Effective Column Length—Tier Buildings

T. R. HIGGINS

The revision of the AISC Specification adopted in November, 1961 contained a large number of new provisions reflecting extensive research sponsored by the Institute during the previous decade. Yet none of these have provoked as much discussion (and misunderstanding) as those in Sect. 1.8 which relate allowable axial stress in compression to an effective column length instead of to the actual unbraced length as in earlier editions.

Oddly enough, the underlying theory here is by no means new. Why, then, is it only now receiving recognition in American specifications? The answer lies in two recent developments both of which place greater emphasis than previously on the possibility of overall frame instability; the introduction of high strength steels and the trend towards light wall constructions.

Evolution of practices in the design of tier building frames, tempered by many decades of experience in the field, was conditioned upon the use of a single grade of structural steel, in buildings with masonry exterior walls. During this development it was generally assumed that, in the absence of a positive system of diagonal bracing, the walls themselves afforded sufficient lateral support to prevent lateral displacement of the columns at the floor levels under any conceivable vertical loading. Therefore, had the working stresses been based upon effective rather than actual unbraced length at this time, it would undoubtedly have been for the purpose of increased economy. In this event the effective length could often have been taken as *less* than the actual unbraced column length.

The recent introduction of very light exterior wall constructions now necessitates a more careful evaluation of the lateral support provided by elements of the building other than the bending stiffness of the steel frame itself. Unless this is adequate, the column working stresses should be based upon an effective length *greater* than the actual unbraced length.

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The importance of lateral support, in preventing overall frame instability at a level of loading lower than that which the same columns can carry when lateral support is present, is increased with the introduction of steels having a yield point substantially higher than that in use for many years. Within the range of slenderness ratios generally encountered in tier building design, the ratio of the axial stress permitted in a laterally supported column, to that which may be permitted in the same column if lateral support is not provided, increases with increased yield point.

It should not be construed from the foregoing remarks that, from now on, all tier buildings, or even very many, must be designed on the basis of overall instability unless provided with an extensive bracing system. As Dr. Galambos will show elsewhere in this *Journal*, the amount of support provided by even a moderate amount of wall area or diagonal bracing is much greater than generally realized.

This article will be concerned with questions which have been asked about provisions of Sect. 1.8 of the Specification, with particular reference to those cases where a multi-story frame must depend entirely upon its own stiffness to inhibit premature sidesway.

SIDESWAY OR LATERAL STABILITY

Before going any further, the term "sidesway", as used in that section, and its opposite number "lateral stability", need to be clearly defined. From questions which have come up it appears that some engineers have construed "sidesway" as synonomous with the lateral displacement D, shown in Fig. 1a, generally referred to as "drift", caused by an applied horizontal force H. Such is not the intent. If the horizontal force H were kept constant but the loads P were increased by some large overload factor the limit would be reached as the frame lurched and fell sideways as a unit. It is this eventuality which is referred to in Sect. 1.8 as "sidesway". Incipient sidesway, as the critical value of P is reached, is indicated in Fig. 1b as an additional differential displacement δD.

To avoid confusion, it is well to think of lateral instability only in terms of *vertical* loading. Thus, it is the application of a critical amount of *vertical* loading on an unbraced frame, concurrently subjected to such bending stress as would be caused by the given horizontal design loads multiplied by the required factor of safety, which produces the instability—not the *side* loading.

In Formula (7a),

$$\frac{f_a}{F_a} + \frac{C_m f_b}{\left(1 - \frac{f_a}{F_e'}\right) F_b} \leq 1.0$$

the effect of drift upon a column's capacity is taken care of by the factor $C_m/(1 - f_a/F_e')$, by which the computed bending stress f_b, caused by the side loading, is amplified.

To the extent that the value of the computed axial stress f_a in this interaction formula is decreased, because of larger and larger concurrent bending stresses, the relative importance of F_a decreases. Hence, the importance of K-values greater than unity also decreases. Therefore, where the design is based upon large side loading, the effective column length may generally be taken as the actual unbraced length with little resulting error.

Likewise, if because of some drift limitation lateral deflection due to side loading, rather than working stress, governs the design, values of K greater than unity generally lose their significance.

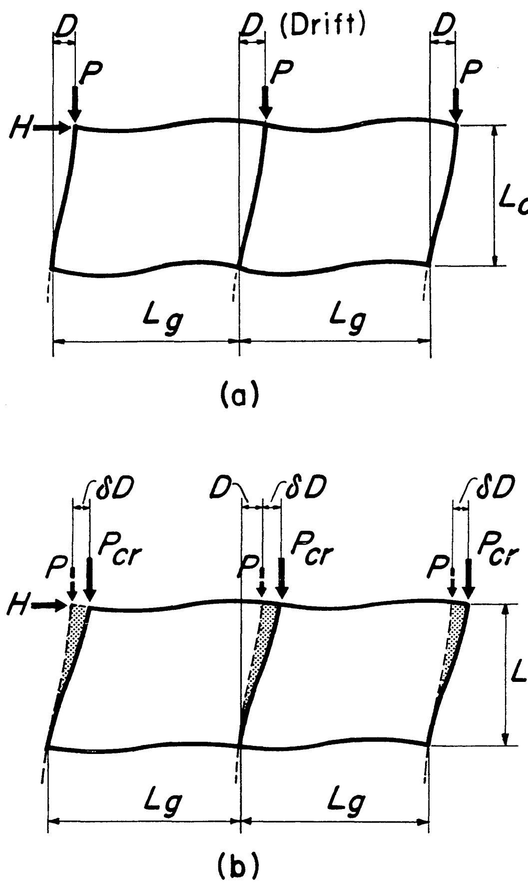

Figure 1

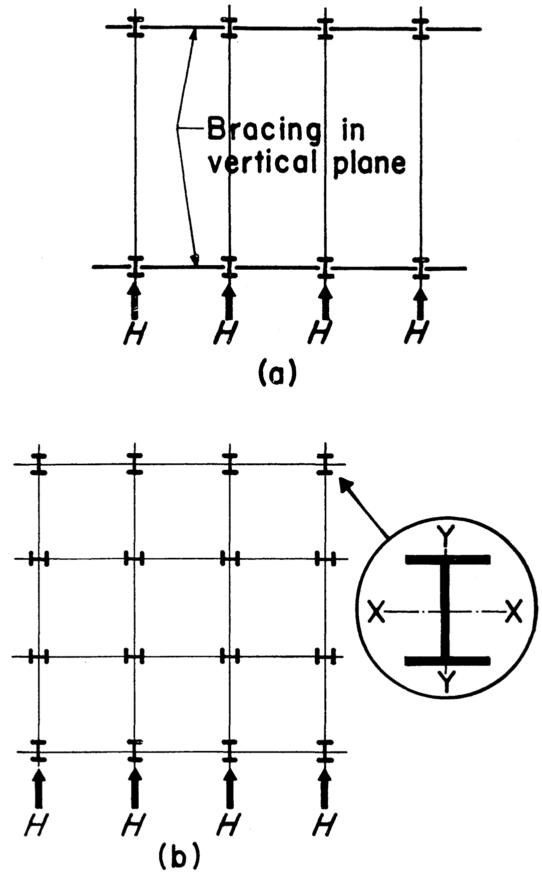

Figure 2

EFFECTIVE SLENDERNESS RATIOS

The column formulas given in Sect. 1.5.1 relate allowable stress to the largest *effective* slenderness ratio. Therefore, when columns are designed in accordance with Sect. 1.8.3, two cases must be considered:

a. The case where all the columns are oriented to resist side-load bending about their strong axis while lateral support is provided with respect to their weak axis, as in Fig. 2a.
b. The case where, regardless of orientations, all of the columns resist side-load bending in proportion to their contribution to the bending stiffness of the entire frame, as in Fig. 2b.

In the first case the value of F_a will usually be controlled by the ratio l/r_y because this ratio, even though based directly upon the actual unbraced length, is likely to exceed Kl/r_x, despite K being greater than 1.0, even by a substantial amount. In the second case either Kl/r_y (a ratio at least somewhat larger than l/r_y), or Kl/r_x, will govern the value of F_a, depending upon the stiffness of the beams normal to the Y- and X-axes and the relative value of r_y and r_x.

Suitable values of K, dependent upon the column-to-beam stiffness ratios G_A and G_B at the upper and lower ends of an unbraced length of column, can be obtained from the nomograph given in the Commentary accompanying the AISC Specification. However, designers familiar with the analysis of high-rise frameworks soon acquire a "feel" for the problem which enables them to make reasonable estimates of these values with, at most, a spot check now and then. Fortunately, when taken in conjunction with the bending stresses resulting from side loading, rather large inaccuracies in the assumed value of K have only a relatively minor effect upon the selection of column sizes as governed by Formula (7a). Furthermore, excess in stiffness resulting from the over-design of one column is available to counteract insufficiency of stiffness in another.

The value of K in a typical tier building braced only by its beam stiffness seldom exceeds 2.0, although higher values are possible. The question is sometimes raised, "How can it exceed 2.0 when this is the value associated with a free-standing flag pole?" The answer, of course, lies in the fact that l is the story height, not the full building height. Because of the bending restraint contributed by the floor beams, the effective unbraced length Kl is always less than the building height. However, if this restraint is relatively weak, it may be more than twice the story height.

Taking L_c as one of several equal story heights and AB as a column length in one of these stories, let us study the effect of beam stiffness upon the value of K as the frame is deflected laterally a differential distance δD under critical vertical loading. To simplify the investigation we will assume a constant column section so that $\Sigma I_c/L_c$ will be the same at A and B.

First consider the impossible situation shown in Fig. 3, where the beams *fully* restrain both ends of AB. Above and below a mid-story height point of contraflexure the elastic line of the column would be one-half that of an equivalent pin-end column having both ends on the same vertical line, that is to say, a hypothetical column whose ends have not been displaced laterally with respect to one another. For this case K equals 1.0.

Next let us look at the case shown in Fig. 4, where the bending stiffness of the beams, $\Sigma I_g/L_g$, at braced points A and B, is just equal to that of the columns for which they provide the restraint. Here, the elastic curve, above

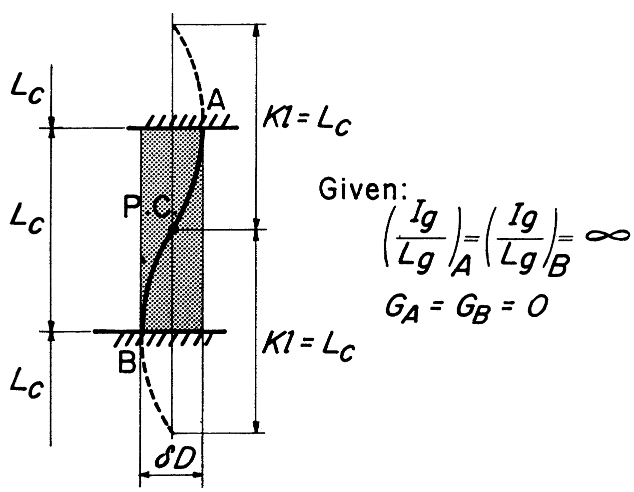

Figure 3

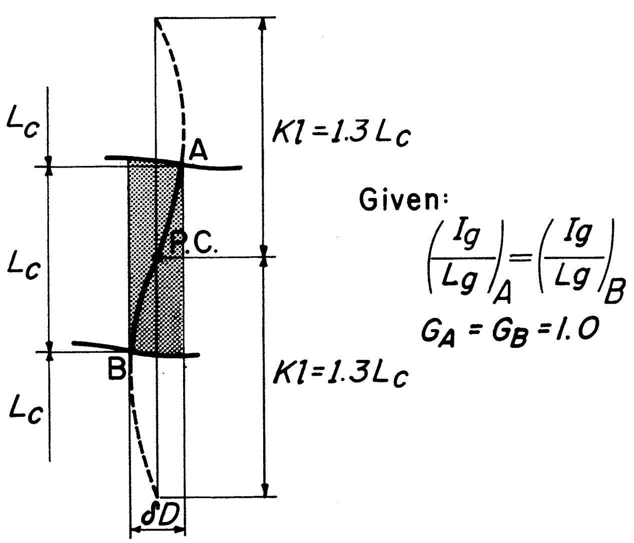

Figure 4

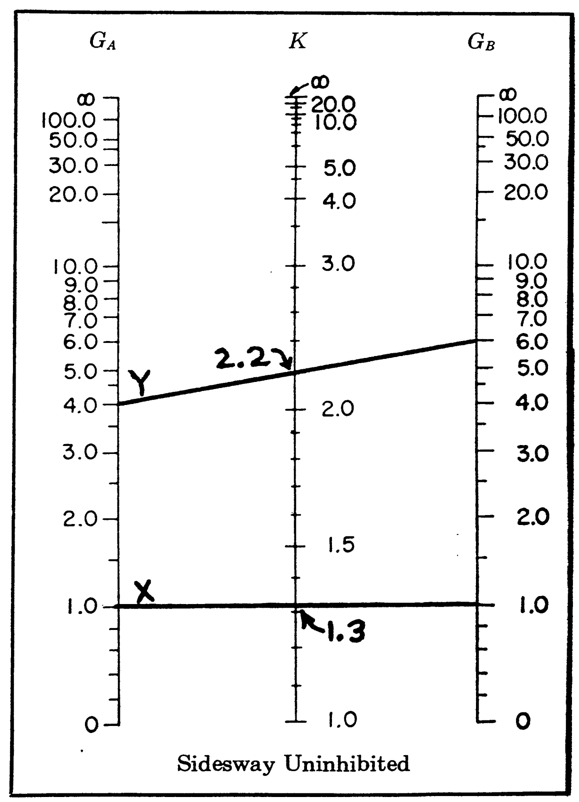

Figure 5

and below the point of contraflexure, is less than one-half that of an equivalent pin-end column whose ends would lie on a common vertical line. $G_A = G_B = 1.0$ (See Fig. 5, line X). From the nomograph it can be determined that $K \cong 1.3$.

Fig. 6 depicts the case where the stiffness of the beams is one-fourth that of the columns at A and one-sixth that of the columns at B. The elastic column curves, above and below the point of contraflexure, constitute even smaller portions of equivalent pin-end columns whose ends would lie on a common plumb line, than in the previous example. Because of the unequal end restraint, the point of contraflexure is above mid-story height and the length of the equivalent pin-end column below this point becomes the effective length. Consulting the nomograph (Fig. 5, line Y) we find that $K \cong 2.2$.

Finally, Fig. 7 covers the impossible case of no beam restraint. According to the nomograph the value of K would be infinite. If, however, the columns were fully fixed at their base their effective length would, of course, not exceed twice the building height.

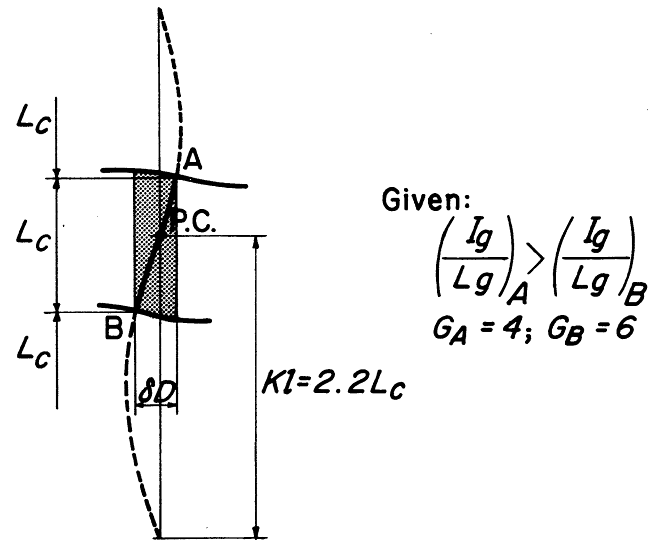

Figure 6

SUMMARY

The trend in modern tier building construction away from heavy masonry exterior walls and substantial tight fitting permanent interior partitions toward light curtain walls and light movable interior partitions with large unobstructed areas, has made it prudent to refine the design procedures for columns. In recognition of this trend and this need, the effective length factor K has been introduced into the AISC Specification. The vast majority of buildings will not be affected by the requirements. Required bracing necessary to eliminate values of K greater than 1.0 is less than generally realized.

In those cases where effective length factors greater than 1.0 must be considered, the nomograph for determining K is simple to use. As designers gain experience in use of the new requirements, they will develop a feel for the problem as they have developed a feel for the design of structural members under the more familiar rules of the past.

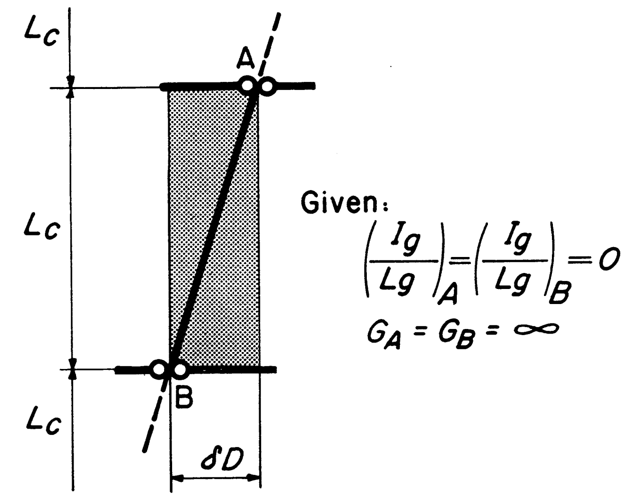

Figure 7