Flexural Strength of WT Sections

DUANE S. ELLIFRITT, GREGORY WINE, THOMAS SPUTO, and SANTOSH SAMUEL

INTRODUCTION

WT sections are commonly used as chord members in lightly and moderately loaded roof and floor trusses. In this application, these members are subjected to combined axial and flexural loads. The design of these sections for flexural loads was not specifically addressed in the ASD Specification. Not until the publication of the LRFD Specification was flexural loading of these sections directly addressed. With the advent of ultimate strength design methods included in the LRFD Specification, it became obvious that these sections could carry increased loads.

This paper will look at design capacity for these sections and will report on laboratory experimentation supporting these limits.

SECTION STRENGTH

A doubly symmetric, compact, braced wide flange section has an allowable moment (ϕM_n) of ϕM_p in LRFD. The mean value for the plastic shape factor (SF) for wide flange sections is 1.12. Considering a live to dead load ratio of 3/1 (effective load factor (LF) of 1.5), the section utilizes:

$$\phi M_n \,/\, LF(M_p)$$

$$0.9(M_p) \,/\, 1.5(M_p) = 0.6$$

or 60 percent of the ultimate capacity of the section, where $M_p = 1.12M_y$.

For weak axis bending of this same section, the design moment is again ϕM_p. The plastic shape factor for a wide flange section in weak axis bending is 1.50. Using the same load ratio, the section utilizes:

$$0.9(M_p) \,/\, 1.5(M_p) = 0.6$$

Duane S. Ellifritt is Crom Professor of Structural Design, Department of Civil Engineering, University of Florida, Gainesville, FL.

Gregory Wine is a major, U.S. Army Corps of Engineers.

Thomas Sputo is consulting structural engineer, Gainesville, FL.

Santosh Samuel is programming engineer, WedgCor Metal Building Systems, Denver, CO.

or 60 percent of the ultimate capacity of the section, where $M_p = 1.50M_y$.

The case for WT sections is a bit different. The design moment for positive bending, where the stem is in tension, is $\phi 1.5M_y$.* The value of M_y used here is the lesser value, considering yielding in the stem, rather than the greater value which considers yielding in the flange. The shape factor for WT sections is a bit variable, but it can be seen from Fig. 1 that the mode (value occurring most frequently among all WT sections) of 1.78 is a reasonably conservative assumed value. Based on this value, $M_p = 1.78M_y$. Using the same load ratio, the section utilizes only:

$$0.9(1.5M_y) \,/\, 1.5(1.78M_y) = 0.51$$

or 51 percent of the ultimate capacity of this section.

How much of the ultimate capacity of a WT would be utilized if the limit for flexural strength were ϕM_p instead of $\phi 1.5M_y$? Assuming the same load ratios as before, the section would utilize:

$$0.9(M_p) \,/\, 1.5(M_p) = 0.6$$

or 60 percent of the ultimate capacity of the section.

Strength is not the controlling factor. The key thing to consider is serviceability under service load conditions. For a wide flange section loaded in strong axis bending, where

*Revision to specification, October 1990. Was formerly ϕM_y.

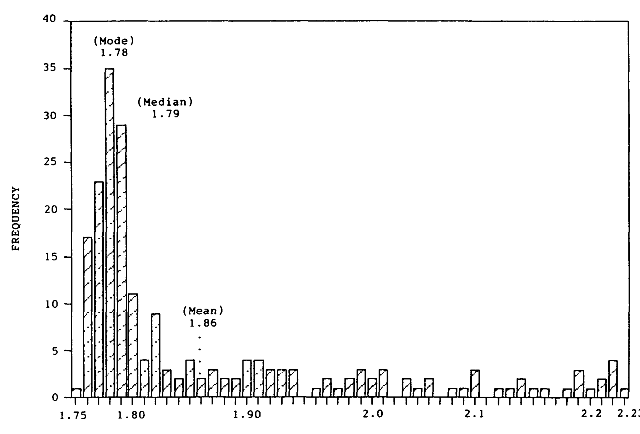

Fig. 1. Variation of shape factor for WT section.

$M_p = 1.12M_y$, the flexural stress in the section at service load is:

$(\phi M_n / LF) / S$

$0.9(1.12M_y) / 1.5S = 0.67F_y$

For weak axis flexure of a wide flange, where M_p = $1.50M_y$, the flexural stress in the section at service load is:

$0.9(1.5M_y) / 1.5S = 0.90F_y$

Using 1.78 as the mean shape factor for WT sections and letting $\phi M_p = 1.78M_y$ for the flexural limit where the stem is in tension, would result in a flexural stress at service load of:

$0.9(1.78M_y) / 1.5S = 1.07F_y$

This would mean that the section has begun to yield in the stem at service load, a condition to be avoided for reasons of deflection control.

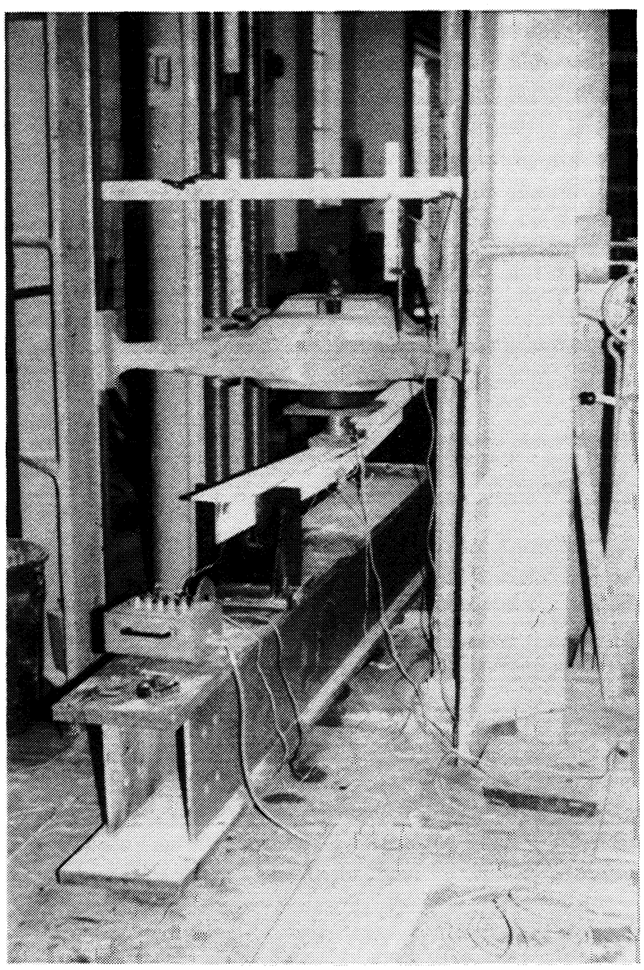

Fig. 2. Test beam in loading fixture.

If the limit is $\phi 1.5M_y$, the flexural stress at service load would be:

$0.9(1.5M_y) / 1.5S = 0.90F_y$

The flexural stress at service load is now less than the yield stress, and the same as for a wide flange section in weak axis bending.

LABORATORY TESTING

Laboratory testing was conducted to study the behavior of WT sections in both positive (stem in tension) and negative (stem in compression) flexure. The specimens were tested over a 7-ft span and were loaded using a mechanically driven universal testing machine, as shown in Fig. 2. Loads were measured using a load cell and deflections were measured using an LVDT.

Negative bending was induced by rotationally fixing the ends of the span to create a fixed ended beam. Schematics of the testing apparatus are shown in Fig. 3 for positive bending and Fig. 4 for negative bending.

Tests 1 through 5 were performed for positive bending and tests 6 through 8 had the ends fixed creating negative bending at the supports. Table 1 lists the cross section measurements and measured yield strengths for all eight specimens. The listed section designations are assumed since they were not initially known and cannot be conclusively determined from the listed measurements. Table 2 lists the calculated cross section properties for all eight sections.

Load versus deflection curves were developed from the experimental measurements of Tests 1 through 5. As expected, the sections, on average, developed the calculated

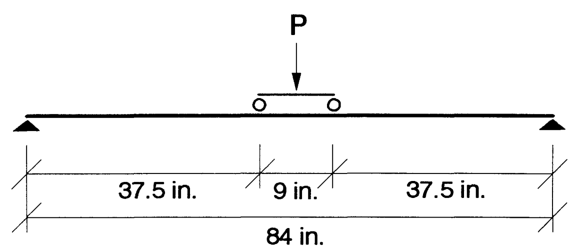

Fig. 3. Postitive bending test.

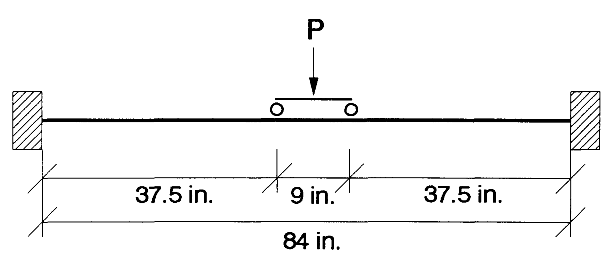

Fig. 4. Negative bending test.

Table 1.
Section Measurements

Test No.	b_f	t_f	d	t_w	F_y	Measured Nominal Section
1	5.71	0.449	5.15	0.287	51.0	WT5×13
2	5.72	0.477	5.10	0.302	51.0	WT5×13
3	6.44	0.360	6.12	0.277	55.7	WT6×13
4	6.52	0.359	6.19	0.275	55.7	WT6×13
5	6.49	0.367	6.08	0.269	55.7	WT6×13
6	6.21	0.415	6.54	0.293	50.0	WT6×15
7	6.13	0.415	6.54	0.270	50.0	WT6×15
8	6.14	0.416	6.55	0.255	50.0	WT6×15

b_f = Flange width, in.
t_f = Flange thickness, in.
d = Total section depth, in.
t_w = Stem thickness, in.
F_y = Yield stress, ksi

Table 2.
Cross-Section Properties

Test No.	I_x	I_y	Z_x	S_{xf}	S_{xs}
1	8.39	6.98	3.68	7.54	2.08
2	8.54	7.46	3.80	7.76	2.14
3	13.28	8.02	4.99	9.31	2.83
4	13.69	8.31	5.07	9.52	2.88
5	12.84	8.38	4.80	9.34	2.73
6	16.96	8.29	6.01	10.94	3.40
7	15.92	7.98	5.56	10.64	3.16
8	15.34	8.03	5.29	10.57	3.01

I_x = X axis moment of inertia, in.^{4}
I_y = Y axis moment of inertia, in.^{4}
Z_x = X axis plastic section modulus, in.^{4}
S_{xf} = X axis section modulus to flange, in.^{3}
S_{xs} = X axis section modulus to stem, in.^{3}

plastic moment capacity, as shown in Table 3. Figure 5 shows a typical failure, including the plastic hinge which was formed. Figure 6 is the moment versus deflection diagram for Test 4. The deflections follow the predicted elastic deflections quite well until the initiation of yielding in the stem at M_y, then begin to gradually increase until the plastic capacity of the section is reached. Based on these measured deflections, it is reasonable to allow a service load of $0.90M_y$ ($0.90F_y$) for serviceability reasons.

Load versus deflection curves were also developed for Tests 6, 7, and 8. These sections failed in the region of negative bending where the stem was in flexural compression. Figures 7 and 8 show a typical failure of the stem through buckling. Figure 9 shows the moment versus deflection diagram for Test 7. Again, the measured deflections generally follow the predicted elastic deflections once the slippage of the specimen in the supports is considered.

Table 4 compares the measured capacities versus calculated capacities. If the provisions of the LRFD specification are followed directly, the stems of all three specimens are slender elements and the capacity of the sections must be reduced according to LRFD Appendix B. But the criteria for tee stems was derived for stems in axial compression, not stems in flexural compression and uses a very conservative assumption for the length of the unstiffened element. It is possible to derive a somewhat less conservative limit for stems in flexural compression, using the same basic criteria, which shows the test specimen stems not to be slender elements. This derivation is shown in Appendix B of this paper.

The collected test data for negative bending is rather slim to base a specification provision on. It is possible, however, to conservatively set initiation of stem yielding (M_y) as an upper limit for strength in negative bending. Further research may reasonably allow a design moment greater than M_y.

Table 3. Test Results for Positive Bending

Test No.	M_n+	M_{ys}	M_p	M_{test}	M_p / M_{test}
1	1,711	105.9	187.8	180.0	0.958
2	1,856	109.0	193.8	187.5	0.967
3	2,061	157.7	278.0	318.7	1.146
4	2,148	160.4	282.6	281.2	0.995
5	2,137	152.0	267.3	288.7	1.080
				Avg.	1.029

$M_n+ = M_n$ from LRFD Eq. F1-15 using $B+$, kip/in.
M_{ys} = Yield moment for yielding of stem, kip/in.
M_p = Plastic moment capacity, kip/in.
M_{test} = Measured test moment, kip/in.

Table 4. Test Results for Negative Bending

Test No.	M_n-	M_{ys}	M_p	M_{test}	M_{ys} / M_{test}
6	322.5	169.9	300.3	168.0	0.989
7	302.5	157.8	278.0	187.5	1.131
8	295.2	150.5	264.4	168.0	1.116
				Avg.	1.078

$M_n- = M_n$ from LRFD Eq. F1-15 using $B-$, kip/in.
M_{ys} = Yield moment for yielding of stem, kip/in.
M_p = Plastic moment capacity, kip/in.
M_{test} = Measured test moment, kip/in.

LRFD SPECIFICATION[1]

LRFD equation F1-15 is the limiting lateral buckling equation for WT strength in both positive and negative bending. This equation is theoretically correct for elastic buckling of tee shaped beams. In positive bending, it is impossible in practice to develop this elastic strength before a plastic hinge is formed. Note the calculated capacities using Eq. F1-15 in Table 3 as compared to the plastic moment capacities. Also note Fig. 10 which shows Eq. F1-15 plotted for a WT6×20. Note that a WT6×20 will not experience lateral buckling with the flange in compression until the unbraced length reaches 75 ft! It would probably be better just to note the capacity in positive flexure to be $1.5M_y$ and to eliminate the use of Eq. F1-15 for positive bending.

It may be possible to exceed the elastic buckling strength in negative bending in some rare cases, however. Table 5 lists the elastic buckling stress computed from F1-15, using C_b = 1.0, for all WT, ST, and MT sections listed in the manual where the buckling stress at a length of 25 times the section depth ($25D$) is 50 ksi or less. Note that this list excludes most sections listed in the *Manual of Steel Construction*.[1,2] For any case where the elastic buckling stress is greater than the yield stress, Eq. F1-15 will not control. For cases where the elastic buckling stress is less than the yield stress, lateral buckling will govern for negative bending, and lesser of M_y and Eq. F1-15 should govern. Slender elements should be considered as recommended in Appendix B of this paper. If slender elements are present, the lesser value of the elastic lateral buckling stress from Eq. F1-15 and Q_sM_y should govern.

ASD SPECIFICATION[2]

Judging from the above, it is unnecessarily conservative to limit the allowable bending stress in the stem for positive bending to $0.66F_y$. A limit of $0.90F_y$ would be more reasonable and in keeping with the LRFD design criteria which allow a service load of $0.90M_y$ at a live/dead load ratio of 3/1. This "high" allowable stress is not an isolated case in steel design specifications. The specifications of the Steel Joist Institute allow a bending stress of $0.90F_y$ for solid round web members in open web steel joists. Rounds have a plastic shape factor of only 1.70 as opposed to an average of 1.78 for WTs.

Negative bending of WTs is not covered in the ASD specification. This question was raised by Milek[6] in 1965. At that time, it was recommended that the allowable bending stress for negative bending be $0.60F_y$. In light of the results shown in Table 5, both lateral buckling and strength should be checked for sections and lengths listed in Table 5, for cases where the elastic buckling stress is less than the yield stress.

It is probably reasonable to allow a compressive stress of $0.60F_y$ on the stem, modified by the applicable slender element modification factors shown in Appendix B of this paper, as a strength limit. One possible method of checking lateral buckling strength would be to use LRFD Eq. F1-15, incorporating a factor of safety of 1.67. As modified, this equation reads:

$$F_b = (34{,}000 C_b \sqrt{I_y J} \,/\, L_b S_x)(\sqrt{1 + B^2} - B)$$

where:

$$B = (2.3d \,/\, L_b) \sqrt{I_y / J}$$

CONCLUSION

Laboratory testing has verified the strength limits for WT sections in both positive and negative bending. Based on this testing, the LRFD design requirements have been validated and recommended modifications to the ASD specification have been noted. Additionally, slender element criteria for tee section stems in flexural compression have been developed and shown in Appendix B of this paper.

ACKNOWLEDGMENTS

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The conclusions and recommendations are those of the authors and do not necessarily reflect the views of the American Institute of Steel Construction.

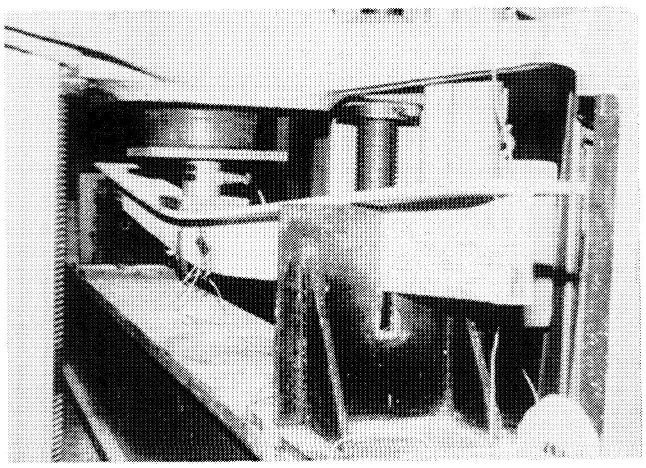

Fig. 5. Plastic hinge formed in positive bending.

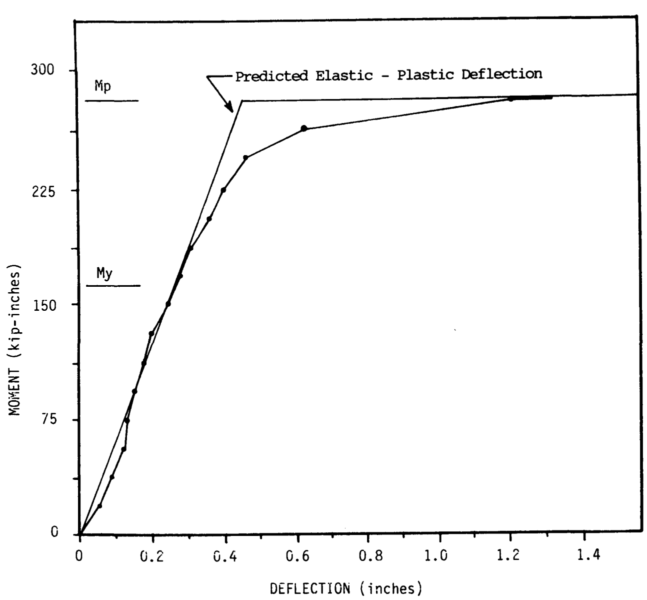

Fig. 6. Moment vs. deflection for Test 4.

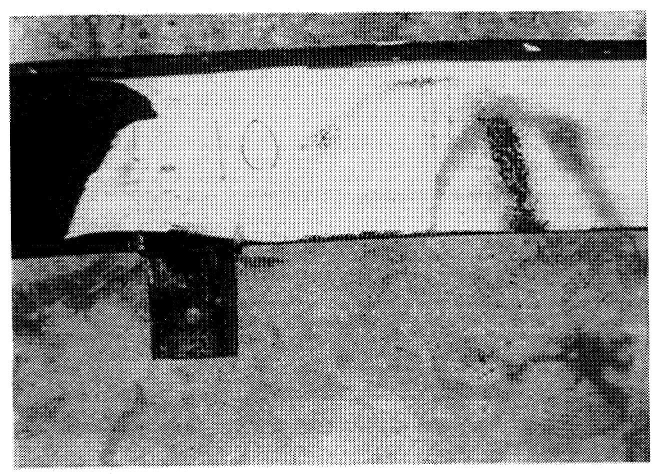

Fig. 7. Local buckling of stem in negative bending.

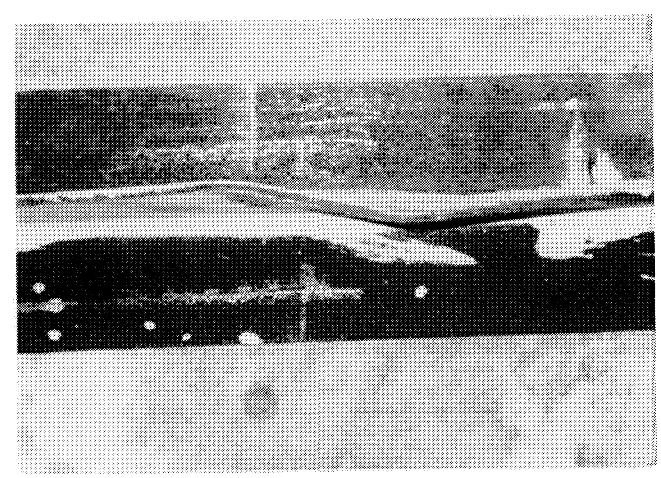

Fig. 8. Lateral displacement of stem in negative bending.

APPENDIX A—REFERENCES

1. *Load and Resistance Factor Design Manual of Steel Construction,* 1st ed., AISC, Chicago, 1986.
2. *Allowable Stress Design Manual of Steel Construction,* 9th ed., AISC, Chicago, 1989.
3. *Standard Specifications, Load Tables, and Weight Tables for Steel Joists and Joist Girders,* Steel Joist Institute, Myrtle Beach, SC, 1990.
4. Salmon, Charles G., and John E. Johnson, *Steel Structures, Design and Behavior,* 2nd ed., Harper and Row, New York, 1980., pp. 310–314, 318–319.
5. Galambos, Theodore V., ed., *Guide to Stability Design Criteria for Metal Structures,* 4th ed., Wiley, New York, 1988, p. 103.
6. Milek, William A., "One Engineer's Opinion," *Engineering Journal,* Vol. 2, No. 3, pp. 103–104.

APPENDIX B—SLENDERNESS LIMITS FOR STEMS OF TEES IN FLEXURAL COMPRESSION

DERIVATION

Both ASD and LRFD set the noncompact or λ_r limit for tee stems as:

$$d / t \leq 127 / \sqrt{F_y}$$

where:

d = section depth
t = stem thickness

This is based on the assumption of a uniformly compressed plate where one unloaded edge is fixed and the other unloaded edge is free. The limit is derived from classical plate buckling theory where k = 1.227 for this case of loads and fixity. See Ref. 4, pages 310–314 for the full background of this derivation.

This limit is not correct for a case where the stem is in flexural compression. Reference 5, page 103, gives a value of k = 1.61 for a case where flexural compression exists. This case assumes no compression at the fixed edge and full compression at the free edge as is shown in Fig. 11.

Starting with the classical plate buckling equation of:

$$F_{cr} = k\pi^2 E / [12(1 - M^2)(b / t)^2$$

AISC provisions require that the critical buckling stress (F_{cr}) be no less than the yield stress, F_y. Substituting in M = 0.3 for steel and E = 29,000 ksi yields:

$$(b / t) \leq 161 \sqrt{k / F_y}$$

This value must be reduced to account for residual stress, post-buckling effects, and imperfections (see Ref. 4). A reduction value of α = 0.7 is used. Therefore:

$$(b / t) \leq 161\alpha\sqrt{k / F_y} \text{ or}$$

$$(b / t) \leq 113\sqrt{k / F_y}$$

Substituting in k = 1.277 produces the limit of $127 / \sqrt{F_y}$ for uniform axial load.

Substituting in k = 1.61 produces a limit of $144 / \sqrt{F_y}$ for flexural compression.

Next, it needs to be determined what is the correct value of b to use. Using the full depth of the section is unnecessarily conservative. Considering that the elastic neutral axis for almost all WT sections is somewhere in the stem near the

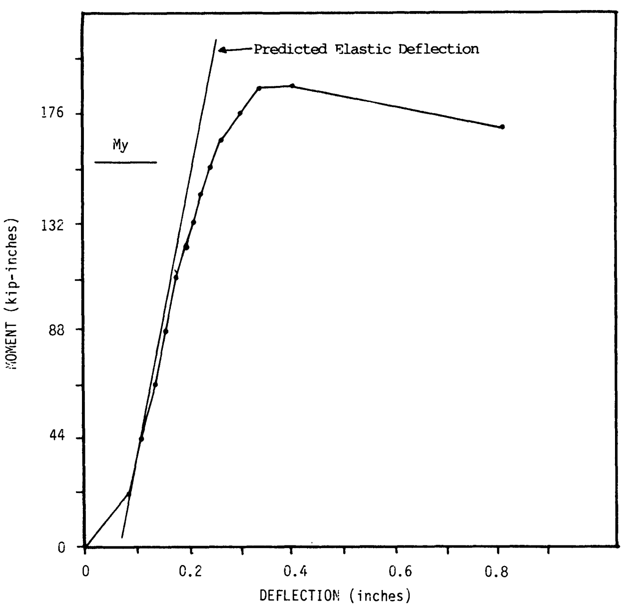

Fig. 9. Moment vs. deflection for Test 7.

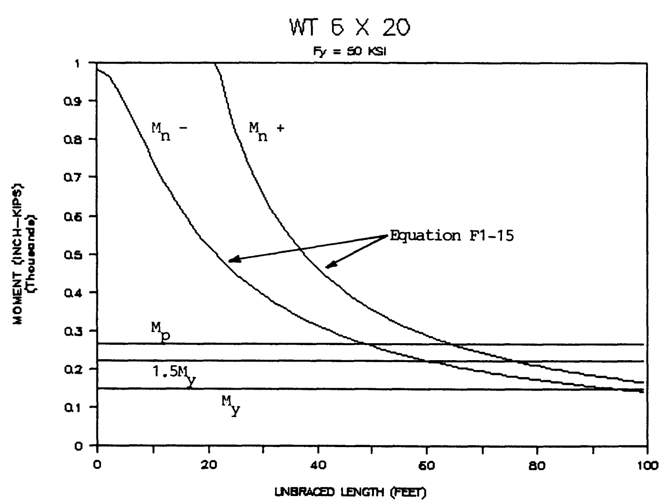

Fig. 10. Flexural strength of WT6×20, F_y = 50 ksi.

Table 5. Elastic Lateral-Torsional Buckling Stress by LRFD F1-15 for various Unbraced Lengths. Only Sections where F_y < 50 ksi at L = 25D Listed.				
	Elastic Lateral Buckling Stress for Negative Bending at Various Unbraced Lengths, ksi			
Section	L = 10D	L = 15D	L = 20D	L = 25D
WT 18.00 × 85.00	73.1	63.6	55.4	48.8
WT 18.00 × 80.00	64.6	56.7	49.7	44.0
WT 18.00 × 75.00	56.5	50.0	44.1	39.2
WT 18.00 × 67.50	43.0	38.5	34.3	30.8
WT 16.50 × 70.50	69.0	60.7	53.5	47.4
WT 16.50 × 65.00	56.9	50.7	45.0	40.2
WT 16.50 × 59.00	45.1	40.6	36.4	32.7
WT 15.00 × 58.00	67.4	59.4	52.3	46.4
WT 15.00 × 54.00	56.3	50.1	44.4	39.6
WT 15.00 × 49.50	46.3	41.6	37.2	33.4
WT 13.50 × 47.00	68.0	60.7	54.0	48.3
WT 13.50 × 42.00	53.0	47.9	43.1	38.9
WT 12.00 × 34.00	55.7	50.4	45.4	40.9
WT 12.00 × 31.00	47.3	40.9	35.6	31.3
WT 12.00 × 27.50	37.4	32.9	28.9	25.6
WT 10.50 × 31.00	46.9	43.5	39.9	36.6
WT 10.50 × 28.50	71.6	61.2	52.8	46.1
WT 10.50 × 25.00	53.3	46.4	40.5	35.7
WT 10.50 × 22.00	40.9	36.2	31.9	28.4
WT 9.00 × 20.00	70.4	61.6	54.1	47.8
WT 9.00 × 17.50	49.7	44.3	39.4	35.2
WT 8.00 × 15.50	66.4	58.8	51.9	46.1
WT 8.00 × 13.00	44.8	40.4	36.3	32.7
WT 7.00 × 11.00	56.9	51.1	45.7	41.0
WT 6.00 × 9.50	67.1	58.3	50.7	44.6
WT 6.00 × 8.00	44.7	39.6	35.0	31.2
WT 6.00 × 7.00	35.2	31.6	28.3	25.3
WT 5.00 × 6.00	50.0	45.4	41.0	37.1
MT 7.00 × 9.00	33.2	28.9	25.3	22.3
MT 6.00 × 5.90	32.0	27.2	23.3	20.3
MT 5.00 × 4.50	41.1	34.6	29.6	25.6
MT 4.00 × 3.25	63.0	52.2	44.0	37.9

flange-stem juncture, a more realistic and still slightly conservative choice would be:

$$b = d - t_f$$

where:

t_f = flange thickness
d = section depth

LRFD Eqs. A-B5-5 and A-B5-6 would need to be modified for the new slenderness limit. They have been modified by proportionally shifting the limits and providing the same values of Q_s at each end of the new range as was previously provided for stems in axial compression. Reference 4, pages 318–319 show this methodology to be in keeping with AISC provisions.

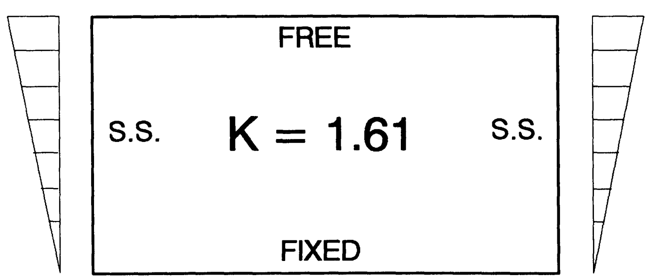

Fig. 11. Tee stem in flexural compression.

As modified they read:
For stems of tees in flexural compression:

When $144 / \sqrt{F_y} < b / t < 203 / \sqrt{F_y}$

$$Q_s = 1.908 - 0.00715(b / t)\sqrt{F_y}$$

When $b / t \geq 203 / \sqrt{F_y}$

$$Q_s = 26{,}780 / [F_y (b / t)^2]$$

The values of Q_s would be applied to the limiting moment, M_y, in the same method as for stems in axial compression.

EXAMPLE

Given:

$d = 6.17$ in.
$t_f = 0.440$ in.
$b_f = 6.52$ in.
$t_w = 0.260$ in.
$F_y = 36$ ksi

Required:

Calculate the slenderness limits for axial compression and for flexural compression using the proposed method.

Solution:

Axial compression:

$127 / \sqrt{F_y} = 21.17$

$d / t_w = 23.73$ therefore slender element

$Q_s = 1.908 - 0.00715(23.73)(6.0) = 0.891$

Proposed flexural compression:

$144 / \sqrt{F_y} = 24.00$

$(d - t_f) / t_w = 22.04$ therefore not a slender element