

A Plastic Method For Unbraced Frame Design

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FREQUENTLY, because of architectural or functional requirements, no structural bracing can be permitted in the plane of a multi-story frame. Furthermore, with the trend in modern building design towards light curtain wall construction, larger window areas, and removable interior partitions, the bracing effect provided by cladding is often small or unreliable. Because of this situation the designer is faced with the design of an unbraced multi-story frame in which the bare structural skeleton of columns and girders must resist all applied gravity and lateral loads.

Certain initial steps in the design of unbraced frames are identical to the same steps in braced frame design.¹ For example, the dimensional layouts in both designs, being based on architectural and functional requirements, will likely be similar. The assignment of load systems and their distribution to the plane frame will be the same. The working loads will also be increased by the same load factors in each design.

However, some of the design problems are unique to the design of unbraced frames. There is a greater dependence on the interaction between columns and girders to create structural integrity. In addition, there is a concern for the sway deflections which can occur and for the secondary overturning effects due to column thrusts acting in displaced positions resulting from the sway deflections. The problem of frame instability under combined gravity and lateral wind loads also becomes of primary importance.^{2, 3, 6}

A practical design method which considers these problems is discussed in this paper.^{4, 5} It is applicable to the design of unbraced multi-story frames which are subjected to combined gravity and lateral loads and to those stories of multi-story frames where the design will be controlled by the combined loading. The design method starts with columns and girders selected initially from a previous preliminary design of the frame.^{1, 7, 8}

Through application of an analysis procedure to be described in this paper, the complete lateral load versus sway deflection behavior of each story can be determined. The preliminary member sizes will then be adequate if the load-deflection behavior of each story is satisfactory. If a revision is required, the analysis will assist in the selection of revised members. A desirable feature of the design method is that any story can be designed separately without a previous design for the rest of the frame. Consequently, a first analysis may be made at widely separated stories to obtain the approximate adequacy of the preliminary member sizes, then a story-by-story design can be made when it appears that only minor adjustments to member sizes will be necessary.

Before discussing the design method further it will be worth-while to review briefly the load-deflection behavior of unbraced frames and a few of the more well-known theories that predict this behavior.⁹

LOAD-DEFLECTION BEHAVIOR

Unbraced frames may have two basic modes of failure. When the frame is symmetrical and is under gravity loads only, it may buckle, as shown in Fig. 1a. That is, the frame remains vertical until a certain critical value of the load is reached, and then it deflects laterally without further increase in load. This mode of failure is likely to be confined to the top few stories of an unbraced frame where gravity loading governs the design.^{10, 11}

When combined gravity and lateral loads are applied, the frame will fail by instability. This is shown in Fig. 1b. This mode of failure is characterized by a gradually increasing lateral deflection, first under an increasing load and later, after the load reaches a maximum, under a diminishing load. The portions of an unbraced frame below the top few stories, in a region where the design is controlled by the combined loading condition, will exhibit this type of behavior. It is this load-deflection behavior which is of interest in this paper, and which the method discussed in this paper attempts to predict.

There are a number of analytical theories which attempt to approximate the load-deflection behavior

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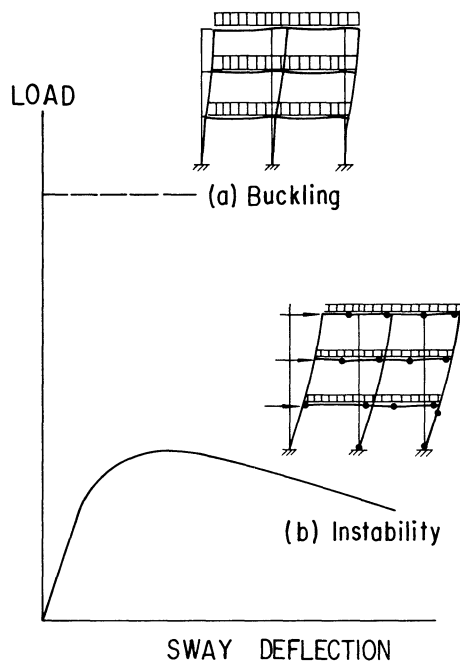


Fig. 1. Buckling and instability modes of frame behavior

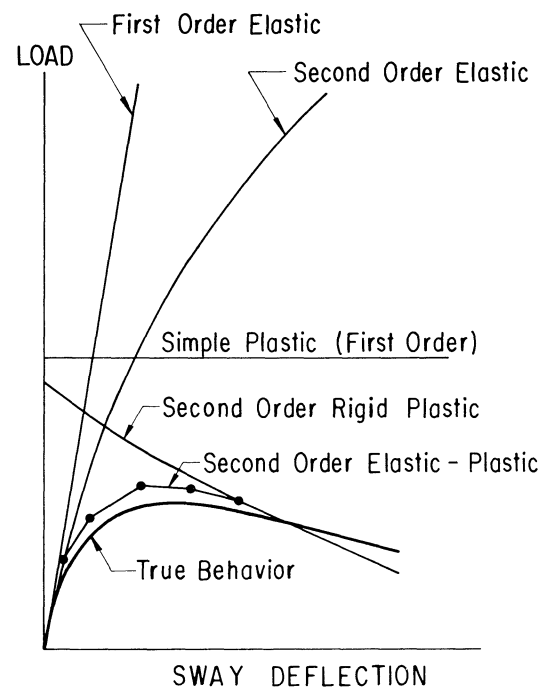


Fig. 2. Load deflection relationships (proportional loading)

of frames under proportional loading, that is, a combined loading in which the gravity and lateral loads increase proportionately. The approximations are shown in Fig. 2.

1. *First order elastic*—The material is assumed to be infinitely elastic and equilibrium is formulated on the undeformed structure. The behavior is linear and neglects the second order overturning moments created by axial thrusts acting through sway displacements.
2. *Second order elastic*—This theory differs from the first one by inclusion of the second order effects.
3. *Simple plastic*—This is a first order rigid-plastic theory. A mechanism forms at the ultimate load and the structure is assumed to be in equilibrium under this load for any magnitude of sway deflection.
4. *Second order rigid-plastic*—This theory differs from the previous one by including the reduction of plastic moment due to axial loads in the columns and by including second order effects.
5. *Second order elastic-plastic*—In this approach the second order elastic analysis is used between the formation of the individual plastic hinges. After the last plastic hinge has formed, this curve coincides with the second order rigid-plastic curve.

Many refinements to the methods of obtaining these load-deflection curves are possible, such as including strain hardening, residual stresses and strain reversal.¹² However, the calculations become very involved as the true behavior is more closely approximated, making

it necessary to resort to electronic computer solutions for even the simplest one- and two-story frames. Although a computer analysis has been prepared at Lehigh University which will determine the second order elastic-plastic load-deflection curve for each story of an unbraced multi-story frame of considerable height, it is highly complex and at present of more value to the researcher than to the designer.¹³ It was the desire to provide the designer with a practical, easy to apply method of designing unbraced multi-story frames which lead to the development of the sway subassembly method, which will now be discussed.

SWAY SUBASSEMBLAGE METHOD

Real frames will in general be subjected to non-proportional loading. That is, the gravity loads will remain virtually constant as the lateral wind loads are applied. The resulting load-deflection curve of the frame will be somewhat different from that obtained from considering proportional loading. However, the curves will exhibit the same shape. Before further discussion of the design method, it will be worth-while then to briefly examine the behavior of an unbraced multi-story frame under gradually increasing lateral loads while the factored gravity loads remain constant.

A three-bay unbraced multi-story frame which is subjected to combined loading is shown in Fig. 3a. Figure 3b shows the bending moment diagrams for the members in the vicinity of floor level n when the frame is subjected to the factored gravity loads only. The initial

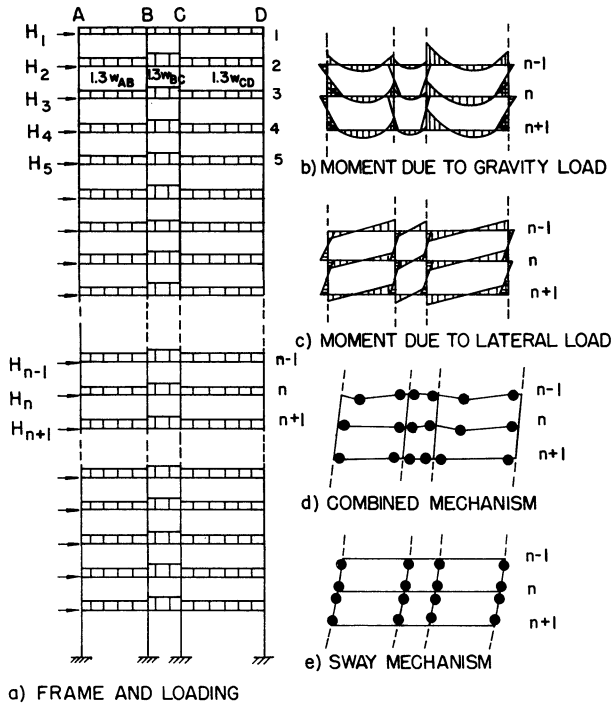


Fig. 3. Possible bending moments and failure mechanisms for a multi-story frame

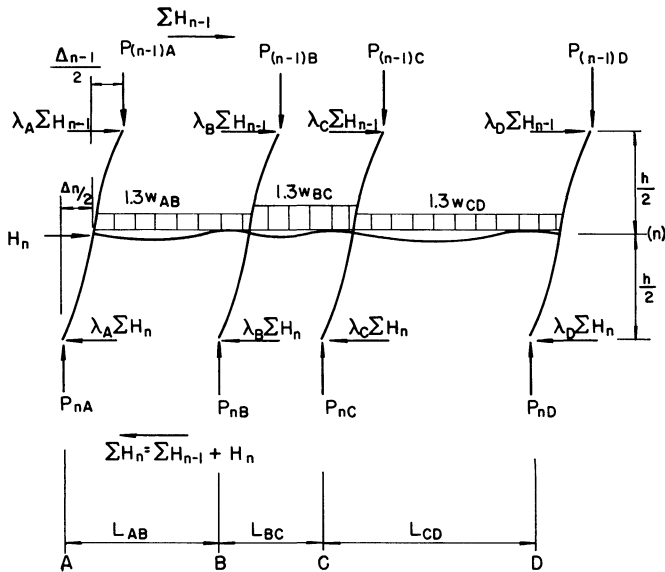


Fig. 4. Columns and girders at level n

application of the lateral loads introduces additional bending moments such as those shown in Fig. 3c. When these two bending moment diagrams are combined, the bending moments at the leeward ends of the girders will increase while those at the windward ends will decrease.

The leeward ends of the girders are therefore the potential locations for the first plastic hinges which form. Similarly the bending moments at the ends of certain columns increase during initial application of the lateral loads, making it possible for the first plastic hinges to also form at the tops and bottoms of these columns. As the lateral loads increase, additional plastic hinges form in other columns and in the girders until eventually one story or a portion of it becomes a mechanism.

Figures 3d and 3e show two possible mechanisms which can develop. In a weak-girder, strong-column design, plastic hinges are likely to form in the girders and the failure mechanism will be that of Fig. 3d.^{3, 14} If a strong-girder, weak-column approach is used in the design, the sway mechanism shown in Fig. 3e will be the probable mechanism.^{14, 15} The design method discussed in this paper is suitable for performing designs using either approach.

The appropriate sway subassemblages to be used in the design procedure can now be developed if we isolate the portion of the frame in the vicinity of level n by cutting through the columns above and below that level. The resulting frame is shown in Fig. 4. In addition we will make the following assumptions:

- Both stories above and below level n are of the same height.
- The point of inflection in each column above and below level n is at mid-height of the column.
- The distribution of the total shear to the columns above and below level n is the same.
- The column axial loads, P , remain constant during application of the lateral loads.

The first assumption is usually satisfied for most frames, and the second assumption becomes exact if both stories fail by sway mechanism. The third assumption is easily justified if the member stiffnesses in the two stories do not differ appreciably. Since the column axial loads actually vary during application of the lateral load, the fourth assumption is not fully justified. However, with a suitable distribution of axial loads, in equilibrium with the gravity loads, a safe design is possible.

Before reducing this frame further to the individual sway subassemblages, we will make the additional assumption that the magnitude of shear in each column above and below a joint, as well as the distribution of total shear to the columns, is such that the upper columns in Fig. 4 apply external moments to each joint which are equal in magnitude to the internal bending moments at the top of each column below the joint. With this assumption we can now reduce the frame to its individual sway subassemblages shown in Fig. 5. These are designated as the windward, interior or leeward sway subassemblages depending on their position in the windward, interior or leeward portions of the frame.

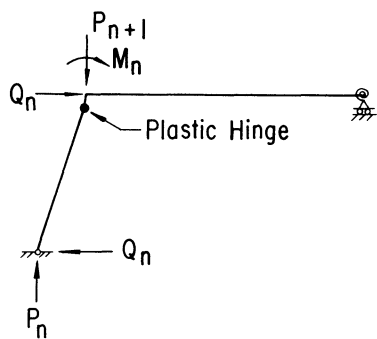


Fig. 7. A rigid-plastic mechanism for a windward sway subassembly

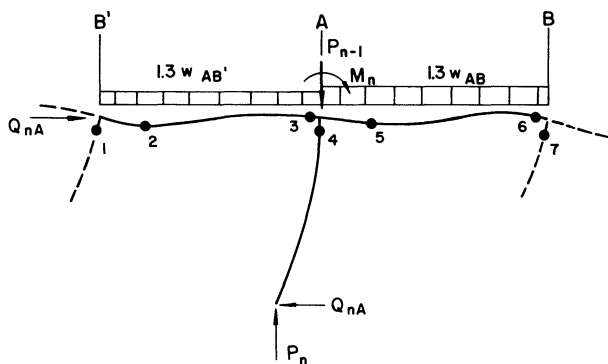


Fig. 8. Plastic hinges which influence restraining coefficients at Joint A

of plastic hinges in the columns and girders of the story, and do not remain constant as was assumed above in the discussion of the restrained column. Therefore the second solution to be obtained will be the moment-rotation characteristics of the real restraint provided by the actual girders at the column top. With initial sway, the restrained column will be subjected to a certain initial rotational restraint which will be a function of the elastic and/or inelastic properties of all the columns and girders in a story. As sway increases, the restraint gradually reduces to zero, because of the successive formation of plastic hinges in the story.

Figure 8 shows a typical interior sway subassembly. The rotational restraint to the far ends of the girders is provided by the columns and girders directly connected to those ends shown dashed. The plastic hinges which are considered effective in reducing the restraint at the top of the restrained column in the subassembly are shown numbered 1 through 7.

As pointed out earlier in this paper, plastic hinges marked 1, 3, 4, 6, and 7 will usually be the first plastic hinges to form and will occur at the ends of the members. Under certain conditions, plastic hinges 2 and 5 may form at the windward ends of the girders instead of somewhere

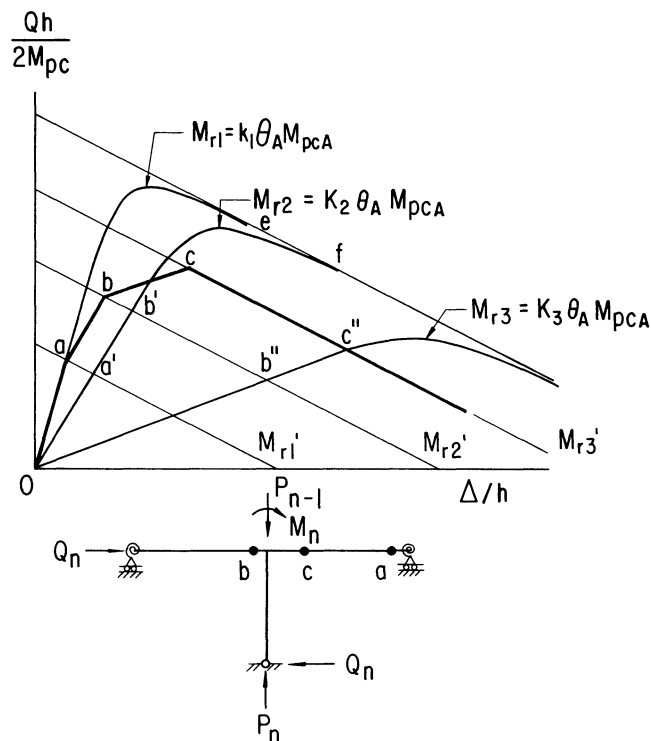


Fig. 9. Typical load-deflection curve for an interior sway subassembly

within the span. The exact position of these two plastic hinges will depend on the ratio of the plastic moment capacity of the girders to the bending moments produced by the gravity loads.¹ Although all of the plastic hinges shown are possible for an interior subassembly, only plastic hinges 4, 5, 6 and 7 can occur in a windward subassembly. Similarly, only plastic hinges 1, 2, 3 and 4 can occur in a leeward sway subassembly.

Because of the definition of a sway subassembly, the formation of plastic hinges 3, 4 and 5 are required to reduce the subassembly to a mechanism. Plastic hinges 1, 2, 6 and 7 will reduce the rotational restraint at the top of the restrained column in the sway subassembly, but their formation alone cannot reduce the sway subassembly to a mechanism.

Reference 4 discusses in detail the method of calculating the value of initial restraint at the column top, the reduction of initial restraint with the formation of plastic hinges, the corresponding joint rotation at the column top at each reduction, and the assumptions involved. With this information determined for each sway subassembly in a story, the load-deflection curve for each subassembly may be constructed with the aid of the prepared design charts mentioned earlier (see Fig. 10).

For example, referring to Figs. 6 and 8, if the initial restraining moment at Joint A is defined as M_{r1} , then this moment can be expressed as a function of the rotation

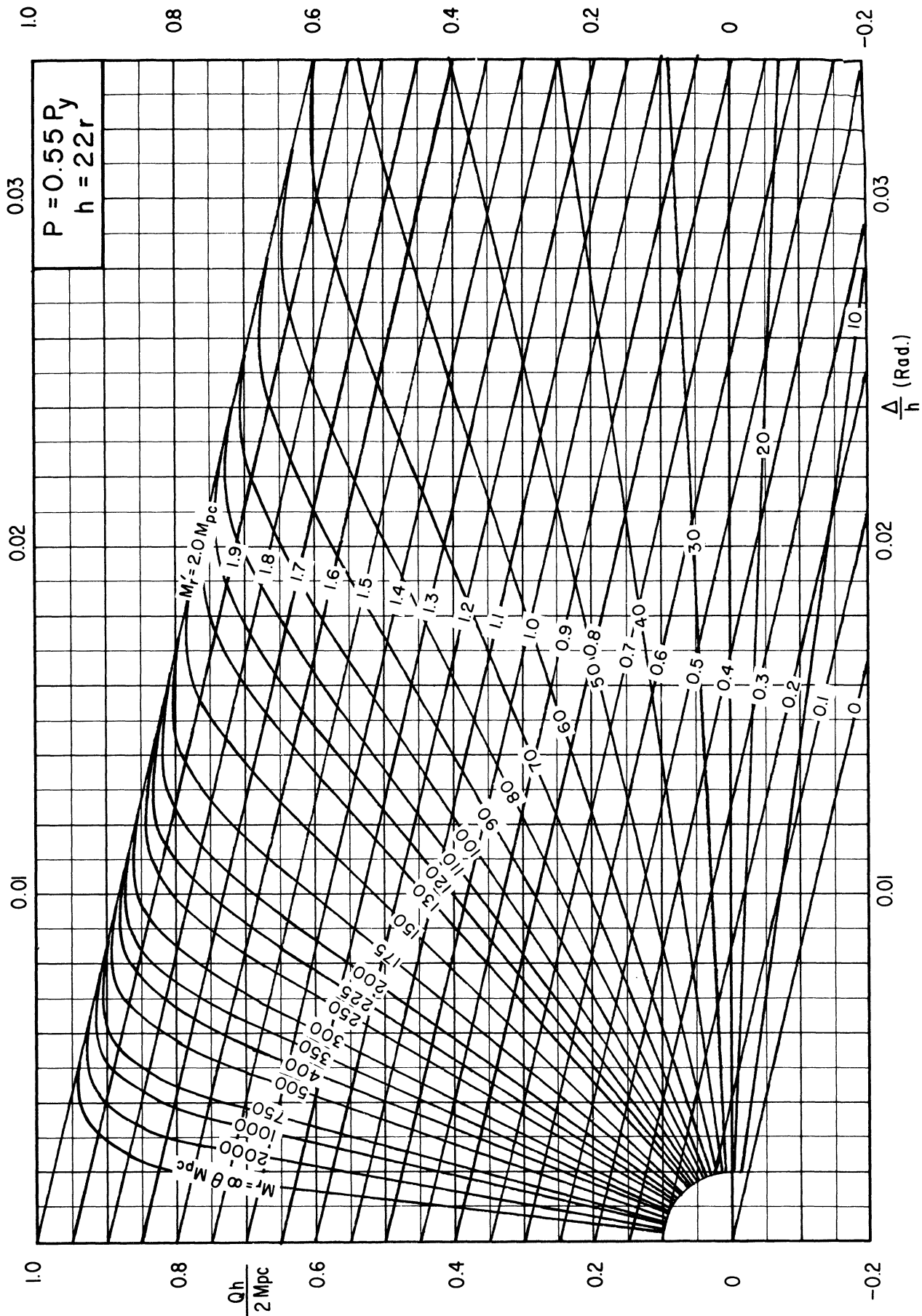


Fig. 10. Typical design chart ($P = 0.55 P_y$ and $h = 22r$)

of Joint **A**, θ_A and of the reduced plastic moment of the column at **A**, M_{pcA} , as follows:

$$M'_{r1} = k_1 \theta_A M_{pcA}$$

where $k_1 M_{pcA}$ is the initial restraint provided by the girders in the sway subassembly. If $k_1 M_{pcA}$ did not change with sway, the complete non-dimensionalized $Qh/2M_{pc}$ vs Δ/h load-deflection curve for the restrained column shown in Fig. 8 would be determined as curve **O-a-e** of Fig. 9. However, suppose a plastic hinge formed in the leeward girder at point **a**, as shown at the bottom of Fig. 9, then the joint rotation θ_A at the formation of this plastic hinge (calculated by the methods of Ref. 4) would be substituted into the previous equation for M'_{r1} to determine the value of restraining moment M'_{r1} at the formation of the first plastic hinge. This corresponds to point **a** on the load-deflection curve **O-a-e**. If a mechanism had formed with the first plastic hinge, the load-deflection curve would follow the straight line second order rigid-plastic mechanism curve from point **a** which is the rigid-plastic mechanism curve for the restrained column with a constant restraining moment M'_{r1} , and shown in Fig. 9.

However, in this example a mechanism does not form at this point. Instead, the restraint at the column top reduces to the value $k_2 M_{pcA}$. Assuming that this reduced value of restraint had existed from the beginning of sway, the complete load-deflection curve for the restrained column would have been as shown by curve **O-a'-b'-f** in Fig. 9. The actual load-deflection curve for the sway subassembly should then follow the curve **a-b** which is parallel to the segment **a'-b'**. If the second plastic hinge were to form at point **b** say, the restraint would further reduce to $k_3 M_{pcA}$. The subassembly curve would then follow the curve **b-c** which is parallel to the segment **b''-c''** corresponding to the restrained column curve **O-b''-c''** with column top restraint $k_3 M_{pcA}$. If the third plastic hinge were to form at point **c**, the subassembly would now be reduced to a mechanism, and the load-deflection curve would follow the second order rigid-plastic curve corresponding to the constant restraining moment M'_{r3} .

These non-dimensionalized load-deflection curves are constructed for each sway subassembly in the story with the aid of design charts.⁵ A typical design chart is shown in Fig. 10. Each chart is prepared for a given axial load ratio P/P_y and slenderness ratio h/r for the restrained column in the sway subassembly.

Figure 11 shows the method of obtaining the load-deflection curve for a story. Before combining the load-deflection curves of each sway subassembly, the ordinate to each curve is multiplied by the value of $2M_{pc}/h$ corresponding to that curve. The resulting Q vs Δ/h curves are then combined as shown in Fig. 11 to

determine the complete load-deflection curve of the story. The procedure at this stage is very similar to the combining process used in braced frame design to determine the moment-rotation curves for the subassemblages used there.

The sequence of formation of plastic hinges in the story (order of formation with increasing sway), maximum shear resistance, shear resistance at formation of a mechanism, deflection index Δ/h at working load, maximum load, mechanism load, etc., may all be obtained from this load-deflection curve. One or more of these may be used to determine the adequacy of the preliminary design of that story.

Reference 4 presents the detailed design of one story of a 24-story 3-bay frame. A summary of the results is presented as follows: Figure 12a shows a portion of the three-bay multi-story frame which is similar to the frame shown in Fig. 3a. The columns and girders at level 20 (20 floors from the top) have been selected from a preliminary design. Although wind from both left and right must be considered for a complete analysis, this example considers only wind from the left. The actual factored

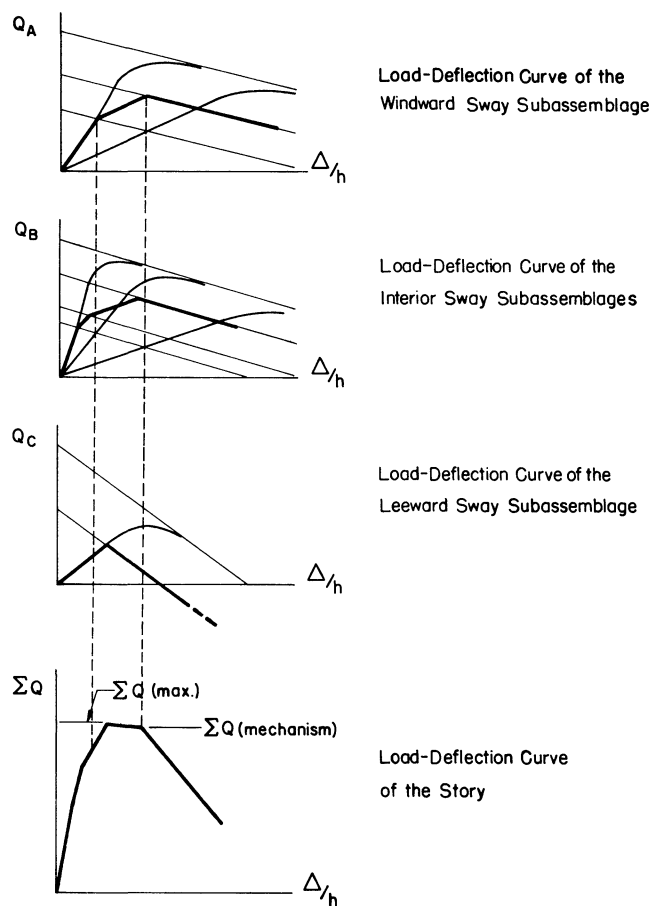


Fig. 11. Determination of the load-deflection curve of a story from the load-deflection curves of the sway subassemblages

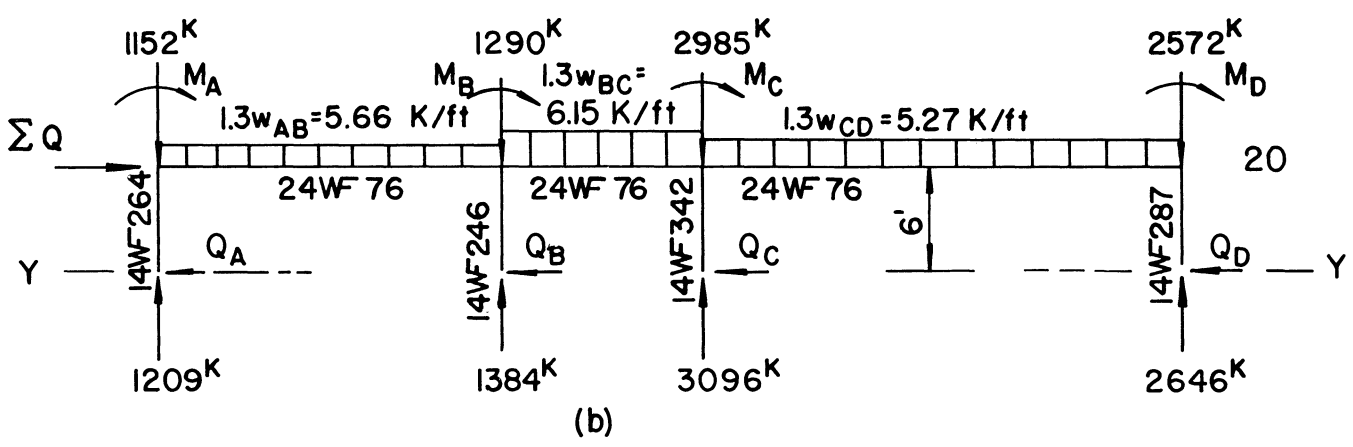
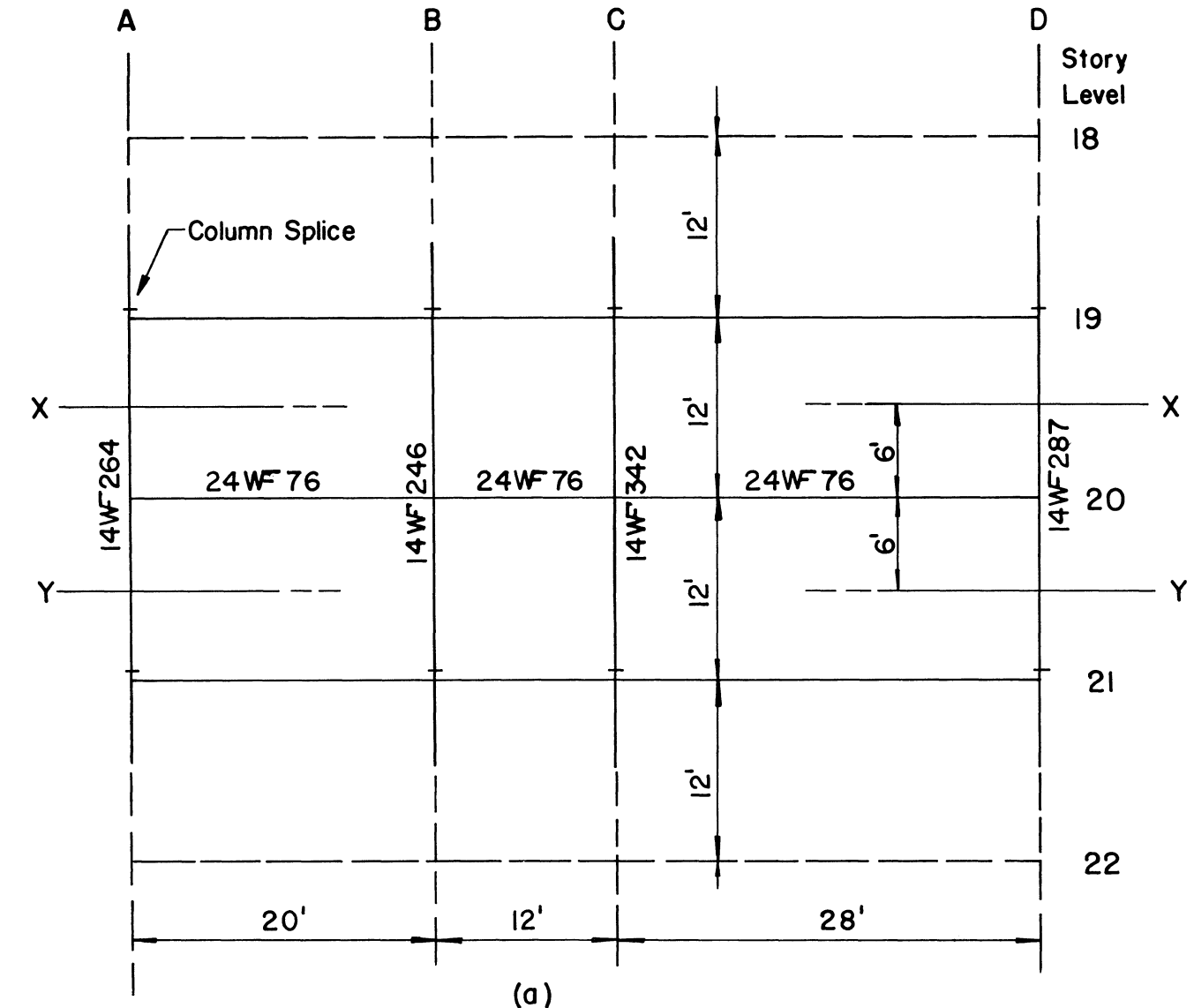


Fig. 12. Portion of a 24-story 3-bay frame designed in reference 4

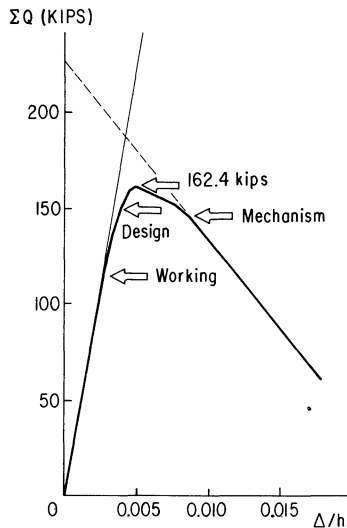


Fig. 13. Load-deflection curve for the portion of the frame designed in reference 4

wind shear between levels 20 and 21 is 148.5 kips (load factor = 1.30). The column axial loads, assumed constant, have been computed as the algebraic sum of the gravity loads on all the girders above level 21, plus the accumulated girder shears corresponding to a combined mechanism in all stories above level 20 under the overturning moment due to wind, plus secondary overturning moments due to axial force (assuming $\Delta/h = 0.020$ in each story at the formation of the combined mechanism).

Figure 13 shows the resulting load-deflection curve. The maximum shear resistance of the story is 162.4 kips which exceeds the factored wind shear of 148.5 kips. It will be noted that the maximum shear resistance occurs well before the point at which a mechanism occurs. At the peak, only four of the six plastic hinges required for a mechanism have formed. This is a typical load-deflection curve for the lower stories of multi-story frames, and illustrates the occurrence of instability of the frame prior to the formation of a mechanism.

The load-deflection curve up to the maximum load was also obtained by the computer analysis mentioned earlier.¹³ Very good agreement was found to exist up to the maximum load, between that load-deflection curve and the one shown in Fig. 13.

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REFERENCES

1. Driscoll, G. C., Jr. Plastic Design of Multi-Story Frames—Lecture Notes, *Lect. 1 & 16, Fritz Laboratory Report 273.20, Lehigh University, 1965.*
2. Yura, J. A. and Galambos, T. V. Strength of Single-Story Steel Frames, *Proc. ASCE Vol. 91 (ST5), p. 81, October, 1965.*
3. Holmes, M. and Gandhi, S. N. Ultimate Load Design of Tall Steel Building Frames Allowing for Instability, *Proc. Instn. Civil Engrs., Vol. 30, pp. 147-166, 1965.*
4. Daniels, J. H. and Lu, L. W. The Subassemblage Method of Designing Unbraced Multi-Story Frames, *Fritz Engineering Laboratory Report No. 273.37, Lehigh University, March, 1966.*
5. Daniels, J. H. and Lu, L. W. Design Charts for the Subassemblage Method of Designing Multi-Story Frames, *Fritz Engineering Laboratory Report, (In preparation).*
6. Horne, M. R. Instability and the Plastic Theory of Structures, *Trans. Eng. Inst. Canada, Vol. 4, No. 2, 1960.*
7. Murray, T. M. and Ostapenko, A. Optimum Design of Multi-Story Frames by Plastic Theory, *Fritz Engineering Laboratory Report No. 273.42, 1966.*
8. Hansell, W. C. Preliminary Design of Unbraced Multi-Story Frames, *Ph.D. Dissertation, Lehigh University, May, 1966.*
9. Ostapenko, A. Plastic Design of Multi-Story Frames—Lecture Notes, *Lecture 13, Fritz Engineering Laboratory Report No. 273.20, Lehigh University, 1965.*
10. Levi, V., Driscoll, G. C., Jr., and Lu, L. W. Structural Subassemblages Prevented from Sway, *Proc. ASCE Vol. 91 (ST5), p. 103, October, 1965.*
11. Levi, V. Multi-Story Frames II, in *Structural Steel Design, ed. L. Tall, Ronald Press, New York, 1964.*
12. Adams, P. F. Plastic Design in High Strength Steel, *Ph.D. Dissertation, Lehigh University, June, 1966.*
13. Parikh, B. P. Elastic-Plastic Analysis and Design of Multi-Story Steel Frames, *Ph.D. Dissertation, Lehigh University, June, 1966.*
14. Horne, M. R. Multi-Story Frames, *Symposium on the Plastic Theory of Structures, British Welding Journal, Vol. 3, No. 8, August, 1956.*
15. Heyman, J. An Approach to the Design of Tall Steel Buildings, *Proc. Instn. Civil Engineers. Vol. 17, pp. 431-454, 1960.*
16. Parikh, B. P., Daniels, J. H. and Lu, L. W. Plastic Design of Multi-Story Frames—Design Aids Booklet, *Fritz Engineering Laboratory Report No. 273.24, Lehigh University, 1965.*