The Warping Constant for the W-Section with a Channel Cap

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It is common practice in crane runway beams to place a channel, open-side down, over the top flange of a W-section, as shown in Figure 1, to increase its lateral stability. This is done because it is not always convenient to brace the compression flange between columns.

The lateral-torsional buckling capacity of a singly symmetric section may be determined by the formulas (Footnote c on page 6-96 of the current LRFD manual[1]) without knowing the warping section constant. However, these formulas were developed from research on three-plate monosymmetric sections.[2,3,4,5,6,7] If one is to develop a similar equation for the case of the W-section with a channel cap, it is necessary to return to the basic theory of lateral-torsional buckling and that means that one must calculate the warping constant (C_{wc}) for the cross-section.

It is not a difficult matter to do this with a computer, but since the program may not be available to everyone, it would be useful to develop a simple empirical equation for preliminary design, making use of the section properties already given in the AISC steel manual.

According to Kitipornchai and Trahair,[3] Equation 1, which provides for calculating the warping section constant, "is exact for an unlipped section and approximate for a lipped section." Their "lipped" section of Figure 2 closely approximates the subject of this paper, the W-section with a channel cap.

$$I_w = a^2 I_{yc} + b^2 I_{yt} \tag{1}$$

where

$a = (1 - \rho)h \qquad b = \rho h$

$h = h_c + e \qquad \rho = I_{yc} / I_y \qquad e = (D_L^2 B_c^2 T_L) / (4\rho I_y)$

I_w = warping section constant
I_y = moment of inertia of the combined section about the axis parallel to web
I_{yc} = moment of inertia of the compression flange about the axis parallel to web
I_{yt} = moment of inertia of the tension flange about the axis parallel to web
h_c = distance from the centroid of the compression flange to the centroid of the tension flange
h = distance from the shear center of the compression flange to the centroid of the tension flange
e = distance from the shear center of the compression flange to the centroid of the compression flange
D_L = the depth of the lip
B_c = the width of the lipped flange
T_L = the thickness of the lip

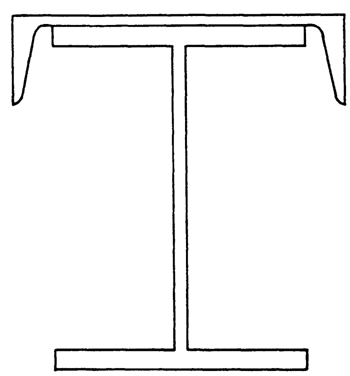

Fig. 1. Crane runway beam.

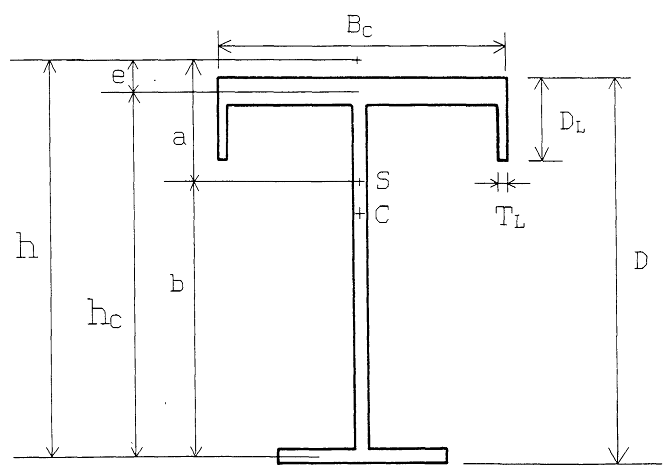

Fig. 2. W-section with lipped flange.

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Table 1.							
W	C or MC	A_c	A_w	C_{wc}	C_w	A_c / A_w	C_{wc} / C_w
36×150	15×33.9	9.96	44.20	146,275	82,200	0.2250	1.780
33×141	15×33.9	9.96	41.60	117,627	64,400	0.2390	1.827
24×84	12×20.7	6.09	24.70	24,215	12,800	0.2470	1.892
36×150	*18×42.7	12.60	44.20	160,492	82,200	0.2850	1.952
33×118	15×33.9	9.96	34.70	91,050	48,300	0.2870	1.885
30×116	15×33.9	9.96	34.20	68,520	34,900	0.2910	1.963
33×141	*18×42.7	12.60	41.60	128,892	64,400	0.3029	2.001
24×68	12×20.7	6.09	20.10	18,386	9,430	0.3030	1.950
21×68	12×20.7	6.09	20.00	13,753	6,760	0.3050	2.034
21×62	12×20.7	6.09	18.30	12,272	5,960	0.3330	2.059
30×99	15×33.9	9.96	29.10	53,937	26,800	0.3420	2.013
27×94	15×33.9	9.96	27.70	43,840	21,300	0.3600	2.058
33×118	*18×42.7	12.60	34.70	98,928	48,300	0.3630	2.048
30×116	*18×42.7	12.60	34.20	74,319	34,900	0.3680	2.129
27×84	15×33.9	9.96	24.80	37,518	17,900	0.4020	2.096
24×84	15×33.9	9.96	24.70	28,050	12,800	0.4030	2.191
18×50	12×20.7	6.09	14.70	6,746	3,040	0.4140	2.219
30×99	*18×42.7	12.60	29.10	58,101	26,800	0.4330	2.168
24×68	15×33.9	9.96	20.10	20,998	9,430	0.4960	2.227
21×68	15×33.9	9.96	20.00	15,750	6,760	0.4980	2.330
14×30	10×15.3	4.49	8.85	2,024	887	0.5070	2.282
21×62	15×33.9	9.96	18.30	13,990	5,960	0.5440	2.347
16×36	12×20.7	6.09	10.60	3,477	1,460	0.5740	2.382
12×26	10×15.3	4.49	7.65	1,459	607	0.5870	2.404
18×50	15×33.9	9.96	14.70	7,711	3,040	0.6780	2.537
14×30	12×20.7	6.09	8.85	2,255	887	0.6880	2.542
12×26	12×20.7	6.09	7.65	1,645	607	0.7960	2.710
16×36	15×33.9	9.96	10.60	4,058	1,460	0.9400	2.779

W—W-section; C—Channel; MC—Miscellaneous Channel
A_c—Area of channel, in.^{2}; A_w—Area of W-section, in.^{2}
C_{wc}—Warping section constant (in.^{6}) for the W-section with a channel cap, which is obtained by using the program as mentioned in this paper
C_w—Warping section constant (in.^{6}) for the W-section alone, which can be found in the AISC steel manual

*MC
Metric Conversion:

To convert	to	Multiply by
inches(in.)	millimeters(mm)	25.4

The authors have enhanced a computer program, which was originally written in BASIC language by Professor Theodore V. Galambos of the University of Minnesota in Minneapolis and converted to FORTRAN language by Dr. Thomas Sputo of the University of Florida, to calculate the exact values of warping section constants (C_{wc}) of the W-section with a channel cap. The user need only input the W-section and channel dimensions. The 28 combined sections shown on pages 1-98 and 1-99 of the AISC LRFD steel manual are shown in Table 1 with their warping section constants (C_{wc}) as calculated by the program described.

The ratios of C_{wc} / C_w are plotted against the ratios of A_c / A_w, as shown in Figure 3 for the 28 combined sections, where C_w, A_c, and A_w are explained in Table 1.

When Equation 1 was applied to the case of the W-section with a channel cap and the results I_w / C_w were compared with the C_{wc} / C_w values obtained from the above-mentioned program, as shown in Figure 4, it can be observed that Equation 1 gives a very conservative estimate (–7 percent to –17 percent) for this case (the W-section with a channel cap). The equation also has the added disadvantage of requiring the user to calculate certain section properties and parameters (I_y, I_{yt}, I_{yc}, ρ, h_c, h, e, a, and b) first.

The purpose of this paper is, therefore, to present a reasonably accurate method of calculating the warping section constant (C_{wc}), using a simple model which can be expressed by Equations 2, 3, or 4 and known section properties that can be found in the AISC steel manual.

The proposed method which seemed to offer the most promise was to use the ratio of the channel area (A_c) to the W-section area (A_w) as the independent variable and plot it against the ratio of the warping section constant (C_{wc}) for the combined section to the warping section constant (C_w) for the W-section alone.

Several curves (or models) were then fit to these data which are shown in Fiqure 3. These curves are represented by the following equations.

$$C_{wc} = C_w\left[1.31 + 2.55\left(\frac{A_c}{A_w}\right) - 1.31\left(\frac{A_c}{A_w}\right)^2 + 0.29\left(\frac{A_c}{A_w}\right)^3\right] \quad (2)$$

$$C_{wc} = C_w\left[1.25 + 2.55\left(\frac{A_c}{A_w}\right) - 1.31\left(\frac{A_c}{A_w}\right)^2 + 0.29\left(\frac{A_c}{A_w}\right)^3\right] \quad (3)$$

$$C_{wc} = C_w\left[1.7\sqrt{\frac{A_c}{A_w}} + 1\right] \quad (4)$$

Equations 2, 3, and 4 were superimposed on the data of Figure 3, and plotted in Figure 5. Equation 2 results in an estimated error of –2.9 percent to +2.9 percent, and Equation 3 with an estimated error of –5.9 percent to 0. Equation 4 is a simplified model with fewer terms involved, with an estimated error of –7.1 percent to +1.5 percent. Equations 2 and 3 were derived based on the multiple linear regression technique of statistics using the data of Figure 3. Equation 4 was also based on the same statistical technique with some further modifications.

Equation 2 is the best fit curve for the data of Figure 3, but does overestimate the warping section constant (C_{wc}) by as much as 2.9 percent. Equation 3 is simply Equation 2 shifted down until all the data points fall above the curve and may be up to 5.9 percent conservative. However, for those who want a formula that they can easily memorize and still get a conservative result within 7.1 percent, Equation 4 is offered.

The Equations 2, 3, and 4 proposed by the authors require no calculation of certain section properties and parameters, and are close to an exact solution. The required parameters for these equations are the channel area (A_c), the area of the W-section (A_w), and its warping section constant (C_w), all of which can be found in the AISC steel manual.

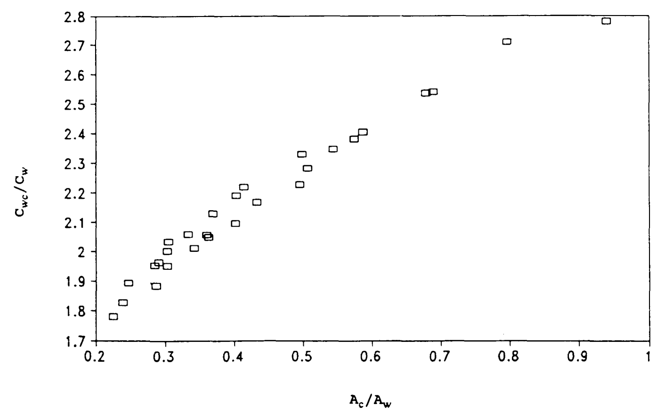

Fig. 3. Plot of C_{wc} / C_w vs. A_c / A_w.

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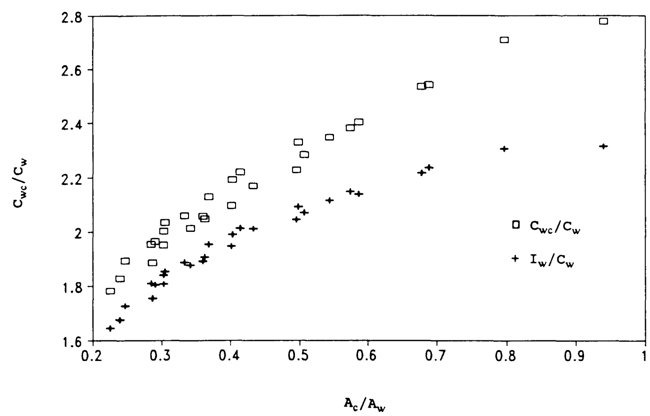

Fig. 4. I_w / C_w compared with the data of Figure 3
C_{wc} is exact for the W-section with a channel cap.
I_w is an approximation from Reference 3 for the W-section with a channel cap (see Equation 1).

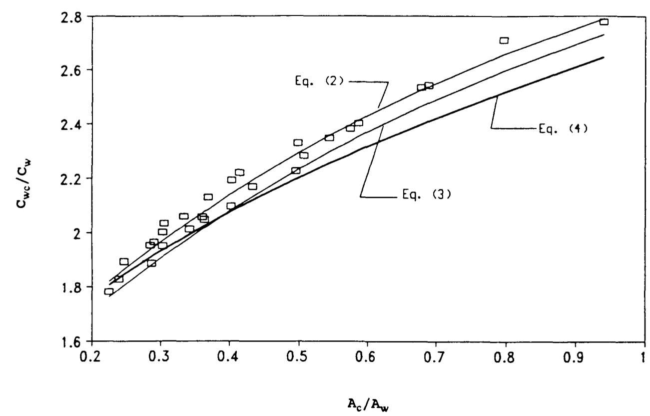

Fig. 5. Equations 2, 3, and 4 fit to the data of Figure 3.