The Significance and Application of C_b in Beam Design

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INTRODUCTION

The basic provisions related to design and evaluation of bending members in the structural steel specifications, either according to Load and Resistance Factor Design (LRFD)[1] or Allowable Stress Design (ASD),[2] typically are first presented from the point of view that the magnitude of bending moment is constant throughout the entire distance between points of lateral support for the compression flange. Then, to account for variations in moment, one multiplies the expression associated with constant moment by a correction factor C_b to arrive at a result which predicts the actual bending strength (or allowable stress) for a specific moment gradient. What one accomplishes is to account for changes that occur in the force within the compression flange of the beam throughout the unbraced length.

A procedure for selecting beams in situations involving non-uniform moment is suggested within the prelude to the charts of design moments in the LRFD manual, but only in extremely brief fashion. The purpose of this paper is to review the principles associated with the application of C_b, and to elaborate on the procedure briefly suggested in the LRFD manual for selecting beams which experience non-uniform moment ($C_b \neq 1$).

BENDING STABILITY

Basic notions of column strength apply to stability-related issues in the strength of sections in bending. With a beam, however, only a portion of the cross section resists the compression. The key issues are still the restraint provided at the boundaries of the element resisting the compression and the distance between the locations of lateral support.

The magnitude of the compressive force within a beam cross section, which will nearly always vary with position along the span, may be determined by inspection of the moment diagram. Since resistance to bending is composed of the internal C (compressive force) and T (tensile force) couple, the magnitude of C at any location along a span equals the applied bending moment divided by the internal moment arm (Fig. 1). Thus, the variation in force within the compression flange has the same shape as the moment diagram.

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Analogies to Single Columns

Three single columns with different variations in axial load are shown in Fig. 2. Each column experiences the same axial compression (equal to the applied force P) within the uppermost segment. There is a difference, however, in the maximum force P that could be applied to each column because the magnitude of axial compression is reduced along the length of the columns in parts (b) and (c). Intuition dictates that the greatest load P may be sustained by the column in Fig. 2(c). By considering free-body diagrams at various positions along the length of the columns, one observes that substantial segments of columns (b) and (c) experience reduced compression, compared to column (a). Furthermore,

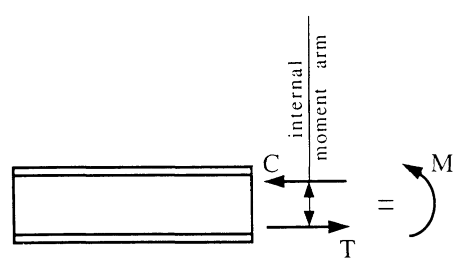

Fig. 1. Internal bending resistance.

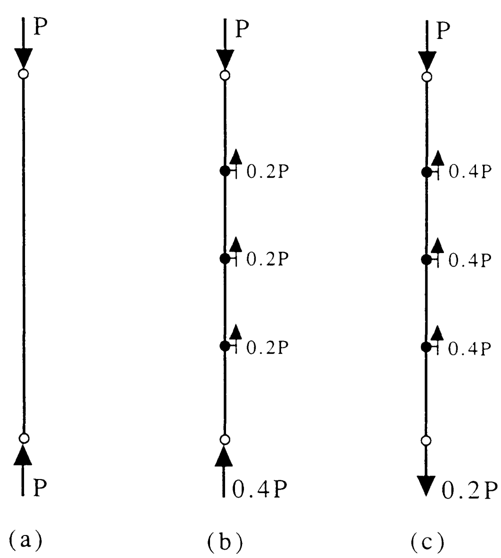

Fig. 2. Columns with varying axial compression.

(c) will compare favorably to (b) because the lowermost segments of (c) experience less compression and, notably, are in tension for the last 25 percent of the column.

The moment diagrams in parts (a), (b), and (c) of Fig. 3 may be associated with the individual columns in the respective parts of Fig. 2. The moment diagrams in Figs. 3(b) and 3(c) would exactly correspond to the respective columns in Fig. 2 only if the opposing mid-length forces were applied as a continuous distribution, but the basic analogy is the same. In part (b) of both Figs. 2 and 3, the force in the compression-carrying element is high at one end of the element and decreases by 60 percent at the other end. Similar behavior is demonstrated in part (c) of Figs. 2 and 3, but the change is more dramatic. At the far end, that part of the element which had been experiencing compression actually changes and becomes a tensile element.

In Figs. 2 and 3, for a given length of compression member, part (c) exhibits the least vulnerability to instability and may be assigned the greatest magnitude of compression/bending. The function of C_b is to take these aspects of behavior into account.

BASIC DESIGN EXPRESSIONS

As mentioned previously, design equations pertaining to beams are first developed from the standpoint of constant bending moment over the unbraced length. In the LRFD specification, bending strength is controlled by either of two equations, Eq. F1-3 or F1-13, depending on whether the yield stress will have appeared within the cross section at the instant a loss in load carrying capacity occurs. The presence of residual stress is taken into account. For the ASD specification, allowable bending stress is most frequently controlled by either Eq. F1-7 or Eq. F1-8, depending on whether lateral or torsional moment strength, respectively, is the more dominant component of bending strength for a given cross section at the instant that instability occurs.

Only the basic form (and not specific terms) of these design equations is relevant to discussing the significance of C_b. For LRFD the basic equation is

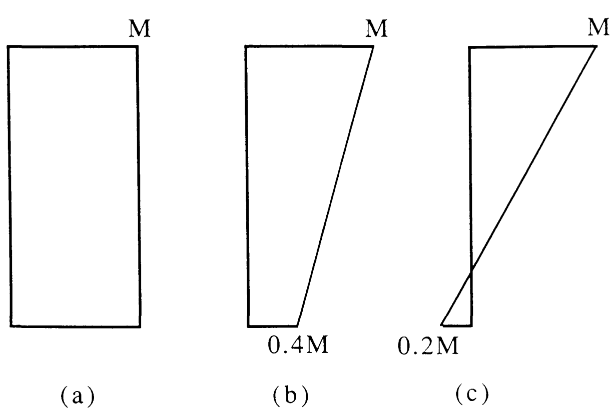

Fig. 3. Segments with varying bending moment.

$$M_n = C_b \, \{\text{LRFD Eq. F1-3 or F1-13, for constant moment}\}$$

$$\leq M_p \qquad (1)$$

and for ASD the following usually applies (unless Eq. F1-6 controls)

$$F_b = C_b \, \{\text{ASD Eq. F1-7 or F1-8, for constant moment}\}$$

$$\leq 0.6F_y \qquad (2)$$

where

$$C_b = 1.75 + 1.05\left(\pm\frac{M_1}{M_2}\right) + 0.3\left(\frac{M_1}{M_2}\right)^2 \leq 2.3 \qquad (3)$$

In Eq. 3, the end moments are considered on an absolute value basis, with M_1 equal to the smaller of the moments at the ends of the unbraced length. Should the moment anywhere within the unbraced length exceed that which occurs at either end, the above expression is disregarded and C_b is assigned a value of one. The matter of the plus/minus sign is considered in the following.

When moment decreases from a value of any magnitude at one end of the unbraced length to zero at the other end, the ratio $M_1 / M_2 = 0$ and $C_b = 1.75$. (Thus, bending strength [or allowable stress] is 75 percent greater than that which could have been achieved had the moment been uniform over the unbraced length, but limited to a result that does not exceed M_p [LRFD] or $0.6F_y$ [ASD].) When the smaller end moment is non-zero, one must decide on a proper sign for the second term in Eq. 3. Situations that provide increased strength compared to zero moment at one end must lead to $C_b > 1.75$, and situations with less strength (more closely resembling uniform moment) should reduce C_b below 1.75, back toward 1.00. Recalling previous discussion associated with Figs. 2 and 3, the decision regarding the proper sign is very straightforward.

For situations of single curvature (moment diagram on only one side of the baseline for the entire unbraced length), the same flange is always in compression. If there is little change in moment over the unbraced length, the compressive force in the flange will be maintained at a fairly constant level. Such a condition is more susceptible to instability, and bending strength should not be increased appreciably from that exhibited when moment is constant. Thus, for single curvature, the sign of M_1 / M_2 is negative.

For double (reversed) curvature (moment diagram changes from one side of the baseline to the other), the flange that experiences compression eventually changes to tension. There is greater stability in such a situation, and the moment that may be applied at one end may be significantly increased from that which could be applied as constant moment (C_b = 1), and beyond that for zero moment at one end (C_b = 1.75). Thus, for double curvature, the sign of M_1 / M_2 is positive.

One may note the same expression is used for C_b in both the LRFD and ASD specifications. Since it is the purpose of this factor to reflect structural behavior and account for the shape of the moment diagram, it is expected that there would be no difference in C_b equations between the specifications.

BEAM DESIGN WHEN $C_b \neq 1$

Both the LRFD and ASD manuals contain a series of "beam curves" that provide invaluable assistance in selecting a section that is suitable for a given combination of bending moment and unbraced length. These curves, designated Beam Design Moments (LRFD) and Allowable Moments in Beams (ASD), apply directly when moment is constant throughout the unbraced length (C_b = 1). The curves can also provide significant assistance when $C_b \neq 1$, after properly accounting for increased strength resulting from non-uniform moment.

ASD Procedures

Designers have extensive experience with the beam curves in the ASD manual and have developed a methodology for selecting beams for situations of non-uniform moment that is consistent with the design equations governing allowable stress. Observing ASD Eq. F1-8 (which usually controls allowable stress, especially for moderate-to-large unbraced lengths L_b)

$$F_b = \frac{12 \times 10^3 C_b}{L_b \dfrac{d}{A_f}} \leq 0.60 F_y \tag{4}$$

it is appropriate to use L_b / C_b and the applied service moment as an entry point to the curves because F_b is linear in that parameter. Should one anticipate that Eq. F1-6 will apply, experience has shown that $L_b / \sqrt{C_b}$ and applied service moment are an appropriate entry point in the curves for finding an acceptable section. These well-established procedures for allowable stress design are illustrated in steel design textbooks (for example, Salmon and Johnson[3]) and require no further treatment here.

LRFD Procedures

For LRFD design, however, it is more appropriate to divide the required moment strength M_u by C_b (rather than dividing it into the unbraced length) and use L_b and M_u / C_b as the entry point in the beam curves. This procedure, suggested on page 3-56 of the LRFD manual, may be justified by observing the equation (LRFD F1-3) which usually governs for the most economical section

$$M_n = C_b \left[M_p - (M_p - M_r)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_p \tag{5}$$

One may divide both sides of the equation by C_b to obtain M_n / C_b, but there is no significance to the parameter L_b / C_b. A typical plot of Eq. 5 with C_b = 1 is shown in Fig. 4. M_p is the fully plastic moment, L_p is the maximum unbraced length at which that moment may be sustained, M_r is the level of bending moment at which yield stress in the flange tips will first appear (taking into account the presence of residual stress), and L_r is the maximum unbraced length at which the latter moment may be sustained. Eq. 5, then, provides a linear interpolation between M_p and M_r for all unbraced lengths exceeding L_p but less than L_r.

It is very important to note that design situations do not always allow one to take full advantage of the computed value of C_b. Although one may multiply the result obtained within the brackets in Eq. 5 by C_b, it is not permissible to assign a nominal strength any higher than the fully plastic moment. Thus, to ensure that an acceptable section has been located when entering the curves with L_b and M_u / C_b, one must verify that the intended section is one that also results in $\phi_b M_p$ exceeding the required strength from factored loads, M_u. Only then will an adequate section have been selected. Example designs are provided in the following, including remarks in Example 3 which provides clarification of errors contained within a design example in the LRFD manual.

Example 1

Given:

For a required design moment of M_u = 500 kip-ft, L_b = 20 ft and C_b = 1.17, use the LRFD beam curves (Beam Design Moments) to select the lightest section of F_y = 36 ksi steel (see Fig. 5).

Solution:

The entry point for the beam curves on page 3-66 of the LRFD Manual is L_b = 20 ft and M_u / C_b = 500 / 1.17 = 427 kip-ft. A W24×84 is found to be the lightest section, providing M_u of 444 kip-ft at L_b = 20 ft, when C_b = 1. Since C_b equals 1.17, the design strength of this section actually.is 444 × 1.17 = 519 kip-ft (> 500 kip-ft required, **o.k.**) as long as $\phi_b M_p \geq$ 500 kip-ft. By inspection of page 3-66 one may note the $\phi_b M_p$ value for a W24×84 exceeds 500 kip-ft (605 kip-ft, actual value),

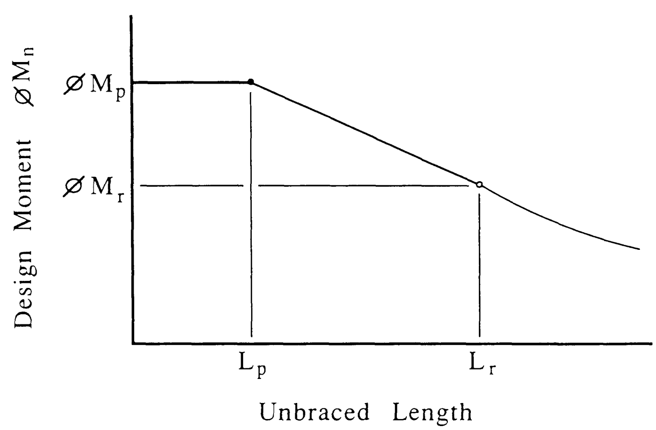

Fig. 4. Design moment strength vs. unbraced length.

verifying that the section is acceptable. Use a W24×84, F_y = 36 ksi.

Example 2

Given:

Repeat Example 1, changing the value of C_b from 1.17 to 1.75 (see Fig. 6).

Solution:

The entry point for the beam curves on page 3-70 of the LRFD manual is L_b = 20 ft and $M_u / C_b = 500 / 1.75 = 286$ kip-ft. Any section listed above and to the right of this entry point is acceptable, if it additionally satisfies $\phi_b M_p \geq 500$ kip-ft. The first two beams encountered are a W16×67 and W21×68, but these sections are unacceptable because they have maximum design strength $\phi_b M_p$ equal to 351 and 432 kip-ft, respectively, and 500 kip-ft is required. Having used M_u / C_b as the effective moment for the entry point, one must not forget that it frequently is not possible to take full advantage of C_b. The problem one encounters is illustrated for the W21×68 in Fig. 7. With C_b = 1 and L_b = 20 ft, the design strength of the beam is 303 kip-ft (page 3-68 of LRFD manual), and 303 × 1.75 > 500. The maximum design strength ($\phi_b M_p$) of the section is only 432 kip-ft, however, and using the full value of C_b elevates the design strength of the W21×68 to an unattainable level.

To find an acceptable beam, though, one need only continue to move straight up along the line L_b = 20 ft, discarding several unacceptable sections (W24×68, W21×73, W16×77, and W18×76), until eventually reaching a W24×76. It provides ϕM_n of 386 kip-ft at L_b = 540 kip-ft, which is greater than the 500 kip-ft required. (Although C_b has a value of 1.75 for the actual moment gradient, the maximum effective value is only 540 / 386 = 1.40.) Use a W24×76, F_y = 36 ksi.

WHEN $C_b > M_p / M_r$

After examining a moderate number of design problems, one concludes that the lightest beam is often a section for which the constant-moment design strength being provided ($\phi_b M_n$ when C_b = 1) exceeds M_r , or, stated differently, $L_b \leq L_r$. The implication of this in design situations is that it may be unnecessary to examine the beam curves whenever $C_b > M_p / M_r$. One need only refer to the tabulation of ascending/descending $\phi_b M_p$ values in the Load Factor Design Selection Table (LRFD manual, pages 3-13–3-17) to find the lightest beam with strength exceeding the required design strength (a procedure recommended in Ref. 3, as well). To be sure the beam is acceptable, though, one must also verify that the tabulated L_r is greater than the unbraced length. (In cases of large C_b, should it happen that the lightest beam from the selection table is one for which $L_r < L_b$ by only a small amount, the beam may still be acceptable. The section will have to be verified, however, either by checking that C_b times the value from the beam curves [with C_b = 1] is greater than the required design strength, or by computing the strength according to LRFD Eq. F1-13. See Example 5.)

Beginning with the expressions for M_p and M_r, and noting that the shape factor (Z / S) for rolled steel beams is approximately 1.12, one may compute

$$\frac{M_p}{M_r} = \frac{F_y Z}{(F_y - F_r)S} = 1.12\left(\frac{F_y}{(F_y - F_r)}\right) \quad (6)$$

where F_r = 10 ksi residual stress for rolled shapes, and then substituting for F_y one obtains M_p / M_r = 1.55 and 1.38 for F_y = 36 and 50 ksi steels, respectively. Then, for example, for situations with F_y = 36 ksi and C_b > 1.55, one may proceed directly to the Load Factor Design Selection Table to select the lightest beam, simultaneously checking that $L_b \leq L_r$. Had one followed this procedure in Example 2 (C_b = 1.75, and > 1.55), a W24×76 (L_r > 20 ft) would be selected very quickly. The advantage of using this table, compared to the procedure previously illustrated, is that no time would be spent in discarding all the unacceptable sections with maximum design strength less than the applied moment due to factored loads.

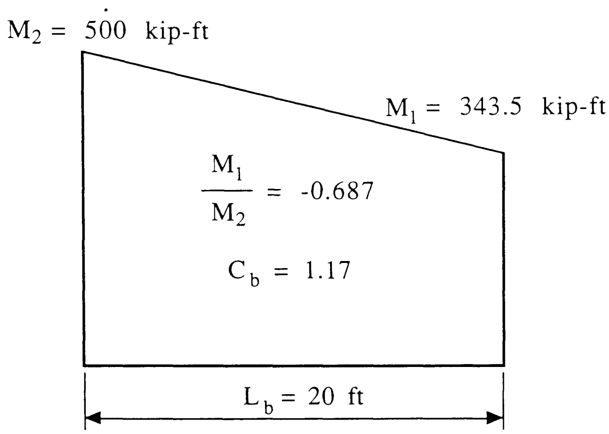

Fig. 5. Moment diagram for Example 1.

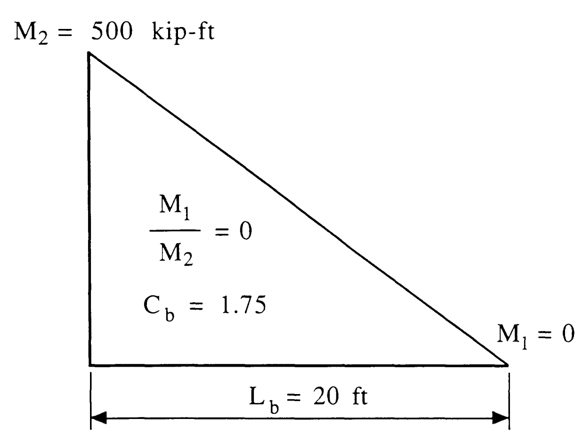

Fig. 6. Moment diagram for Example 2.

Example 3

Given:

Select the lightest section for the situation described in the example problem appearing on page 3-56 of the LRFD manual. The required design moment M_u = 352 kip-ft, L_b = 15 ft, and C_b = 1.75 (see Fig. 8).

Solution:

With C_b > 1.55 (M_p / M_r for 36 ksi steel), one may proceed directly to the Load Factor Design Selection Table, searching for $\phi_b M_p$ > 352 kip-ft and L_r > 15 ft. Use a W24×55, F_y = 36 ksi ($\phi_b M_p$ > 352 kip-ft and L_r = 16.6 ft).

If one elects to use the beam curves on page 3-70 of the LRFD manual, the entry point is L_b = 15 ft and M_u / C_b = 352 / 1.75 = 201 kip-ft. Similar to the situation illustrated in Example 2, several beams must be discarded because their maximum design strength $\phi_b M_p$ is less than M_u = 352 kip-ft. In the example in the LRFD manual, although inadequate sections are appropriately disregarded, the lightest section was not selected. A W21×62 was selected instead of a W24×55. While sorting through the various beams that must be discarded when using an effective moment (M_u / C_b) obtained with a large C_b, it is easy enough to overlook a section that is actually satisfactory. On the other hand, by using the selection table, as illustrated previously within this example, the path to finding the W24×55 is very direct. (Incidentally, on page 3-70 at L_b = 15 ft and C_b = 1, ϕM_n for a W24×55 is 242 kip-ft, and 1.75 ×242 > 352 kip-ft, **o.k.**)

A comment is in order regarding a statement appearing in the example in the LRFD manual. With regard to using an entry point based on an effective moment, the example states, "Any beam listed above and to the right of the point satisfies the design moment." It should read, instead, "Any beam listed above and to the right of the point, *and with* $\phi_b M_p > M_u$, *will have a strength which exceeds* the design moment." Although the example acted in accordance with the latter version of the statement, it is misleading to have it printed according to the former.

Another clarification is appropriate for the example in the LRFD manual, related to the sentence, "The design moment for a W21×62 with an unbraced length of 15 ft is 314 kip-ft and $\phi_b M_p$ is 389 kip-ft." Following "314 kip-ft" one should insert "(if C_b were 1)." With C_b equal to anything greater than 389 / 314 = 1.24, and L_b = 15 ft, the design moment for a W21×62 is 389 kip-ft.

Example 4

Given:

Select the lightest beam of F_y = 36 ksi steel for a required design moment of M_u = 406 kip-ft, L_b = 16 ft, and C_b = 1.75 (see Fig. 9).

Solution:

With C_b > 1.55 and 36 ksi steel, one may proceed directly to the Load Factor Design Selection Table, searching for $\phi_b M_p \geq$ 406 kip-ft and $L_r \geq$ 16 ft. One obtains a W24×62, with $\phi_b M_p$ = 413 kip-ft and L_r = 17.2 ft.

Alternatively, electing to use the beam curves, one enters

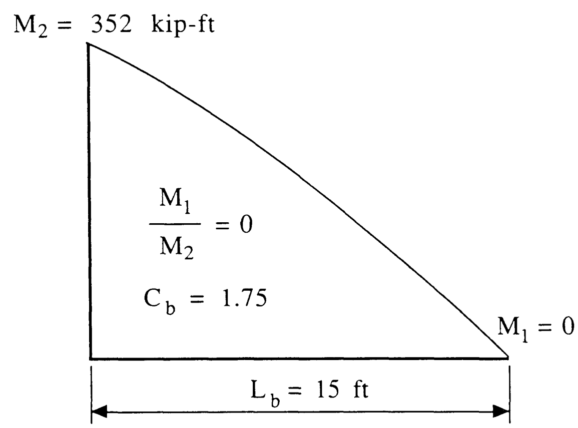

Fig. 8. Moment diagram for Example 3.

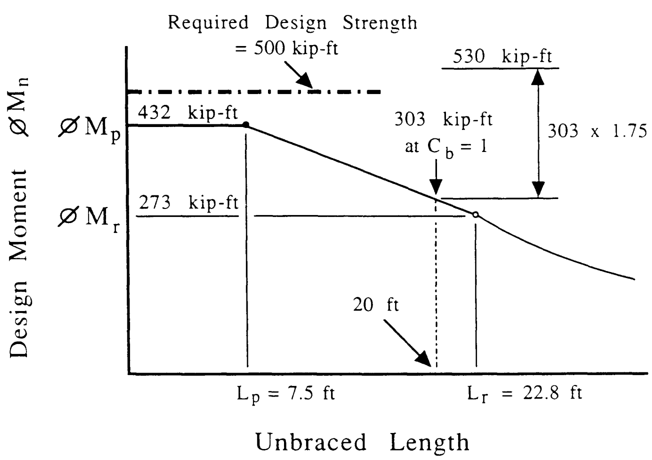

Fig. 7. Bending strength for W21×68 in Example 2.

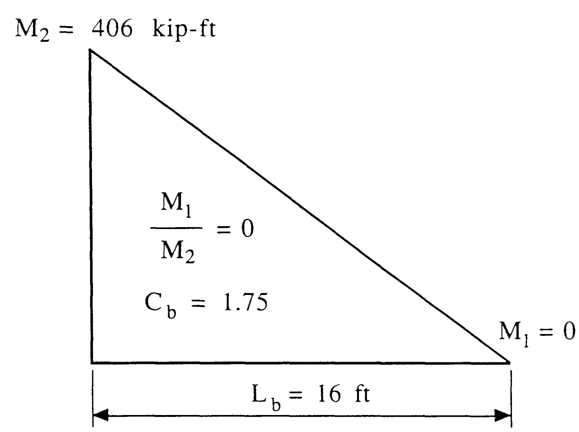

Fig. 9. Moment diagram for Example 4.

page 3-70 in the LRFD manual with L_b = 16 ft and M_u / C_b = 406 / 1.75 = 232 kip-ft. Checking all sections drawn with a solid line (to designate the lightest section available in a particular range of moments) while progressing upward along the line L_b = 16 ft, W18×55, W18×60, and W21×62 must all be disqualified because these sections have maximum design strength $\phi_b M_p$ < 406 kip-ft. The first satisfactory solid line beam encountered is a W21×68. A W24×62 is a lighter beam, though, as determined earlier in this example. This illustrates that when using an entry point based on an effective moment (with $C_b \neq 1$), beams designated with dashed lines in the beam curves cannot be overlooked as candidates for the lightest section. Use a W24×62, F_y = 36 ksi.

Example 5

Given:

Repeat Example 4, increasing the unbraced length from 16 to 18 ft (M_u and C_b still equal 406 kip-ft and 1.75, respectively).

Solution:

Now, when finding the W24×62 in the selection table, one observes that the unbraced length of 18 ft exceeds L_r = 17.2 ft. The design strength provided, which will be less than $1.75\phi M_r$, may still be acceptable, but it has to be verified because $L_b > L_r$. From the beam curves, for L_b = 18 ft and C_b = 1, $\phi_b M_n$ = 239 kip-ft. Accounting for C_b, 1.75 × 239 = 418 kip-ft, but the limit = ϕM_p = 413 kip-ft. This exceeds the required design strength of 406 kip-ft, so the W24×62 is still acceptable at L_b = 18 ft.

SUMMARY OF LRFD DESIGN PROCEDURE FOR $C_b \neq 1$

1. When $C_b \geq M_p / M_r$ (1.55 and 1.38 for F_y = 36 and 50 ksi, respectively), use the Load Factor Design Selection Table for beam design, searching for sections satisfying $\phi M_p \geq$ required design strength and $L_r \geq$ unbraced length.
2. For design situations where $C_b < M_p / M_r$, or L_b is greater than many of the L_r values tabulated in the selection table, enter the beam curves (Beam Design Moments section beginning on page 3-57 in the LRFD manual) with L_b and an effective moment equal to M_u / C_b. Any beam above and to the right of the entry point will be acceptable, provided it satisfies $\phi_b M_p > M_u$.

REFERENCES

1. American Institute of Steel Construction, "Load and Resistance Factor Design Specification for Structural Steel Buildings (September 1, 1986)," *Manual of Steel Construction: Load and Resistance Factor Design,* 1st ed., Chapter 6, 1986.
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3. Salmon, Charles G. and Johnson, John E., *Steel Structures: Design and Behavior,* Harper and Row, New York, 1990, p. 562.