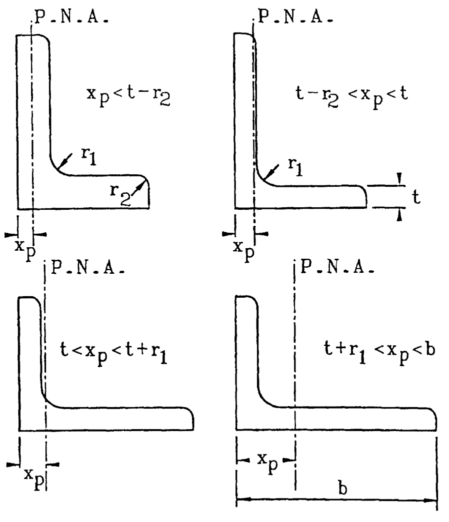

Fig. 2. Different possible locations of plastic neutral axis.

$$Z_y = ht\left(x_p - \frac{t}{2}\right) + \frac{t(b - x_p)^2}{2} + \frac{t(x_p - t)^2}{2} -$$
$$0.2146r_2^2(x_p - t + 0.2234r_2) +$$
$$A_1x_{A1} + A_2(x_p - t - x_{A2}) -$$
$$0.2146r_2^2(b - x_p - 0.2234r_2) \quad (17c)$$

Case (d): $(t + r_1) < x_p$,

$$x_p = \frac{1}{2}\left(b + t - h - \frac{0.2146r_1^2}{t}\right) \quad (16d)$$

$$Z_y = ht\left(x_p - \frac{t}{2}\right) + \frac{t(b - x_p)^2}{2} + \frac{t(x_p - t)^2}{2} -$$
$$0.2146r_2^2(x_p - t + 0.2234r_2) +$$
$$0.2146r_1^2(x_p - t - 0.2234r_1) -$$
$$0.2146r_2^2(b - x_p - 0.2234r_2) \quad (17d)$$

X-axis

Case (a): $y_p < (t - r_2)$,

$$y_p = \frac{1}{2b}[(h - t + b)t + 0.2146(r_1^2 - 2r_2^2)] \quad (18a)$$

$$Z_x = \frac{by_p^2}{2} + \frac{b(t - y_p)^2}{2} + t(h - t)\left(\frac{h + t}{2} - y_p\right) -$$
$$0.2146r_2^2(t - y_p - 0.2234r_2) +$$
$$0.2146r_1^2(t - y_p + 0.2234r_1) -$$
$$0.2146r_2^2(h - y_p - 0.2234r_2) \quad (19a)$$

Case (b): $(t - r_2) < y_p < t$,

$$c_1 = t - y_p;\ c_2 = \sqrt{c_1(2r_2 - c_1)};\ \sin\alpha_c = \frac{c_2}{r_2}$$

$$A_3 = \frac{\alpha_c r_2^2}{2} - \frac{c_2(r_2 - c_1)}{2};\ A_2 = c_1r_2 - A_3;\ A_1 = 0.2146r_2^2 - A_2$$

$$y_{A3} = \frac{r_2^3}{12A_3}(6\alpha_c - 3\sin 2\alpha_c - 4\sin^3\alpha_c)$$

$$y_{A2} = \frac{1}{2A_2}(c_1^2r_2 - 2A_3y_{A3})$$

$$y_{A1} = \frac{1}{A1}(0.04794r_2^3 - A_2y_{A2}) - c_1$$

$$y_p = \frac{1}{2b}[(h + b - t)t + 0.2146(r_1^2 - r_2^2) + A_1 - A_2] \quad (18b)$$

$$Z_x = \frac{by_p^2}{2} + \frac{b(t - y_p)^2}{2} + t(h - t)\left(\frac{h + t}{2} - y_p\right) +$$
$$0.2146r_2^2(t - y_p + 0.2234r_1) -$$
$$A_1y_{A1} - A_2(t - y_p - y_{A2}) -$$
$$0.2146r_2^2(h - y_p - 0.2234r_2) \quad (19b)$$

Case (c): $t < y_p < (t + r_1)$,

$$c_1 = y_p - t;\ c_2 = \sqrt{c_1(2r_1 - c_1)};\ \sin\alpha_c = \frac{c_2}{r_1}$$

$$A_3 = \frac{\alpha_c r_1^2}{2} - \frac{c_2(r_1 - c_1)}{2};\ A_2 = c_1r_1 - A_3;\ A_1 = 0.2146r_1^2 - A_2$$

$$y_{A3} = \frac{r_1^3}{12A_3}(6\alpha_c - 3\sin 2\alpha_c - 4\sin^3\alpha_c)$$

$$y_{A2} = \frac{1}{2A_2}(c_1^2r_1 - 2A_3y_{A3})$$

$$y_{A1} = \frac{1}{A_1}(0.04794r_1^3 - A_2y_{A2}) - c_1$$

$$y_p = \frac{1}{2t}[(h - b + t)t + A_1 - A_2] \quad (18c)$$

$$Z_x = bt\left(y_p - \frac{t}{2}\right) + \frac{t(h - y_p)^2}{2} + \frac{t(y_p - t)^2}{2} -$$
$$0.2146r_2^2(y_p - t + 0.2234r_2) +$$
$$A_1y_{A1} + A_2(y_p - t - y_{A2}) -$$
$$0.2146r_2^2(h - y_p - 0.2234r_2) \quad (19c)$$

Case (d): $(t + r_1) < y_p$,

$$y_p = \frac{1}{2}\left(h + t - b - \frac{0.2146r_1^2}{t}\right) \quad (18d)$$

$$Z_x = bt\left(y_p - \frac{t}{2}\right) + \frac{t(h - y_p)^2}{2} + \frac{t(y_p - t)^2}{2} -$$
$$0.2146r_2^2(y_p - t + 0.2234r_2) +$$
$$0.2146r_1^2(y_p - t - 0.2234r_1) -$$
$$0.2146r_2^2(h - y_p - 0.2234r_2) \quad (19d)$$

Although some of the above formulas seem similar to each other, the designer must be cautious since the input parameters for each of them are different and depend upon the case being considered. Since the above formulas for plastic section properties depend upon the location of plastic neutral axis, the designer must assume a value (not just the region as is the case with rectangular elements assumption) for the distance of neutral axis from the edge. This value is used to estimate the plastic section properties. If the computed location of neutral axis is acceptably close to the assumed value, the iteration can be terminated.

There are three different ways to calculate the torsion constant of steel angles. The most prevalent method is to assume that the angle is a centreline element and neglect the effect of the fillet and leg thickness which gives,

$$J = \frac{t^3}{3}(h + b - t) \tag{20a}$$

If the effect of the thickness is included, there will be a reduction in the value of J. For practical width thickness ratios of angle legs,

$$J = \frac{t^3}{3}(h + b - t) - 0.209t^4 \tag{20b}$$

If the effect of thickness and the fillet are included, (El-Darwish and Johnston, 1965) the value of J increases significantly.

$$J = \frac{t^3}{3}(h + b - t) + \alpha_a D_a^4 - 0.315t^4 \tag{20c}$$

where

$$\alpha_a = 0.0728 + 0.0571\frac{r_1}{t} - 0.0049\frac{r_1^2}{t^2}$$
$$D_a = 0.343r_1 + 1.172t$$

It must be noted that Equation 20c is only an average plot for the range of parameters considered in the original derivation. Practical steel angles represent the lower end of the calibration curve in the work of El-Darwish and Johnston (1965) and hence are not 'exact'.

The effect of warping constant C_w on the compressive strength of rolled sections is only of the order of two percent to five percent at slenderness ratios below 40. For higher (and more practical) slenderness ratios, the effect is close to zero. Since the warping constant has only a marginal effect on the compressive strength, the extra effort involved in accounting for the effect of corner radii in the value of C_w is not warranted.

$$C_w = \frac{t^3}{36}\left[\left(b - \frac{t}{2}\right)^3 + \left(h - \frac{t}{2}\right)^3\right] \tag{21}$$

$$\bar{r}_o^2 = (c_x - 0.5t)^2 + (c_y - 0.5t)^2 + (I_z + I_w)/A \tag{22}$$

$$H = 1 - [(c_x - 0.5t)^2 + (c_y - 0.5t)^2]/\bar{r}_o^2 \tag{23}$$

INFLUENCE OF CORNER RADII

As mentioned earlier, the North American practice with regard to the properties of steel angles is different from the world practice as adopted by the International Standards Organization (ISO 657-1: 1989 (E)). Both the North American and the international practices recognize that the corner properties, especially the toe radii are difficult to standardize. In North America, the fillet radius for various sections are often listed by individual rolling mills in their brochures and other literature. The toe radius, on the other hand is not widely available in published literature. However, the international practice (ISO 657-1: 1989 (E)) takes the theoretical nominal toe radius as being equal to half the fillet radius of the angle. The North American practice ignores the corner radii in the calculation of geometrical properties on the grounds that the nominal values are not standardized. As mentioned earlier, this rule is not followed for the calculation of the torsion constant (AISC-ASD, 1989).

For standard sections listed by AISC-ASD (1989), Table 1 shows the effect of including the corner radii on the cross-sectional properties. The fillet radii used in these calculations is the same as that used by AISC-ASD (1989) for calculating the torsion constant. The toe radius is taken as half of the fillet radius as per the usual international practice. The shape of the toe is assumed to be that shown in Figure 1. Equation 20a is used for calculating the torsion constant by neglecting the fillet effect. If the more exact Equation 20b is used instead, the change due to inclusion of fillet effect will be even more substantial. The effect of corner radii on cross-sectional properties of angles is comparable to the that on other hot-rolled shapes.

Metric Section Properties

Recently, the metric system was designated in the U.S. as the preferred measurement system for trade and commerce (Omnibus Trade and Competitiveness Act of 1988—Public Law 100-418 specified 1992 as the deadline for the conversion from U.S. customary units to metric system). In this connection, it should be noted that the properties calculated using the formulas given above for nominal sizes in Table 1 and those listed in ASTM A6M-90a and AISC (1991) differ slightly from each other. ASTM A6M-90a lists area and mass of steel angles in SI units by converting the corresponding values in U.S. customary units published in ASTM A6. These values have originally been rounded off in U.S. customary units before the conversion. Hence the ASTM section properties in SI units are subjected to rounding off at two stages, viz., once when the properties are calculated in inch-pound units and a second time after they have been converted to SI units.

The properties published by AISC in SI units differ slightly from those by ASTM. Some of the metric properties published by AISC (1991) seem to have been computed by using metric sizes. The metric sizes are derived by converting the imperial sizes to metric and rounding off the leg widths to the nearest millimeters and the thicknesses to the nearest tenth of a millimeter. The properties calculated using these metric sizes are to be rounded off again at the end. Hence both the ASTM and AISC properties seem to have been rounded off more than once at different stages of calculation.

It also appears that different conversion factors were used for different sections. For example, the mass per meter length of L8×6×⅞ in. section is 58.2 kg as per AISC (1991) which seems to have used a factor of 1.488 to convert the mass in lb/ft into kg/m units. The same section has 57.9 kg/m as per ASTM A6M (1990) using a factor closer to 1.481. But ASTM A6M uses 1.488 factor for L8×8×⅞ as does AISC. The effect of multiple rounding off and the use of different conversion factors for different sections can result in some confusion for the designers. In another example, the mass per meter length of L8×6×9⁄16 in. is 38.1 kg as per ASTM A6M, 38.2 kg as per AISC (using a size of 203×152×25.4 mm) and 38.3 kg as per the nominal size without considering the effect of corner radii (using 8×6×9⁄16 in. for all calculations and converting and rounding at the end). The mass of the same section computed by including the effect of corner radii is 38.4 kg per meter length. Such confusion exists for large as well as small sizes of sections, e.g., L2½×2 ½×⅜ (with masses 8.8 kg/m and 8.7 kg/m by AISC and ASTM respectively). While the differences are not large, it may be pointed out that consistency between popularly used specifications helps standardization.

An additional point to be noted is that the value of "*k*" used in detailing, which is taken as the sum of the thickness of leg and the largest fillet radius (for all suppliers) rounded to the nearest millimeter. The fillet radius used to calculate torsion constant for 9×9 in. angle in AISC-ASD (1989) is ⅝-in. while the radius used to calculate the value of *k* is ½-in. However, *k* should be based on the maximum fillet radius (≥ ⅝-in.) while torsion constant should be based on the minimum theoretical fillet radius.

The metric system was introduced into the Canadian structural steel design practice in mid 1970s. However, steel angle sections produced in Canada today are still in imperial sizes. On a similar basis (although the statutory requirement in the U.S. necessitates the use of SI units for all computations), it is conceivable that imperial size sections will be the only sections produced in the U.S. for several years to come. In such a case, it appears that the logical method of calculating the section properties is to use the imperial nominal sizes for all computations, convert the results to metric and round off only at the end of all calculations.

DISCUSSION

Results of Table 1 show a clear decrease in the moments of inertia of the section due to the inclusion of corner radii. The sectional area and torsion constant, on the other hand, show a clear increase. The increase in the area results in a proportional increase in the load carrying capacity of the section under tensile loads. The increase in torsion constant results in an appreciable increase in the torsional-flexural buckling capacity of the angle member. The loss in minimum moment of inertia although compensated to some extent by the increase in cross-sectional area, might result in a slight reduction in flexural buckling capacity for some sections. The loss in the minimum radius of gyration, although small, might discourage some designers from using the section properties of angles which are computed by considering the effect of fillet and toe radii. However, it should be noted that the corner radii—though difficult to standardize for all rolling mills—are present in every hot-rolled steel angle and as such should be considered directly or indirectly. The fillet and toe radii in other types of sections which have comparable effects on the corresponding section properties have been routinely included in design computations for a long time. The basic design strengths for steel angles have been formulated in a manner very similar to that for other types of sections. The formulas for tensile and compressive strengths of different sections including steel angles are also very similar to each other. Hence it is rational to think that in the interest of standardization and uniformity, the effect of toe and fillet radii should be considered for all properties of steel angles as well. The lowest and highest fillet radii for North American rolling mills are already known with reasonable certainty. The corresponding toe radii can be taken as per the international practice. The international practice, as mentioned earlier, conservatively assumes that the toe radius is half of the corresponding fillet radius.

SUMMARY

The current practice regarding the sectional properties of hot-rolled 90° steel angles is to ignore the effect of corner radii except in the case of the beneficial effect on the torsion constant. In this paper, it has been shown that the effect of corner radii increases the cross-sectional area and can be on the unconservative side for moments of inertia. The torsion constant of angles however, increases substantially by the inclusion of corner radii. The AISC ASD (1989) takes into account the effect of fillet radius for the calculation of torsion constant only. In the interest of standardization and consistency, it may be recommended that the effect of corner radii be included in the calculation of all section properties of hot-rolled steel angles. The metric section properties published by AISC and ASTM seem to have been estimated by rounding off the numbers more than once. The logical method for computing the metric section properties at present in North America appears to be, to use imperial sizes for all computations, convert the results to metric at the end, and round off as desired. The formulas for section properties including the effect of corner radii can be readily used for many purposes including software development.

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Table 1.
Effect of Fillet and Toe Radii on the Geometric Properties of Hot-Rolled Steel Angles
(% Change Due to the Inclusion of Radii)

Size and Thickness, mm	Size and Thickness, in.	r_1, in.	r_1, mm	r_2, mm	% Change Due to Inclusion of Fillet and Toe Radii			
					Area	I_{max}	I_{min}	Tors. Const.
229×102×15.9	9×4×5/8	5/8	15.9	7.9	0.5	–0.5	–1.9	8.1
229×102×14.3	9×4×9/16	5/8	15.9	7.9	0.6	–0.6	–2.0	8.6
229×102×12.7	9×4×1/2	5/8	15.9	7.9	0.7	–0.6	–2.1	9.3
203×203×28.6	8×8×1 1/8	5/8	15.9	7.9	0.3	–0.6	–0.4	5.1
203×203×25.4	8×8×1	5/8	15.9	7.9	0.3	–0.7	–0.3	5.3
203×203×22.2	8×8×7/8	5/8	15.9	7.9	0.3	–0.8	–0.2	5.6
203×203×19.0	8×8×3/4	5/8	15.9	7.9	0.4	–1.0	–0.1	5.9
203×203×15.9	8×8×5/8	5/8	15.9	7.9	0.4	–1.2	0.0	6.5
203×203×14.3	8×8×9/16	5/8	15.9	7.9	0.5	–1.3	0.1	6.9
203×203×12.7	8×8×1/2	5/8	15.9	7.9	0.5	–1.5	0.3	7.5
203×152×25.4	8×6×1	1/2	12.7	6.4	0.2	–0.4	–0.4	4.5
203×152×22.2	8×6×7/8	1/2	12.7	6.4	0.2	–0.5	–0.4	4.7
203×152×19.0	8×6×3/4	1/2	12.7	6.4	0.3	–0.6	–0.3	4.9
203×152×15.9	8×6×5/8	1/2	12.7	6.4	0.3	–0.7	–0.2	5.3
203×152×14.3	8×6×9/16	1/2	12.7	6.4	0.4	–0.8	–0.2	5.5
203×152×12.7	8×6×1/2	1/2	12.7	6.4	0.4	–0.9	–0.1	5.9
203×152×11.1	8×6×7/16	1/2	12.7	6.4	0.5	–1.1	0.0	6.4
203×102×25.4	8×4×1	1/2	12.7	6.4	0.2	–0.3	–1.0	5.4
203×102×19.0	8×4×3/4	1/2	12.7	6.4	0.3	–0.4	–1.1	5.8
203×102×14.3	8×4×9/16	1/2	12.7	6.4	0.4	–0.5	–1.2	6.5
203×102×12.7	8×4×1/2	1/2	12.7	6.4	0.5	–0.6	–1.2	7.0
178×102×19.0	7×4×3/4	1/2	12.7	6.4	0.3	–0.5	–1.1	6.4
178×102×15.9	7×4×5/8	1/2	12.7	6.4	0.4	–0.6	–1.1	6.9
178×102×12.7	7×4×1/2	1/2	12.7	6.4	0.5	–0.8	–1.1	7.6
178×102×9.5	7×4×3/8	1/2	12.7	6.4	0.7	–1.1	–1.1	9.2
152×152×25.4	6×6×1	1/2	12.7	6.4	0.2	–0.6	–0.5	5.4
152×152×22.2	6×6×7/8	1/2	12.7	6.4	0.3	–0.7	–0.4	5.6
152×152×19.0	6×6×3/4	1/2	12.7	6.4	0.3	–0.8	–0.4	5.8
152×152×15.9	6×6×5/8	1/2	12.7	6.4	0.4	–1.0	–0.2	6.2
152×152×14.3	6×6×9/16	1/2	12.7	6.4	0.4	–1.1	–0.1	6.5
152×152×12.7	6×6×1/2	1/2	12.7	6.4	0.5	–1.3	0.0	7.0
152×152×11.1	6×6×7/16	1/2	12.7	6.4	0.5	–1.5	0.1	7.5
152×152×9.5	6×6×3/8	1/2	12.7	6.4	0.6	–1.7	0.3	8.4
152×152×7.9	6×6×5/16	1/2	12.7	6.4	0.7	–2.1	0.5	9.7
152×102×22.2	6×4×7/8	1/2	12.7	6.4	0.3	–0.6	–1.1	6.9
152×102×19.0	6×4×3/4	1/2	12.7	6.4	0.4	–0.7	–1.0	7.1
152×102×15.9	6×4×5/8	1/2	12.7	6.4	0.5	–0.9	–1.0	7.6
152×102×14.3	6×4×9/16	1/2	12.7	6.4	0.5	–1.0	–1.0	8.0
152×102×12.7	6×4×1/2	1/2	12.7	6.4	0.6	–1.1	–0.9	8.5
152×102×11.1	6×4×7/16	1/2	12.7	6.4	0.6	–1.3	–0.9	9.1
152×102×9.5	6×4×3/8	1/2	12.7	6.4	0.7	–1.5	–0.8	10.2
152×102×7.9	6×4×5/16	1/2	12.7	6.4	0.9	–1.8	–0.7	11.7
152×89×12.7	6×3 1/2×1/2	1/2	12.7	6.4	0.6	–1.0	–1.4	8.9
152×89×9.5	6×3 1/2×3/8	1/2	12.7	6.4	0.8	–1.3	–1.5	10.7
152×89×7.9	6×3 1/2×5/16	1/2	12.7	6.4	0.9	–1.5	–1.5	12.4

Table 1. (cont.) Effect of Fillet and Toe Radii on the Geometric Properties of Hot-Rolled Steel Angles (% Change Due to the Inclusion of Radii)								
Size and Thickness, mm	Size and Thickness, in.	r_1, in.	r_1, mm	r_2, mm	% Change Due to Inclusion of Fillet and Toe Radii			
					Area	I_{max}	I_{min}	Tors. Const.
127×127×22.2	5×5×⅞	½	12.7	6.4	0.3	−0.8	−0.8	6.9
127×127×19.0	5×5×¾	½	12.7	6.4	0.4	−1.0	−0.7	7.1
127×127×15.9	5×5×⅝	½	12.7	6.4	0.5	−1.2	−0.6	7.6
127×127×12.7	5×5×½	½	12.7	6.4	0.6	−1.5	−0.4	8.5
127×127×11.1	5×5×7/16	½	12.7	6.4	0.6	−1.7	−0.2	9.1
127×127×9.5	5×5×⅜	½	12.7	6.4	0.7	−2.0	0.0	10.2
127×127×7.9	5×5×5/16	½	12.7	6.4	0.9	−2.5	0.2	11.7
127×89×19.0	5×3½×¾	7/16	11.1	5.6	0.4	−0.7	−1.1	7.1
127×89×15.9	5×3½×⅝	7/16	11.1	5.6	0.4	−0.8	−1.0	7.5
127×89×12.7	5×3½×½	7/16	11.1	5.6	0.5	−1.1	−1.0	8.2
127×89×11.1	5×3½×7/16	7/16	11.1	5.6	0.6	−1.2	−0.9	8.7
127×89×9.5	5×3½×⅜	7/16	11.1	5.6	0.7	−1.4	−0.8	9.6
127×89×7.9	5×3½×5/16	7/16	11.1	5.6	0.8	−1.7	−0.7	10.9
127×89×6.4	5×3½×¼	7/16	11.1	5.6	1.0	−2.2	−0.6	13.1
127×76×15.9	5×3×⅝	⅜	9.5	4.8	0.3	−0.5	−1.1	6.4
127×76×12.7	5×3×½	⅜	9.5	4.8	0.4	−0.7	−1.1	6.9
127×76×11.1	5×3×7/16	⅜	9.5	4.8	0.5	−0.8	−1.1	7.3
127×76×9.5	5×3×⅜	⅜	9.5	4.8	0.5	−0.9	−1.0	7.9
127×76×7.9	5×3×5/16	⅜	9.5	4.8	0.6	−1.1	−1.0	8.8
127×76×6.4	5×3×¼	⅜	9.5	4.8	0.8	−1.3	−1.0	10.4
102×102×19.0	4×4×¾	⅜	9.5	4.8	0.3	−0.7	−0.7	6.2
102×102×15.9	4×4×⅝	⅜	9.5	4.8	0.3	−0.8	−0.6	6.4
102×102×12.7	4×4×½	⅜	9.5	4.8	0.4	−1.0	−0.5	6.9
102×102×11.1	4×4×7/16	⅜	9.5	4.8	0.5	−1.2	−0.4	7.3
102×102×9.5	4×4×⅜	⅜	9.5	4.8	0.5	−1.4	−0.2	7.9
102×102×7.9	4×4×5/16	⅜	9.5	4.8	0.6	−1.7	0.0	8.8
102×102×6.4	4×4×¼	⅜	9.5	4.8	0.8	−2.2	0.2	10.4
102×89×12.7	4×3½×½	⅜	9.5	4.8	0.4	−1.1	−0.7	7.4
102×89×11.1	4×3½×7/16	⅜	9.5	4.8	0.5	−1.2	−0.6	7.8
102×89×9.5	4×3½×⅜	⅜	9.5	4.8	0.6	−1.4	−0.4	8.5
102×89×7.9	4×3½×5/16	⅜	9.5	4.8	0.7	−1.8	−0.2	9.4
102×89×6.4	4×3½×¼	⅜	9.5	4.8	0.8	−2.2	0.0	11.1
102×76×12.7	4×3×½	⅜	9.5	4.8	0.5	−1.0	−1.0	8.0
102×76×11.1	4×3×7/16	⅜	9.5	4.8	0.5	−1.2	−0.9	8.5
102×76×9.5	4×3×⅜	⅜	9.5	4.8	0.6	−1.4	−0.9	9.1
102×76×7.9	4×3×5/16	⅜	9.5	4.8	0.7	−1.7	−0.7	10.2
102×76×6.4	4×3×¼	⅜	9.5	4.8	0.9	−2.1	−0.6	12.0
89×89×12.7	3½×3½×½	⅜	9.5	4.8	0.5	−1.2	−0.8	8.0
89×89×11.1	3½×3½×7/16	⅜	9.5	4.8	0.5	−1.4	−0.7	8.5
89×89×9.5	3½×3½×⅜	⅜	9.5	4.8	0.6	−1.6	−0.5	9.1
89×89×7.9	3½×3½×5/16	⅜	9.5	4.8	0.7	−1.9	−0.3	10.2
89×89×6.4	3½×3½×¼	⅜	9.5	4.8	0.9	−2.5	0.0	12.0
89×76×12.7	3½×3×½	⅜	9.5	4.8	0.5	−1.2	−1.0	8.7
89×76×11.1	3½×3×7/16	⅜	9.5	4.8	0.6	−1.4	−1.0	9.2
89×76×9.5	3½×3×⅜	⅜	9.5	4.8	0.7	−1.6	−0.8	9.9
89×76×7.9	3½×3×5/16	⅜	9.5	4.8	0.8	−2.0	−0.6	11.0
89×76×6.4	3½×3×¼	⅜	9.5	4.8	1.0	−2.5	−0.4	12.9

Table 1. (cont.)
Effect of Fillet and Toe Radii on the Geometric Properties of Hot-Rolled Steel Angles
(% Change Due to the Inclusion of Radii)

Size and Thickness, mm	Size and Thickness, in.	r_1, in.	r_1, mm	r_2, mm	% Change Due to Inclusion of Fillet and Toe Radii			
					Area	I_{max}	I_{min}	Tors. Const.
89×64×12.7	3½×2½×½	5⁄16	7.9	4.0	0.4	–0.8	–1.1	7.3
89×64×11.1	3½×2½×7⁄16	5⁄16	7.9	4.0	0.4	–0.9	–1.0	7.6
89×64×9.5	3½×2½×3⁄8	5⁄16	7.9	4.0	0.5	–1.0	–1.0	8.1
89×64×7.9	3½×2½×5⁄16	5⁄16	7.9	4.0	0.6	–1.3	–0.9	8.8
89×64×6.4	3½×2½×¼	5⁄16	7.9	4.0	0.7	–1.6	–0.8	10.1
76×76×12.7	3×3×½	5⁄16	7.9	4.0	0.4	–0.9	–0.8	7.3
76×76×11.1	3×3×7⁄16	5⁄16	7.9	4.0	0.4	–1.1	–0.7	7.6
76×76×9.5	3×3×3⁄8	5⁄16	7.9	4.0	0.5	–1.3	–0.6	8.1
76×76×7.9	3×3×5⁄16	5⁄16	7.9	4.0	0.6	–1.6	–0.4	8.8
76×76×6.4	3×3×¼	5⁄16	7.9	4.0	0.7	–2.0	–0.2	10.1
76×76×4.8	3×3×3⁄16	5⁄16	7.9	4.0	1.0	–2.7	0.2	12.7
76×64×12.7	3×2½×½	5⁄16	7.9	4.0	0.4	–0.9	–1.1	8.1
76×64×11.1	3×2½×7⁄16	5⁄16	7.9	4.0	0.5	–1.1	–1.1	8.4
76×64×9.5	3×2½×3⁄8	5⁄16	7.9	4.0	0.5	–1.3	–1.0	8.9
76×64×7.9	3×2½×5⁄16	5⁄16	7.9	4.0	0.6	–1.6	–0.8	9.7
76×64×6.4	3×2½×¼	5⁄16	7.9	4.0	0.8	–2.0	–0.6	11.1
76×64×4.8	3×2½×3⁄16	5⁄16	7.9	4.0	1.1	–2.7	–0.3	13.9
76×51×12.7	3×2×½	5⁄16	7.9	4.0	0.5	–0.8	–1.6	9.1
76×51×11.1	3×2×7⁄16	5⁄16	7.9	4.0	0.5	–1.0	–1.7	9.4
76×51×9.5	3×2×3⁄8	5⁄16	7.9	4.0	0.6	–1.1	–1.7	9.9
76×51×7.9	3×2×5⁄16	5⁄16	7.9	4.0	0.7	–1.4	–1.7	10.8
76×51×6.4	3×2×¼	5⁄16	7.9	4.0	0.9	–1.7	–1.6	12.3
76×51×4.8	3×2×3⁄16	5⁄16	7.9	4.0	1.2	–2.3	–1.5	15.4
64×64×12.7	2½×2½×½	3⁄16	4.8	2.4	0.2	–0.4	–0.5	4.6
64×64×9.5	2½×2½×3⁄8	3⁄16	4.8	2.4	0.2	–0.5	–0.4	4.8
64×64×7.9	2½×2½×5⁄16	3⁄16	4.8	2.4	0.3	–0.7	–0.3	5.0
64×64×6.4	2½×2½×¼	3⁄16	4.8	2.4	0.3	–0.8	–0.1	5.4
64×64×4.8	2½×2½×3⁄16	3⁄16	4.8	2.4	0.4	–1.2	0.1	6.2
64×51×9.5	2½×2×3⁄8	3⁄16	4.8	2.4	0.2	–0.5	–0.6	5.4
64×51×7.9	2½×2×5⁄16	3⁄16	4.8	2.4	0.3	–0.7	–0.5	5.6
64×51×6.4	2½×2×¼	3⁄16	4.8	2.4	0.4	–0.8	–0.4	6.1
64×51×4.8	2½×2×3⁄16	3⁄16	4.8	2.4	0.5	–1.1	–0.2	7.0
51×51×9.5	2×2×3⁄8	3⁄16	4.8	2.4	0.3	–0.7	–0.7	6.2
51×51×7.9	2×2×5⁄16	3⁄16	4.8	2.4	0.3	–0.8	–0.6	6.4
51×51×6.4	2×2×¼	3⁄16	4.8	2.4	0.4	–1.0	–0.5	6.9
51×51×4.8	2×2×3⁄16	3⁄16	4.8	2.4	0.5	–1.4	–0.2	7.9
51×51×3.2	2×2×⅛	3⁄16	4.8	2.4	0.8	–2.2	0.2	10.4
44×44×6.4	1¾×1¾×¼	3⁄16	4.8	2.4	0.5	–1.2	–0.8	8.0
44×44×4.8	1¾×1¾×3⁄16	3⁄16	4.8	2.4	0.6	–1.6	–0.5	9.1
38×38×6.4	1½×1½×¼	3⁄16	4.8	2.4	0.5	–1.3	–1.2	9.6
38×38×4.8	1½×1½×3⁄16	3⁄16	4.8	2.4	0.7	–1.8	–1.0	10.8
32×32×6.4	1¼×1¼×¼	3⁄16	4.8	2.4	0.7	–1.6	–1.9	11.8
32×32×4.8	1¼×1¼×3⁄16	3⁄16	4.8	2.4	0.9	–2.2	–1.7	13.2
29×29×3.2	1⅛×1⅛×⅛	⅛	3.2	1.6	0.6	–1.7	–0.6	9.5
25×25×3.2	1×1×⅛	⅛	3.2	1.6	0.7	–1.8	–1.0	10.8

Note: 1 inch = 25.4 mm $r_2 = 0.5 \times r_1$