Application of Tuned Mass Dampers To Control Vibrations of Composite Floor Systems

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INTRODUCTION

Although the incidence of floor vibration problems appears to be on the rise,[1,2] the use of mechanical damping devices to control vibrations is limited. In a recent survey of vibration control methods, Murray[3] reports that passive-mechanical damping methods, including viscous damping, visco-elastic damping, and tuned-mass dampers, have often gone untried outside the laboratory or have had marginal impact in actual buildings. This is particularly unfortunate because mechanical dampers can sometimes control floor vibrations more cheaply than structural stiffening, and are often the only viable means of vibration control in existing structures.

This paper details the successful implementation of a tuned-mass damping system to reduce the steady-state vibrations of the longspan, cantilevered, composite floor system at the Terrace on the Park Building in New York City. The experience with this implementation suggests that tuned mass dampers (TMDs) can be successfully employed to control steady-state vibration problems of other composite floor systems. The potential for general application of TMDs in composite floor systems is discussed, and areas for further research are suggested.

BACKGROUND

The Terrace on the Park Building was designed by The Port Authority of New York and New Jersey as its exhibition building for the 1964 Worlds Fair (Fig. 1). The building features elliptical promenade and roughly-rectangular ballroom levels, both suspended six floors above the ground on four steel supercolumns. The columns support a cross-shaped pattern of floor-girders and an elliptical ring girder, which in turn support a radial set of cantilevered floorbeams (Fig. 2). The floorbeams span between the floor and ring girders, and cantilever from the ring girder to the face of the building (Fig. 3). The ballroom sub-floor is a reinforced concrete deck-formed slab, resting on top of and periodically welded to the floor-beams.

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At the close of the Fair, the Authority turned over the building to the New York City Department of Parks and Recreation, which leased it to a private caterer to generate income for the city. The caterer partitioned the ballroom level symmetrically into four dining/dancing halls at the corners of the building, each served by an existing, central kitchen area. Individual halls were arranged with dining tables near the kitchen (and the center of the building); bandstands and dance floors were located at the tip of the cantilevered floors (Figs. 2 and 3).

As soon as the building's cantilevered main floors were used as dining and dance halls, guests complained about the structure's vibrations. Preliminary vibration tests performed during dance events showed that the floor accelerations and displacements sometimes reached 0.07G* and 0.13 inches, respectively. Observations of sloshing waves in cocktail glasses and chandeliers that bounced to the beat of the band gave credence to these measurements. Observations made and complaints logged aside, the measured vibration—as interpreted by the modified Reiher-Meister scale[4] or more recent work by Allen[1]—are generally recognized as unacceptable for dining/dance floors. Floor displacements of 0.13 inches are considered "Strongly Perceptible," as measured on the modified Reiher-Meister scale; Allen's recommendations

* A "G" is equal to the acceleration of a body in a vacuum due to the force of gravity. One G = 32.2 ft/second.^{2}

Fig. 1. Terrace on the Park Building—general view.

limit acceptable floor accelerations in combined dining/dancing environments to about 0.03G.

Preliminary free vibration tests of the structure found the first natural frequency of a typical quadrant of the ballroom level floor (corresponding to one dining/dance hall) to be about 2.3 Hz. This very low frequency is well below the recommended levels for floors whose vibrations are controlled by structural stiffness,[1,3] and corresponds closely to the beat of many dances.[5]

Besides the low frequency of the ballroom-level floors, their vibrations were being exacerbated by the location of the dance floors, which maximized the amount of vibrations that dancers were causing (Figs. 3 and 5). Moving the location of the dance floors toward the center of the building clearly would reduce the structure's vibrations. This remedy was completely unacceptable to the caterer, who made the sensible point that, located between the kitchen and dining areas, the dance floors would block movement between the two and obstruct the exits.

In 1988, after studying various structural stiffening schemes they could not afford to construct, the Parks Department decided to explore solving the vibration problem with mechanical damping devices. The tuned mass damper (TMD) solution was developed by Weidlinger Associates and Professor Vaicaitis after we performed a detailed study of the structure's dynamic characteristics, the forcing function shaking it, and an assessment of various nonstructural remedies.

DYNAMIC CHARACTERISTICS OF THE STRUCTURE

First, we began analytical studies of the building's floor system to determine its dynamic characteristics. A preliminary calculation of the first resonant frequency of the longest cantilevered floorbeams (shown on Fig. 3), was performed, using an equation by Murray and Hendrick:[6]

$$f = K[gEI_t / WL^3]^{1/2}, \text{ (Hz)} \quad (1)$$

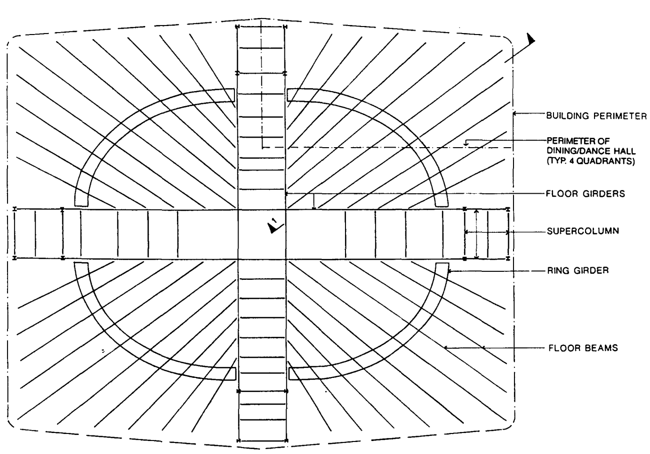

Fig. 2. Ballroom (6th floor) plan.

where:

f = the frequency of vibration of the floor
K = a coefficient depending on ratio of overhang to backspan [tabulated in Ref. 6]
g = 386.4 in/s^2
E = modulus of elasticity
I_t = transformed moment of inertia
W = weight supported by tee beam
L = length of cantilever

Assuming composite action of the floorbeam and concrete deck, Eq. 1 agreed with the earlier rough measurements taken at the structure, which showed that the floor's first natural frequency of vibration was about 2.3 Hz. Although for most of their length, the bottom flanges of the floorbeams are in compression, the composite floorbeam assumption made sense because the deck was significantly reinforced, its steel underside was frequently welded to the floorbeams, and the ratio of live load to dead load was very small, reducing the tendency for the concrete to crack and act independent of the floorbeams.

Next, a detailed, finite element model of a typical floor quadrant (corresponding to one dining/dance hall) was created, to determine the fundamental floor frequency more accurately, compute the associated mode shape, and see if higher floor frequencies and mode shapes were being excited. The floorbeams were modeled with composite bending properties and the concrete deck was modeled with plate elements. The mass included all the structural loads, nonstructural loads such as windows, mullions, partitions, and hung ceilings, and about 15 percent of the 100 psf, code-prescribed live load.

Free vibration analysis of this model showed that the reinforced concrete deck and ring girder tied the floor together, making an entire quadrant of the ballroom level vibrate as a unit. The fundamental mode shape described a continuously deformed floor, with maximum deflection at the extreme cantilevered corner, and monotomically decreasing in deformations toward the ring and floor girders. The first frequency of the floor system was predicted at 2.22 Hz. The second resonant floor frequency was found at 3.9 Hz.

While the structure was being examined analytically, we also measured the natural frequencies of each floor quadrant (corresponding to one dining/dance hall) of the actual structure, the mode-shape associated with the first natural fre-

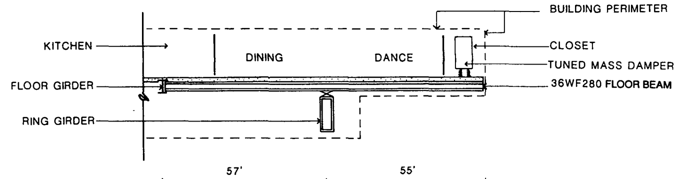

Fig. 3. Section through ballroom floor (Section 1 on Fig. 2).

Quadrant (Dining/Dance Hall)	Fundamental Frequency (Hz)	Second Frequency (Hz)	Damping % of Critical
Rose	2.23	3.75	2.8
Paradise	2.31	—	3.0
Crystal	2.27	3.75	3.0
Regency	2.46	—	3.6
Computer model	2.22	3.91	—

Table 1. Experimentally Determined Floor Frequencies and Damping

quency, and the damping in the first mode. Using a variable speed, largemass shaker, our prediction of the floor's resonant frequencies was confirmed. By simultaneously recording accelerations at a number of locations along the floor, we also confirmed the computer model's prediction of the first mode shape. Using the half power method,[7] the damping in the first mode was determined. The measured frequencies and experimentally obtained damping values for each floor quadrant are given in Table 1. The floors were typically covered with wood, and supported a lightweight steel-panel building-envelope system from the bottom flanges of the floor-beams.

The most important empirical data was obtained during actual dancing. Spectral transforms of the acceleration time-histories obtained during dancing showed that each floor quadrant was vibrating almost exclusively at its first mode (Fig. 4). This result substantially simplified our later analyses and helped us determine an appropriate damping method.

The peak root mean square (RMS) acceleration we measured at the extreme cantilevered corner of a dining/dance hall was 0.06 G, recorded during a rock and roll dance. Assuming the floor to be vibrating in its first mode, we used this measured peak acceleration to determine the maximum floor displacement at the same location. With the floor vibrating in its first mode, both the displacement, $y_2(t)$, and acceleration, $\ddot{y}_2(t)$, of the tip of the floor are essentially sinusoidal functions in time. Their maximums are related by:

$$|y_{2max}| = |\ddot{y}_{2max}| / \omega^2 \quad (2)$$

where:

ω = the frequency of vibration of the floor, in radians per second

$|y_{2max}|$ = the maximum tip displacement at this frequency

$|\ddot{y}_{2max}|$ = the measured RMS floor acceleration

This gave an estimated maximum floor displacement of about 0.11 inches corresponding to the measured 0.06G peak RMS acceleration.

ASSESSMENT OF MECHANICAL VIBRATION CONTROL SYSTEMS

The decision to employ tuned mass dampers was influenced by the functional layout and geometry of the structure; the client's budget; the fact that the floors were being excited primarily at their first resonant frequencies; the large amplitudes of floor motion; and the light structural floor damping.

Simple Passive Dampers

Simple passive dampers, including viscous, friction, and visco-elastic systems, rely on a damper mounted between a vibrating structure and a stationary object to dissipate vibration energy as heat. As the two systems move relative to each other, the simple passive damper is stretched and compressed, reducing the vibrations of the structure by increasing its effecting damping. At the Terrace, there was no non-moving element nearby to attach a damper to, so these systems were rejected.

Tuned Mass Dampers

Tuned mass dampers (TMDs) work by fastening a mass-block to a structural component (such as a floor) via a spring (Fig. 3). This system is set up so that, when the floor vibrates at a resonant frequency (which could be caused by dancing, for example), it induces analogous movement of the mass

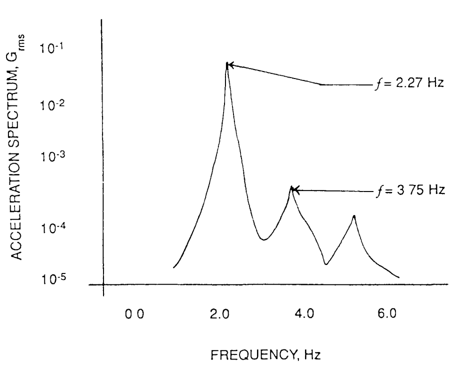

Fig. 4. Typical spectral response (floor excited by dancing).

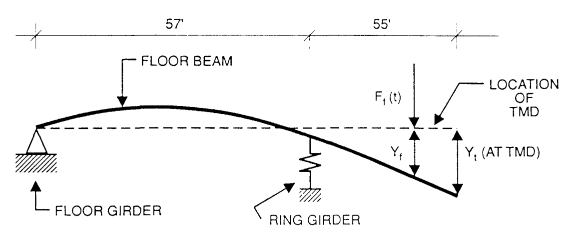

Fig. 5. Floor deflection in first mode shape (Section 1 in Fig. 2).

where:

$F_f(t)$ = idealized, periodic forcing function on dance floor

Y_t = deflection of tip of floor in first mode

Y_f = deflection of floor under forcing function

block and spring. By the conservation of energy, the TMD motion in turn reduces the amplitude of the floor's vibration. A damping device (dashpot) is usually connected in parallel with the spring between the mass-block and floor, increasing the TMD's effectiveness over a range of frequencies and taking a small amount of mechanical energy out of the system as heat.

Because each TMD is "tuned" to a particular resonant frequency, individual TMDs need to be installed for each excited floor frequency. Because they rely only on floor vibrations to operate, they do not need to be fastened to a nearby stationary object. By the same token, TMDs are most effective when located where the floor's amplitudes are the greatest.

TMDs were considered the only viable passive damping system to employ at the Terrace because they did not require fastening to a nearby stationary object. They were also particularly well suited to the Terrace because there was only one floor frequency per ballroom to damp, reducing the required number of TMDs, and the TMDs could be installed at locations where the floor amplitudes were largest (Fig. 5), maximizing their efficiency.

Active Mass Dampers

Active mass dampers, which are computer controlled and can also be configured to work without relying on the relative motion between the floor and a stationary object, were also considered. These systems, currently the subject of much research for controlling wind and earthquake induced vibrations,[8] are a generally attractive solution to vibration problems because they are so effective. These systems were rejected for the Terrace on the basis of their high installation cost, and their need for regular continuing maintenance, which could not be ensured over the life of the structure.

DESIGN OF THE TUNED MASS DAMPERS

The TMD design process began by creating an "equivalent-displacement" one-degree-of-freedom system, representing the dynamic behavior of one point of a typical floor quadrant when vibrating in its first mode. The one-mode model was justified by the experimental data taken in each floor quadrant, which (as noted above) showed that the ballroom floors were vibrating almost exclusively in their first mode. A TMD was then added to this model, creating a two degree of freedom system. The performance of this system, representing an actual floor quadrant and TMD, was used to optimize each TMD's mass, spring stiffness, and damping.

Equivalent Displacement, One Degree of Freedom Floor Model

Figure 5 shows, for a typical quadrant, the line of maximum floor deflection in the first mode (cut at section 1 in Fig. 2). This characteristic mode shape and its associated frequency provided the basis for the equivalent, one degree of freedom (1 DOF) floor-vibration model shown in Fig. 6. To calibrate the 1 DOF system, we required that its free vibrations have the same period as a typical floor quadrant's, vibrating in its first mode. This requirement is stated mathematically by:

$$\sqrt{(k_2 / m_2)} = \omega_{f1} \qquad (3)$$

where k_2 and m_2 are as defined in Fig. 6 and ω_{f1} is the first resonant frequency of the floor, in radians per second (rad/sec).

The calibration for mass and stiffness was completed by dictating that the maximum dynamic displacement of the 1 DOF system would be the same as the tip of the floor constrained to vibrate in its first mode shape, while being forced by a periodic, concentrated load at its tip; i.e., $y_{2\,\max}$ (Fig. 6) = $y_{t\,\max}$ (Fig. 5). Using the free-vibration computer model, the 1 DOF system's mass, m_2, was found by:

$$m_2 = u / d_t^2, \text{ (kips} \times \text{sec}^2\text{/in)} \qquad (4)$$

where u is the mass-normalized generalized mass of the first mode of the floor system, and d_t is the associated modal displacement at the tip of the floor. (This equation is derived in Appendix A.)

As calculated by Eq. 4, m_2 is called the "equivalent-displacement generalized floor mass." Using this value for m_2, k_2 was found from Eq. 3.

We also computed k_2 and m_2 from our experimental data. First, we assumed the floor would respond only in its first mode when shaken by a harmonic forcing function of a

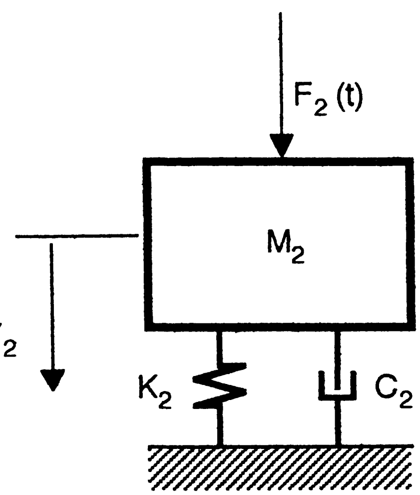

Fig. 6. 1 DOF floor model.

where:

m_2 = displacement normalized generalized mass of floor system in first mode

k_2 = displacement normalized generalized stiffness of floor system in first mode

c_2 = damping in first mode

$F_2(t)$ = idealized, periodic, dance-floor equivalent forcing function

$y_2 = y_t$ = deflection of tip of floor in first mode

Table 2. Stiffness and Mass of 1 DOF Floor Models				
	k_2 (kips/in.)	m_2 (kips)	c_2 (kips*s/in.)	ζ_2 (% of Critical)
Equations 4, 3, 6 (Computer)	197	389	.874 (Determined from choice of ζ_2)	3.1% (Chosen to match experimental data)
Equations 12, 10 (Experimental)	205	406	.912 (Determined by ζ_2)	3.1%

frequency equal to the floor's first resonant frequency [$F(t) = F_o \sin(\omega_{f1} t)$, where ω_{f1} is the first resonant frequency of the floor quadrant (rad/sec) and t = time (sec)]. In this case, the floor behaves as a one degree of freedom system, whose steady-state response is given by :

$$y_2(t) = F_o \sin(\omega_{f1} t + h) / (2\zeta k_2) \quad \text{(Ref. 7)} \tag{5}$$

where:

F_o = the amplitude of the forcing function driving the floor at its cantilevered tip

y_2 = the peak floor response measured at the same location

ω_{f1} = the resonant frequency of the floor

h = a phase angle

k_2 = the equivalent displacement generalized stiffness of the floor

ζ = the measured damping of the floor, expressed as a percent of the floor's critical damping, c_c*

from which:

$$|y_{2max}| = F_o / (2\zeta k_2), \quad \text{(in.)} \tag{8}$$

and:

$$k_2 = F_o / (2\zeta |y_{2max}|), \quad \text{(kips/in.)} \tag{9}$$

and m_2 is then found from Eq. 3.

The damping included in the 1 DOF model (ζ) was 3.1 percent, corresponding to the average of the four experimentally determined values given in Table 1. This is a bit lower than what would be expected based on published values.[4,9] Using Eqs. 6 and 7, the absolute floor damping, c_2, was found to be 0.874 kip-seconds/in.

k_2, c_2 and m_2, computed both analytically and experimentally, are given in Table 2. The computer generated values were used in the subsequent analysis and design work.

The 1 DOF system's forcing function, $F_2(t)$, was also calibrated to approximate the effect of dancing on the actual structural floor. The function was assumed to be sinusoidal (which is arguably a fair approximation for dancing[1]), i.e.:

$$F_2(t) = F_o \sin(\bar{\omega} t), \quad \text{(kips)} \tag{10}$$

The force amplitude (F_o) was adjusted so that at frequencies ($\bar{\omega}$) close to the beat of previously measured dancing at the Terrace, the maximum steady-state acceleration of the 1 DOF model would match the RMS peak acceleration at the tip of the actual floor during an instrumented dance event.

Two Degree of Freedom, Floor-TMD Model

After the equivalent-displacement 1 DOF system was developed, tuned mass dampers were added, creating a two degree of freedom (2 DOF) system (Fig. 7). Using this system, the TMD parameters of mass (m_1), stiffness (k_1), and damping (c_1), were optimized to reduce the dynamic displacement of the floor (y_2), due to the forcing function $F_2(t)$, representing dancers on the real structural floor.

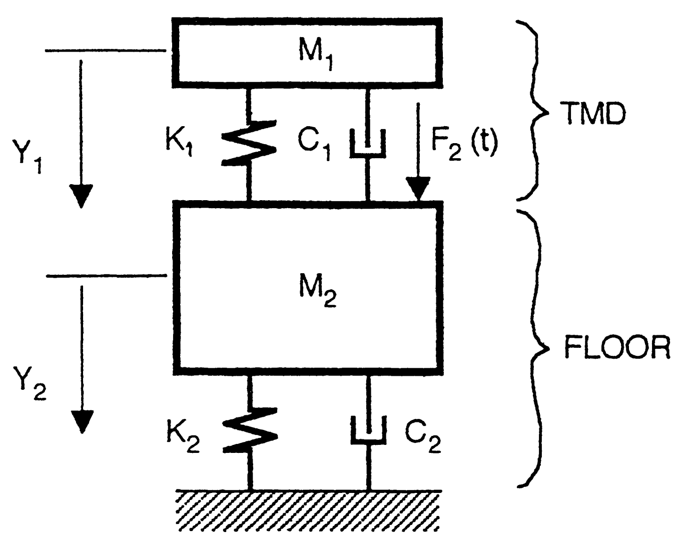

Fig. 7. 2 DOF floor-TMD model.

where:

m_1 = *mass of TMD*

k_1 = *TMD spring stiffness*

c_1 = *TMD damping*

y_1 = *displacement of TMD*

* For the 1 DOF floor model, ζ and c_c are related by:

$$\zeta = c_2 / c_c \quad \text{(Ref. 7)} \tag{6}$$

where

$$c_c = 2\sqrt{(k_2 / m_2)} \quad \text{(kip*sec/in.)} \tag{7}$$

Table 3. Summary of TMD Parameters*			
Quantity	**Trial Optimum Value**	**Initial Construction Value**	**Final Tuned Value**
Mass, m_1 (kips)	18.0 (Controlled by 20 kip floor beam capacity)	19.0	18.4
Damping, c_1 (kips*s/in.)	0.19 Equation 13	0.19	0.15
Spring stiffness, k_1 (kips/in.)	8.3 Equations 12, 11	8.8	8.8 (Cannot be field adjusted)
* Values are presented for the Rose floor quadrant, whose measured natural frequency without TMDs installed was 2.23 Hz. Results for other quadrants are similar.			

The TMDs needed to minimize the floor's vibrations without using so much mass that the existing floorbeams would be overstressed. Although to a point TMDs become more effective with increased mass,[10] calculations showed that the floorbeams supporting the TMDs would be overstressed with masses greater than about 20 kips located at tips. Therefore, 18 kips became our trial-optimal TMD mass. This corresponds to a mass ratio (m_1 / m_2) of about 4.6 percent.

Because each actual ballroom floor was responding primarily in its first mode shape, the TMDs needed to be operating near the associated resonant frequency to maximize the amount of energy shifted from the vibrating floor to themselves. Various approaches to optimizing a TMD's natural frequency have been reported.[11,12] As a start point, we used the approach outlined by Reed,[12] in which the natural frequency of the TMD attached to a fixed base is denoted ω_1. Then:

$$\omega_1 = \sqrt{(k_1 / m_1)}, \quad \text{(rad/sec)} \tag{11}$$

where k_1 and m_1 are the spring-stiffness and mass, respectively, of the TMD.

And Reed's optimum value for ω_1 is given by:

$$\omega_{1,optimum} = 1 / [1 + (m_1 / m_2)], \quad \text{(rad/sec)} \tag{12}$$

where m_2 is the equivalent-displacement generalized floor mass defined above.

With m_1 and m_2, determined, $\omega_{1optimum}$ was found by Eq. 12, and k_1 was determined by Eq. 11. We also used Reed's method for obtaining a trial value of optimum damping, c_1:

$$c_{1,optimum} = \sqrt{2m_1k_2 / [1 + (m_1 / m_2)]}, \quad \text{(kip} \times \text{sec/in.)} \tag{13}$$

The trial-optimum values, k_1, c_1, and m_1, computed using Eqs. 11 through 13, are summarized in Table 3.

Starting with the maximum safe mass and predicted-optimum values for c_1 and k_1, the 2 DOF model of the floor-TMD system (Fig. 7) was analyzed. The system's equations of motion are:

$$\begin{pmatrix} m_1 & 0 \\ 0 & m_1 \end{pmatrix}\begin{pmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{pmatrix} + \begin{pmatrix} c_1 & -c_1 \\ -c_1 & c_1+c_2 \end{pmatrix}\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} + \begin{pmatrix} k_1 & -k_1 \\ -k_1 & k_1+k_2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ F_2(t) = F_o \sin(\overline{\omega}\, t) \end{pmatrix} \text{kips} \tag{14}$$

Where $\overline{\omega}$ is the frequency of the forcing function (rad/sec).

These equations were used to: check the validity of the TMD parameters given in Eqs. 12 and 13; predict the reduction in floor acceleration caused by the TMDs; and estimate the maximum accelerations and relative displacements of the TMD mass (m_1). Because Eq. 14 cannot be solved modally (due to the high damping in the system), they were integrated numerically with the Runge-Kutta fourth order method.[13] For values of $\overline{\omega}$ between 1 and 8 Hz, time histories were produced and maximum values of y_2, $\ddot{y}_2$, and $y_2 - y_1$ were recorded.

Because it was our experience that TMDs needed to be adjusted in the field, we designed the actual TMDs to be "tuned" for frequency and damping after installation. This was done by varying the TMDs' mass (m_1) with 200 pound steel plates, and adjusting its damping (c_1) with variable energy dissipation dashpots. Two types of variable energy dissipation viscous dashpots were tested at the Carleton Lab of Columbia University's Engineering School (Fig. 8), and found to need a minimum stroke (in the form of enough relative floor-TMD mass movement) of about 0.05 inches to be effective. In practice, the relative motion between the TMD and floor ($y_2 - y_1$ in Fig. 7) is reduced with increasing TMD mass (m_1) and increased damping (c_1). To obtain a desired stroke, it was found by manipulating m_1 and c_1 in Eq. 14 that the TMDs performed better if their damping was slightly decreased than if their mass was reduced. Thus, ensuring the stroke of the TMDs was large enough effectively put an upper bound on their damping.

The TMD stiffness, k_1, was limited by the properties of commercially available springs. The spring stiffness, of course, could not be modified in the field, which did not pose much of a problem because the natural frequency of the

TMDs was controlled by adjusting their mass, as described above.

The TMD parameters, m_1, c_1, and k_1, which were used for their initial construction, are given in Table 3. These values were adjusted from the trial-optimum values as required by the constraints on spring stiffness, damping, and mass noted above. The corresponding predicted performance of the TMDs is shown in Fig. 9. Each point on the graph represents maximum steady state floor displacement corresponding to the calibrated forcing function operating at frequency ω. The curve predicted that the TMDs would reduce dance-induced floor vibration by a maximum of 70 percent, corresponding to dancing at about 2.2 Hz.

PERFORMANCE OF THE AS-BUILT SYSTEM

In 1991, one TMD was installed in the corner closet of each dining/dance hall (Fig. 3). A typical system is shown in Fig. 10. Each TMD was tuned for optimum frequency and damping by using a variable-speed, large mass shaker to excite the floor at a range of frequencies while monitoring both floor and TMD accelerations. During an actual dance event, floor accelerations were monitored first with the dampers locked into place, then free to move. The final, "tuned" TMD parameters are summarized for one floor quadrant in Table 3. Results for other quadrants are similar. The results of the shaker and dance-event tests are given in Figs. 11 and 12 respectively. Our measurements of TMD performance during dance events showed that the TMDs reduced ballroom floor vibrations by at least 60 percent. The difference between the predicted 70 percent reduction and the 60 percent in-situ performance is ascribed to the difference between the actual and analytical forcing functions (dancers and a sine-wave, respectively), and the floor's vibrations in its second mode shape, which the TMDs were not designed to reduce. No floor vibration complaints have been reported to us since the TMDs were installed.

The cost of constructing the four TMDs was $220,000. This is less than 15 percent of the estimated construction cost of structural stiffening (with new columns between the ballroom floors and the ground) recommended for the Terrace in 1987.[14]

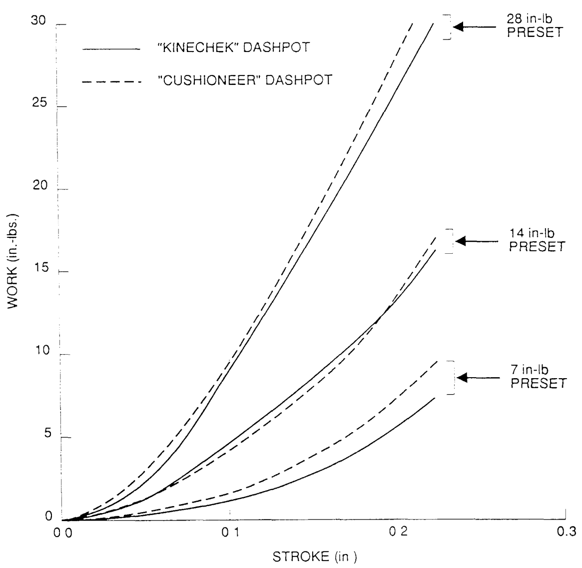

Fig. 8. Viscous dashpot work per stroke at various energy absorption settings. ("Kinechek" and "Cushioneer" refer to the manufacturer's proprietary names of tested models. The energy absorbed by the dashpots per stroke is adjustable. Different "preset" curves correspond to different dashpot settings.)

SUMMARY—CONCLUSIONS

The TMD implementation described in this paper demonstrates their successful use in substantially reducing the vibrations of an existing composite floor system. The critical reasons for the success of the system are: its tunability, which helped ensure that the theoretically predicted performance could be approximated by the actual as-built system; and the cost of the system, which was about an order of magnitude less than the cost of recommended structural corrective measures.

Although the methods used to analyze the case-study floor system and design its TMDs are very general, and can be applied in principle to many composite floor systems, the effective use of TMDs in structures with higher damping values and lower maximum floor displacements may prove troublesome. It has been claimed that it is generally difficult to make TMDs useful in structures with high natural damping. The adjustable viscous dashpots used in this case-study perform marginally at small strokes, suggesting they would not perform adequately in floors whose amplitudes are small. However, other types of damping, which are field tunable and

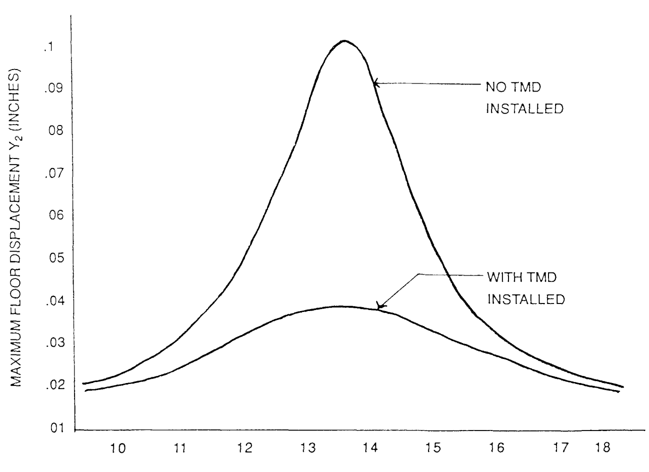

Fig. 9. Maximum, steady-state floor amplitudes at tip of floor, as predicted by Eq. 14. (TMD parameters correspond to "initial construction values" in Table 3.)

may perform well at small amplitudes, have been used in TMD applications,[15] and warrant further study. It should also be noted that, although floor frequency itself should not impact the viability of TMDs, most composite floors have frequencies much higher than the fundamental floor frequency at the Terrace. This may affect the choice of hardware in other installations, including the types of springs and dashpots used.

The success of the field-tuned case-study system presented in this paper, and the small number of mechanical damping systems installed in actual buildings today, suggest that damping systems are not being used as often as they possibly should be. Increased use of passive damping systems requires that structural engineers better understand their overall performance, and the limitations of their actual components (such as dashpots). With this in mind, further research in the performance of passive damping devices in actual floor systems is recommended in the following areas:

- In-depth studies of TMD dashpots, including linear viscous and Coulomb friction types.
- Analysis of tuned-in-the-field TMD effectiveness in floor systems with smaller dynamic displacements.
- Analysis of tuned-in-the-field TMD effectiveness in reducing transient vibrations.
- Comparison of the effectiveness of TMDs and other passive and active damping systems, in controlling both transient and steady-state vibrations, in terms of both performance and cost.

APPENDIX A—DERIVATION OF EQ. 4

Applying free vibration analysis techniques to a finite element model of the floor system, the following quantities can be computed:[16]

ω_1 = the first resonant floor frequency
$\vec{d}$ = the associated mode-shape column vector
M = the mass matrix of the floor system
K = the stiffness matrix of the floor system
u = the generalized mass of the first mode = $\vec{d}^T M \vec{d}$
z = the generalized stiffness of the first mode = $\vec{d}^T K \vec{d}$

Leaving damping aside for simplicity, if the floor is moving in only its first mode, forced by the function $F(t)$, at a particular node, n, then it can be shown[7,17] that the floor movement at any point is described by the equations:

$$u\ddot{\alpha} + z\alpha = d_n F_n(t) \quad \text{(A1)}$$

$$\vec{x} = \vec{d}\,\alpha \quad \text{(A2)}$$

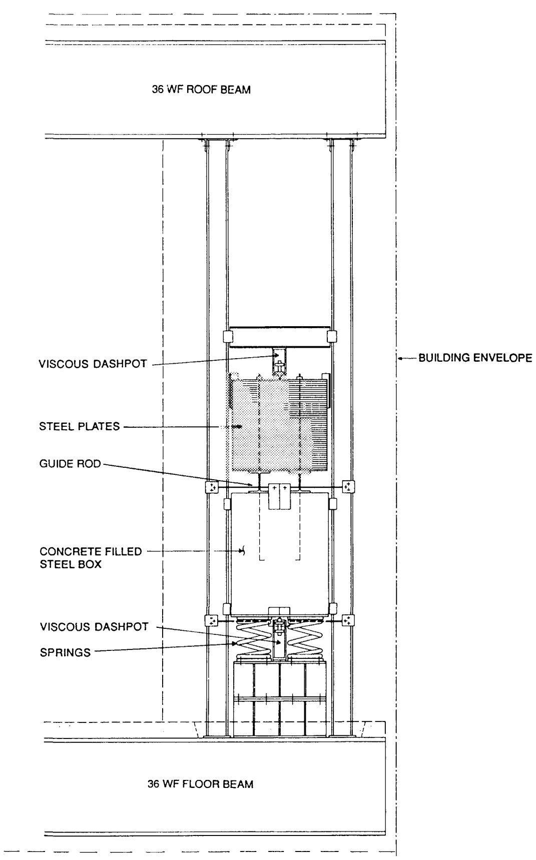

Fig. 10. TMD elevation.

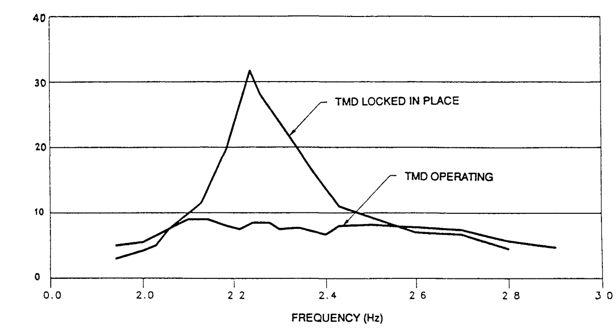

Fig. 11. Peak RMS floor response at tip of floor, subject to sinusoidal forcing function. Test of actual floor system with field-tuned TMD (Crystal Quadrant).

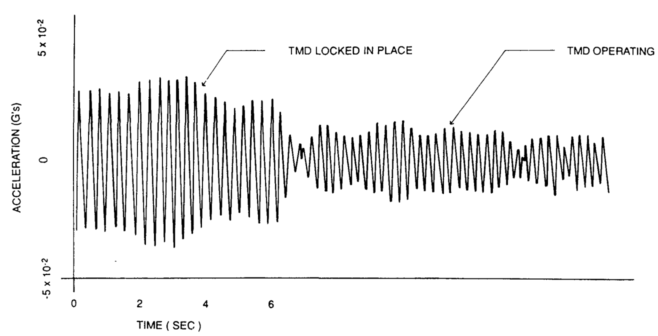

Fig. 12. Measured floor acceleration at tip of floor due to dancing with field-tuned TMD.

Where α is called the generalized coordinate of the first mode, and $\vec{x}$ is the vector of nodal coordinates from the finite element formulation.

The equation of motion of the 1 DOF, equivalent displacement model (Fig. 6) is:

$$m_2 y_2 + k_2 y_2 = F(t) \tag{A3}$$

By definition of the 1 DOF model, $k_2 / m_2 = z / u$. By specifying that $y_2 = x_n$ when $F(t) = f_n(t)$, these constraints lead to:

$$y_2 = d_n \alpha \tag{A4}$$

$$u\ddot{y}_2 + z y_2 = d_n^2 F_n(t) = d_n^2 F(t) \tag{A5}$$

Dividing this by d_n^2, and comparing to Eq. A3 yields $m_2 = u / d_n^2$. Noting that, in the case of the Terrace, $d_n = d_t$, and making this substitution yields Eq. 4.

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