Design Aids—Minimum Depth and Allowable Bending Stress

Paper presented by GEORGE FROYTON (April, 1966, issue)

Discussion by **ADOLPH A. MARRONE**

THE PAPER on "Design Aids—Minimum Depth and Allowable Bending Stress" by George Froyton which appeared in the April, 1966, issue of your JOURNAL was of particular interest to me because for the last five years I have been using the accompanying chart of my own design to accomplish the same purpose as Mr. Froyton's charts.

I have been impelled to submit mine because it has several advantages:

(a) Its dimensionless scales make it applicable to any span with great accuracy.
(b) It allows for the fact that deflection limitations are usually based on the live load deflection only.

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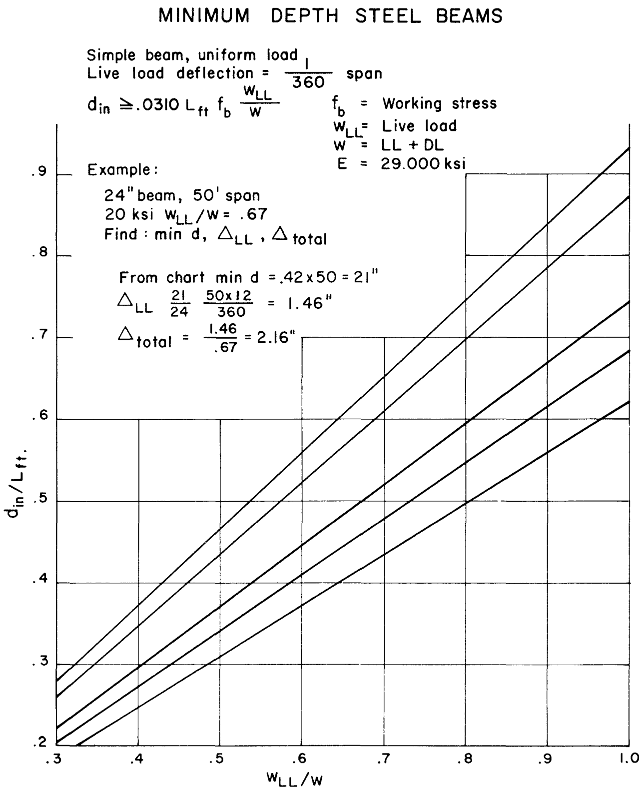

The chart is based on the following theory:

The live load deflection at the center of a simple beam uniformly loaded is

$$\Delta_{LL} = \frac{5}{384} \frac{w_{LL} l^4}{EI} \tag{1}$$

$$f_{LL} = \frac{M_{LL}\, c}{I} = \frac{w_{LL} l^2}{8} \frac{d}{2I} \tag{2}$$

Solve Equation (2) for w_{LL} and substitute in Equation (1)

$$\frac{\Delta_{LL}}{l} = \frac{5}{384} \frac{l^3}{EI} \frac{16 I f_{LL}}{l^2 d} = \frac{1}{4.8} \frac{f_{LL} l}{Ed} \tag{3}$$

$$d = \frac{f_{LL} l}{4.8 E \left(\dfrac{\Delta_{LL}}{l} \right)} \text{ for any material} \tag{4}$$

Using $\Delta_{LL}/l = 1/360$ for the maximum permissible deflection and limiting our study to steel with E = 29,000 ksi and converting to feet for the span by substituting $12L$ for l, Equation (4) becomes

$$d \geq \frac{f_{LL} 12L}{4.8 \times 29{,}000 \ \dfrac{1}{360}} = 0.0310 L f_{LL}$$

$$d \geq 0.0310 L f_b \left(\frac{f_{LL}}{f_b} \right) \qquad \text{also} \ \frac{f_{LL}}{f_b} = \frac{w_{LL}}{w}$$

$$\frac{d}{L} \geq 0.0310\, f_b \left(\frac{w_{LL}}{w} \right) \text{ for steel} \tag{5}$$

The chart is a plot of Equation (5).

Equation (4) is general and the writer has used it to construct minimum depth beam charts for other materials, such as wood.

Equations (4) and (5) show that the depth is inversely proportional to the deflection, directly proportional to the bending stress and directly proportional to the live load to total load ratio. If these simple properties are kept in mind the chart will become a great convenience to the hurried designer, and among other things will completely eliminate solving the unwieldy deflection formula given by Equation (1).

For example, a beam is required to carry a LL of 18^k and a DL of 14^k on a 42 ft span. Find the minimum depth required if the live load deflection must not exceed 1/300 of the span.

$$f_b = 24 \text{ ksi} \qquad \frac{I}{c} \text{ req'd} = \frac{32 \times 42}{8} \left(\frac{12}{24} \right) = 84 \text{ in.}^3$$

Enter chart with $w_{LL}/w = 18/32 = 0.563$ and obtain $d_{\min} = 0.420 \times 42 \text{ ft} \times 300/360 = 14.7$ in.

Now suppose it has been decided to use an 18WF50; what are the LL and DL deflections?

$$\frac{I}{c} \text{ furnished} = 89.0 \text{ in.}^3$$

$$\Delta_{LL} = \frac{14.7}{18} \times \frac{12 \times 42}{300} \left(\frac{84}{89} \right) = 1.295 \text{ in.}$$

$$\Delta_{DL} = \frac{14}{18} \times 1.295 = 1.01 \text{ in.}$$

NOMENCLATURE

d = Depth of beam, in.
Δ_{LL} = Deflection due to live load, in.
l = Span, in.
L = Span, ft
f_{LL} = Bending unit stress due to live load, kips per sq in.
f_b = Working unit stress, bending, kips per sq in.
w_{LL} = Live load, kips per ft
w = Total load, kips per ft