Application of Second-Order Elastic Analysis in LRFD: Research to Practice

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INTRODUCTION

The AISC Load and Resistance Factor Design Specification,[1] states, "In structures designed on the basis of elastic analysis, M_u may be determined from a second-order elastic analysis using factored loads." At the present time, a designer who wishes to use this type of analysis has many choices of methods, all of which may be termed "second-order elastic." This paper compares and contrasts several of the current most commonly used methods (B_1/B_2 approaches and a number of P-Delta approaches) to matrix analysis approaches based on stability function and geometric stiffness formulations. The matrix approaches are comprehensive in that they account for both P-δ and P-Δ effects and place essentially no constraints on the manner in which the structure is modeled for analysis. As these approaches increasingly become available in commercial software, they will provide a powerful facility for the analysis and design of ordinary as well as irregular and complex three-dimensional structures.

In the last twenty to twenty-five years, a large amount of research has been devoted to the nonlinear elastic and inelastic analysis of frame structures. In parallel with the development of more sophisticated analysis methods, the speed, memory capacity, and advanced graphics capabilities of personal and workstation computers continue to increase each year. The capabilities of these new machines have made it possible to employ analysis and design techniques that in the recent past were generally impractical for most engineering firms. As an example, for certain types of building systems, it is now commonplace to perform a three-dimensional linear elastic analysis which accounts for the interaction between shear walls and various other types of structural framing. With each new generation of personal computers and workstations, there are fewer and fewer constraints on analysis technique and size of the analysis model.

At a research level, the current technology for direct analysis of second-order elastic effects is well developed. This technology provides advantages over hand methods which amplify the loads or member forces from linear elastic analysis, especially for the analysis and design of complex structural systems. Furthermore, the current AISC LRFD Specification[1] gives explicit permission for the engineer to compute load effects from a direct second-order elastic analysis. Second-order elastic analysis is the "preferred" method in the Canadian Limit States Design Specification.[2] However, this type of analysis has yet to gain wide acceptance among the design community, and to the knowledge of the authors, at most only a few of the current commercial programs provide a "comprehensive" second-order elastic analysis capability (i.e., one in which both member curvature, P-δ, and chord rotation, P-Δ, effects are accounted for in a general fashion). Conversely, research papers sometimes do not address important practical aspects of applying comprehensive procedures in an actual design setting.

The goals of this paper are: (1) to highlight the advantages and limitations of some of the more comprehensive approaches for second-order elastic analysis of frame structures, (2) to outline the type of analysis/design procedure perceived by the authors to be necessary for these types of analysis to be applied optimally in a practical design setting, and (3) to indicate the computer-aided design software capabilities needed to support this type of procedure. Of course, any type of second-order analysis involves more computational effort than conventional linear elastic analysis. However, the authors believe that comprehensive matrix analysis provides the greatest promise for achieving both simplicity and generality in the calculation of second-order forces for frame design. By providing a complete summary of what the use of comprehensive second-order elastic analysis might entail, the authors hope to demonstrate that comprehensive second-order elastic analysis can be as efficient, and in certain respects, simpler to perform than other more approximate second-order methods.

The authors also hope to stimulate discussion on how second-order elastic analysis methods may best be adapted to design practice and applied in the context of current and future design specifications. Capabilities for handling aspects such as flexible joints, semi-rigid connections, and structural walls should be included in any analysis/design software which claims to be comprehensive. The scope of this paper is limited to rigid-jointed frames with or without leaned columns in order to focus on the aspects of handling second-order effects in frame members.

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The next section provides basic definitions and fundamental background material on "state-of-the-art" matrix methods as well as conventional approaches for calculation of second-order elastic load effects. The third section of the paper attempts to summarize the advantages of comprehensive second-order elastic matrix analysis. The fourth and fifth sections address a number of analysis and design related issues which should be considered in the use of any type of second-order elastic analysis, and particularly for comprehensive matrix analysis approaches. The sixth section lists two possible step-by-step analysis/design procedures based on comprehensive second-order elastic analysis. Conclusions are provided in the final section.

FUNDAMENTAL BACKGROUND

Types of Analysis

To clarify the terms used, several types of analysis will be reviewed briefly. Figure 1 contains schematic representations of load-displacement diagrams obtained for a frame by each type. The elastic critical load is calculated by an eigenvalue analysis. This load may be used as a basis for calculating effective length factors as well as estimating moment amplification effects.[3,4] Linear elastic (first-order elastic) analysis is of course the current staple of the profession. One of its distinguishing characteristics is that equilibrium is formulated on the undeflected geometry. Although this type of analysis provides a simple estimate of the distribution of forces in the structural system, it does not provide any information on the strength or stability of the frame.

In geometric nonlinear or second-order elastic analysis, equilibrium is formulated on the deformed configuration of the structure. When derived on a consistent mechanics basis, this type of analysis includes both P-Δ (chord rotation) and P-δ (member curvature) effects. The P-Δ effect reduces the element flexural stiffness against sidesway. The P-δ effect reduces the element flexural stiffness in both sidesway and non-sidesway modes of deformation. These two effects are illustrated for an arbitrary beam-column subjected to sidesway in Fig. 2. Actually, few programs can model the P-δ effects precisely unless the members are subdivided into a number of elements (particularly if the members have some initial out-of-straightness). Second-order elastic analysis accounts for elastic stability effects, but it does not provide any direct information with regard to the actual inelastic strength and stability of the frame. Therefore, in any design based on this type of analysis, these aspects must be accounted for in the Specification equations for member proportioning.

Material nonlinear or first-order inelastic analysis includes the effects of member yielding in some way (typically by a plastic hinge type of model), and full nonlinear or second-order inelastic analysis includes both geometric and material nonlinear effects, as the name implies. Second-order inelastic analysis is the only method which attempts to be completely rigorous in the solution of the frame performance. However, since the technology for performing second-order inelastic analysis is not readily available for design office use at the present time (1990), the LRFD equations for beam-column design are based on the use of the maximum second-order elastic moments within the span of the member (including both P-Δ and P-δ effects).

A wide range of analysis approaches may be found in the literature which involve different simplifications of the structural response and account primarily for P-Δ effects. Many

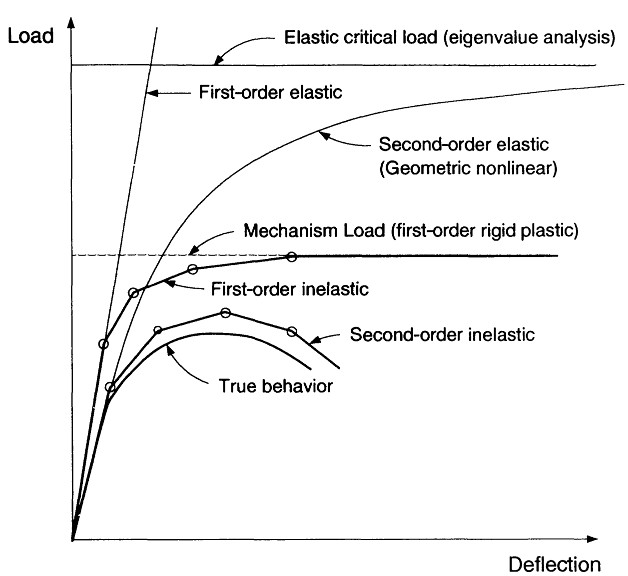


Fig. 1. Types of analysis.

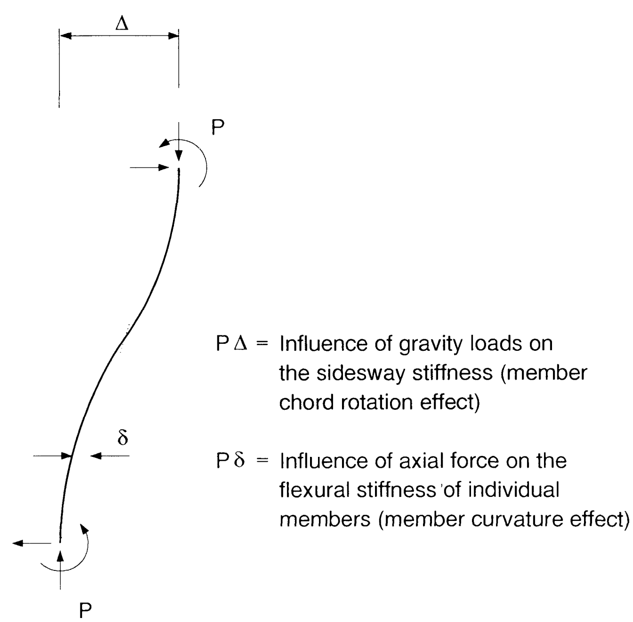


Fig. 2. Second-order effects in building frames.

of the P-Δ methods are geared toward use with linear elastic computer programs. In Ref. 5, Gaiotti and Smith review a number of these methods and categorize them in terms of their ease of use, the types of calculations involved (hand or computerized), and their range of applicability for different types of structures. In most practical designs, the P-δ effects are small (on the order of about two to four percent of the total member forces) and may justifiably be neglected in the overall frame analysis. However, in extreme cases, the amplification of moments and deflections due to P-δ effects can be as large as twenty percent of those due to the P-Δ effects[5] and should therefore be considered in the analysis.

Second-order matrix analysis programs that account comprehensively for both P-Δ and P-δ effects in general types of structures are certain to become more readily available in the near future. What may be surprising to many engineers is that these procedures may be formulated in such a way that the solution effort required to do a comprehensive analysis is comparable to that of a conventional P-Δ type of analysis. Nevertheless, there are complications involved with using any type of second-order elastic analysis in design. In this paper, the authors will focus on how these complications may be alleviated when comprehensive matrix analysis procedures are used.

Matrix Analysis Approaches For Second-Order Elastic Analysis

Two basic methods are commonly employed for the matrix formulation of second-order elastic frame elements: the stability function approach and the finite element (or geometric stiffness) approach. The fundamental difference between these two methods is that the stability function procedure is based directly on the governing differential equations of an initially straight, elastic beam-column. Conversely, the geometric stiffness approach is based commonly on an assumed cubic polynomial variation of the transverse displacements along the element length. Therefore, the stability function approach provides an "exact" solution for the second-order elastic behavior (assuming small strains and neglecting effects of bowing on member "axial" deformation), whereas the geometric stiffness method involves an approximation of the small strain, large rotation, large displacement response.

For either approach, the element stiffness equations may be expressed as

$$[K]\{u\} + \{F_f\} = \{F\} \tag{1}$$

where $[K]$ is the element stiffness matrix, $\{u\}$ is the element nodal displacement vector, $\{F_f\}$ are element fixed-end forces, and $\{F\}$ are the nodal forces at the element ends. The degrees of freedom of the two-dimensional element are standard and are shown in Ref. 6. For this case, if the stability function approach is employed, the stiffness matrix may be written as

$$[K] = \frac{EI}{L}\begin{bmatrix} A/I & 0 & 0 & -A/I & 0 & 0 \\ & 12\phi_1/L^2 & 6\phi_2/L & 0 & -12\phi_1/L^2 & 6\phi_2/L \\ & & 4\phi_3 & 0 & -6\phi_2/L & 2\phi_4 \\ & \text{sym.} & & A/I & 0 & 0 \\ & & & & 12\phi_1/L^2 & -6\phi_2/L \\ & & & & & 4\phi_3 \end{bmatrix} \tag{2}$$

and the fixed-end force vector corresponding to a uniformly distributed lateral load w is

$$\{F_f\} = \{0 \;\; wL/2 \;\; (wL^2/12)\phi_5 \;\; 0 \;\; wL/2 \;\; -(wL^2/12)\phi_5\}^T \tag{3}$$

The terms ϕ_1 through ϕ_5 in the above equations are related to the elastic beam-column stability functions[7] and are described in Ref. 6. All these terms are functions of $L\sqrt{P/EI}$, where P is the absolute value of the axial force in the member. Different functions must be used for ϕ_1 through ϕ_5 for the tension and compression cases. This results in complications for situations in which the element forces change sign during the loading process. Also, these functions are indeterminate when P is exactly equal to zero, in which case, the values for ϕ_1 through ϕ_5 should be taken as 1.0. That is, the stiffness matrix and fixed-end force vector commonly employed for first-order elastic analysis are recovered if the axial force is zero.

If the finite element approach is employed for the two-dimensional formulation, the resulting element stiffness $[K]$ may be subdivided into two separate matrices. The first matrix is often referred to as the elastic or small-displacement stiffness matrix $[K_e]$. This matrix is equivalent to Eq. 2 if all the terms ϕ_1 through ϕ_4 are set equal to 1.0. The second matrix is referred to as the geometric stiffness and may be written as

$$[K_g] = P\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & 6/5L & 1/10 & 0 & -6/5L & 1/10 \\ & & 2L/15 & 0 & -1/10 & -L/30 \\ & \text{sym.} & & 0 & 0 & 0 \\ & & & & 6/5L & -1/10 \\ & & & & & 2L/15 \end{bmatrix} \tag{4}$$

where P is taken as negative in compression. The fixed-end force vector obtained by the finite element approach is equivalent to Eq. 3 with ϕ_5 equal to one. That is, the fixed-end forces obtained by this method are unaffected by the amount of axial force in the element.

Since the finite element approach involves some approximation of the analytical second-order elastic response, its range of applicability must be understood. Figure 3 is a plot of the error in the finite element stiffness terms as a function of P/P_e. Based on this figure, it can be stated that if P/P_e is less than about 0.4 (where $P_e = \pi^2EI/L^2$), the largest error in any of the terms of ($[K_e] + [K_g]$) is less than one percent.[8,9] That is, if it is desired to use the geometric stiffness approach for cases in which P/P_e exceeds 0.4, the corresponding members must be subdivided into two or more elements to limit the error in stiffness to less than one percent.

For an axially loaded strut with full rotational and lateral restraint at its ends, the elastic critical load is $4P_e$. If a member is assumed to be loaded at this extreme and it is subdivided into three equal length finite elements, then the P/P_e for each element is only $4/9 = 0.44$. Therefore, it should be rarely necessary to use more than three elements per member to model the second-order elastic stiffness.[8,9] Of course, for overall frame analysis, one element per member is desirable from an efficiency as well as a modeling viewpoint. In many types of frames, P/P_e is less than 0.4 in all the members and this is possible. For instance, it would be extremely rare for a beam-column in an unbraced frame to support a design axial load greater than $0.4P_e$. Even if the loads at elastic buckling of the frame were considered, a member's effective length factor from a system buckling analysis would have to be less than $K = 1.58$ for P/P_e to be greater than this value. Also, when P/P_e is greater than 0.4, the finite element matrices still represent the P-Δ effect in an exact manner. The approximations are associated only with the P-δ amplification.

It is important to note however that the fixed-end moments of Eq. 3 are somewhat more sensitive to values of P/P_e. Figure 4 is a plot of the error in the finite element fixed-end forces as a function of P/P_e for one, two, and three elements per member. If one element is used for the member, the value of ϕ_5 is 1.073 at $P/P_e = 0.4$, and therefore the error in the finite element fixed-end moments is approximately seven percent. These results demonstrate that if accurate second-order elastic moments are desired in members subjected to transverse loading, either the stability function approach must be employed or the member must be discretized into a number of finite elements. Of course, the "exact" stability function approach is based on the assumption of small deformations (i.e., $\sin(\theta) = \theta$ and $\cos(\theta) = 1$ are assumed, where θ is the rotation relative to the member chord at any cross-section along the member length).

Table 1.
Advantages to the Stability Function and Finite Element Approaches

Stability Function Approach:

1. Represents P-δ effects exactly for members with small deformations.
2. Accounts for axial force effects on the values of fixed-end forces.

Finite Element Approach:

1. Only one function needs to be employed for each of the terms of the element matrices. Tension, compression, and zero axial force are represented by the same matrix terms.
2. More easily extended to general three-dimensional analysis.

The main advantages of the stability function and finite element approaches are summarized in Table 1. One main advantage of the finite element approach is that it may be extended to three-dimensional frame analysis in a more straightforward manner than the stability function approach. Torsional stability functions are derived and presented in Ref. 10, but few researchers have attempted to utilize these functions for matrix analysis. There are two primary difficulties in extension to three-dimensional analysis: the representation of the torsional response, including the effects of warping in open thin-walled cross-sections, and the correct representation of the effects of moderately large three-

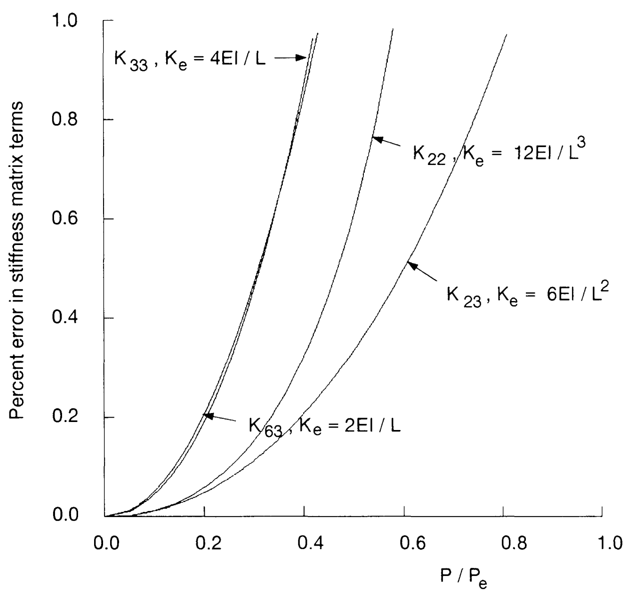


Fig. 3. Accuracy of stiffness matrix terms—finite element procedure.

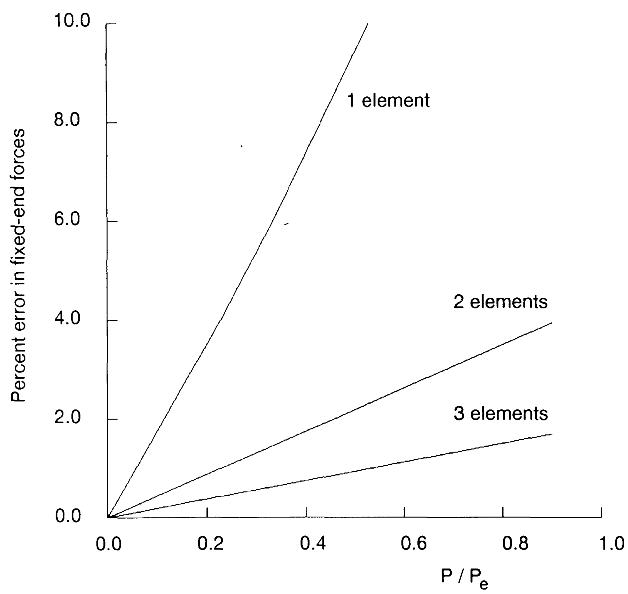


Fig. 4. Accuracy of fixed-end forces for a uniformly distributed transverse loading—finite element procedure.

dimensional rotations. In Ref. 11, Yang reviews the existing literature before 1984 and discusses a number of alternative finite element formulations which address these aspects. Recent work regarding large rotations has been performed by a number of investigators including Rankin and Brogan.[12] There is evidence that the proper handling of large three-dimensional rotations may have a noticeable effect on the solution of problems involving moderate rotations (as in the case of building frames). Accurate finite element based analysis of torsional rotations and member lateral-torsional effects generally requires the use of multiple elements per member.

The inclusion of member lateral-torsional stability effects in the overall three-dimensional analysis of frameworks is currently an area where additional research is needed. In some types of building structures, these effects may be noticeable but small, and thus they might be treated in an approximate way or neglected for overall frame analysis. For example, Ref. 13 indicates that in tests of isolated beam-columns subjected to equal end moments that cause single curvature about the strong-axis, and in which almost full rotational restraint about the weak axis is achieved at the member ends, it is usually possible to reach the inelastic strength predicted based on the assumption of in-plane behavior. Often, structural members may be designed such that elastic or inelastic lateral-torsional instability does not occur prior to factored design loads being reached. Furthermore, many three-dimensional systems are primarily composed of a number of separate but interacting two-dimensional frames.

The second-order effects associated with torsional deformation of the members are most likely small compared to the P-Δ and P-δ effects. However, the representation of the elastic torsional stiffness of the members may be of greater significance. Most of the matrix analysis programs currently available for three-dimensional analysis do not consider the effects of warping on the torsional stiffness. That is, the elastic torsional stiffness of the element is taken as GJ/L, based on the assumption of uniform torsion.

ADVANTAGES OF SECOND-ORDER MATRIX ANALYSIS

The most important advantage of second-order elastic matrix analysis approaches is their generality. Since these procedures can accurately represent the second-order amplification of forces and displacements for arbitrary structural configurations and loadings, they can serve as a powerful tool for design of the irregular and complex three-dimensional structures that are sometimes encountered. For example, two-dimensional idealizations of a structure's primary lateral resisting systems, coupled with standard approximate methods of accounting for second-order effects due to gravity loads, work well for many regular structures. However, they tend to break down for asymmetrical buildings where torsional and other three-dimensional effects may have a significant influence.

The difficulty in accounting for the geometric nonlinear effects in such structures is exemplified when one considers the details of how to apply the current LRFD B_1/B_2 approach. First, this approach only amplifies the moments, whereas second-order elastic analysis provides the complete amplified load effects. Second, for such buildings, the B_2 factor can be cumbersome to calculate, and the procedure used to perform this important task at each floor can be ambiguous. For example, it may be unclear whether to use the major or the minor axis effective length factor when computing ΣP_e for columns whose webs are skewed with respect to the principal directions of the wind load (see Fig. 5). Alternately, the engineer may be uncertain as to the value of Δ_{oh} which should be used if torsional deformation of the structural system occurs. The use of a second-order elastic analysis eliminates the need for the engineer to make intricate decisions about the use of approximate techniques and formulas. With a comprehensive matrix analysis approach, the computation of second-order load effects is relegated to the computer, and the number of tedious hand calculations and analysis decisions is reduced.

Although the P-Δ effects in two- and three-dimensional building structures can be handled properly and efficiently by a number of procedures,[5, 14–17] many of the P-Δ techniques are dependent upon the idealization of the floors as rigid diaphragms. The comprehensive analysis methods discussed in this paper can be used with or without a rigid floor model. Although the rigid floor assumption is often applicable, it may be important to model the flexibility of the floor diaphragm in certain types of buildings (or in certain floors of a complex building structure). Currently, a number of frame modeling packages allow the definition of flexible floors.[18] If the engineer desires to employ this level of sophistication in the structural model, the analysis capabilities described can accommodate this behavior. Also, the comprehensive matrix approaches discussed here account for

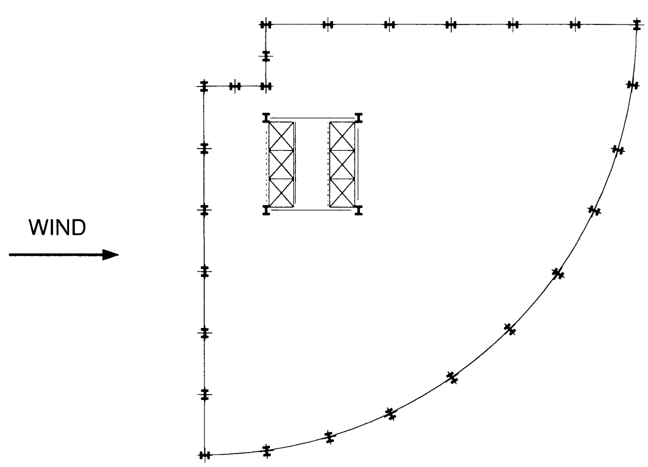


Fig. 5. Floor plan with columns skewed with respect to principal wind directions.

P-δ effects in a simple and direct way in the model of each element.

SECOND-ORDER ANALYSIS ISSUES

The main issues which must be addressed for the efficient and effective use of any type of second-order elastic analysis in design can be summarized as: (1) reduction of computational effort, (2) alleviation of solution complexity, and (3) calculation of second-order effects in preliminary analysis. Each of these issues is discussed below with an emphasis on how they may be addressed when comprehensive matrix analysis is used.

Computational Effort of the Analysis

In the linear elastic LRFD analysis procedure, only two analyses (the NT and the LT cases) need to be performed to consider all the necessary load combinations for design. If a matrix analysis is employed, all the nominal load types can be included as multiple right-hand side vectors in the global solution of the NT and LT analyses. Then, the member forces for design can be obtained by superposition using factored combinations of the nominal effects from each load type with appropriate amplification by B_1 and B_2 terms.

Neither the comprehensive matrix analysis approaches discussed in this paper nor the simpler P-Δ analysis procedures[5, 14–17] can match the computational efficiency of this linear elastic approach. In other words, since the second-order behavior is nonlinear, there is no avoiding the fact that the use of any type of second-order elastic analysis will require some extra computational effort. Since superposition is in general not valid, this extra effort depends in part on the number of separate load combinations considered in the design. In general, a separate analysis is required for each load combination. However, in some cases only a few controlling load combinations may need to be analyzed and the extra effort associated with this aspect would be small.

Also, the computational effort depends heavily on the efficiency of the nonlinear global solution techniques employed. The following sub-section outlines what the authors believe to be one of the most efficient nonlinear solution techniques for comprehensive second-order elastic analysis of building frames. This is followed by a sub-section that describes a way by which a limited use of superposition may be employed with second-order elastic analysis.

One-Increment Newton-Raphson Solution Procedure

Compared to linear elastic analysis, there is little extra computational effort involved with the calculation of the stability function terms of the element stiffness matrices if the stability function approach is used, or of the additional geometric stiffness matrices $[K_g]$ if a finite element approach is employed. However, the solution for second-order elastic forces and displacements in highly nonlinear structures can require significant additional computer time. The second-order forces in the structure depend nonlinearly on the displacements. Therefore, in the general case, the global forces and displacements must be solved for iteratively (using, for example, a Newton-Raphson algorithm[19]) to guarantee the satisfaction of equilibrium in the deformed configuration. For severely nonlinear problems, the loading must often be applied in small increments to achieve reasonably fast convergence of the iterative steps within each increment.

Fortunately, in most building structures, the nonlinearity associated with second-order elastic effects is not severe. For example, the effects of member bowing* and large rotations of the member chord on the member axial deformation are often considered negligible.[20,21] It can be shown that for structural steel frames, if the story sway displacements are restricted to less than $L/400$, the relative vertical displacements of the beam-column ends due to chord rotation and member curvature effects are roughly 3 orders of magnitude less than the member axial deformations associated with the uniaxial yield strain, $\epsilon_y L$. Even if the story sway displacements are only restricted to $L/200$, the relative displacements due to chord rotation and member curvature effects are still two orders of magnitude less than $\epsilon_y L$. If these effects are disregarded, then the resulting behavior usually allows for fast convergence of the Newton-Raphson algorithm. In the absence of these effects, the nonlinear terms of the frame element incremental force-displacement relationships (derived either by the stability function or the geometric stiffness approach) are associated only with changes in the member axial force during the increment.[22] Thus, if there is no change in the member axial force during an increment, the force-displacement relationships are completely linear.

In the experience of the authors, if axial deflections associated with member bowing and large chord rotations are neglected, a one-increment Newton-Raphson procedure is very efficient for building type structures. For the formulation and implementation discussed in Ref. 22, the nonlinear solution usually converges in either one or two iterations. The significance of this aspect is that the second-order analysis for any one of the load combinations would require only about one to two times the computational effort of a linear elastic NT-LT analysis. Also, the time required for this comprehensive analysis would be comparable to that required for an iterative P-Δ method[16] (which is not as easily generalized for three-dimensional analysis, and which does not accurately model the P-δ effects). An example solution for a 10-story planar frame, generated using the STAND program,[23] is shown in Fig. 6. A 10-increment "predictor-corrector" solution is shown which depicts the nature of the nonlinearity in the structure. For this example, the one-increment Newton-Raphson algorithm converges to the equilibrium solution in two iterations.

* The member bowing effect is defined as the longitudinal shortening of a member due to bending action.

Superposition

Probably the most serious factor which influences the computational effort of second-order elastic analysis is the lack of the general ability to use superposition. Theoretically, a separate nonlinear analysis must be performed for each factored load combination considered in the design. Thus, comprehensive matrix approaches provide a rigorous solution of the elastic structural response at the expense of having to execute and manage the results from possibly a large number of separate analyses.

Fortunately, superposition may still be employed in a limited sense if the gravity loading is applied to the structure first and then held constant during the application of lateral load, and if it is assumed that: (1) changes in the axial forces in frame members during the application of lateral loads do not significantly affect the structure stiffness,* and (2) the effects of member bowing and large rotation of the member chords are negligible. These conditions are necessary to linearize the solution of the system response to lateral loads.[22] That is, if the gravity load is applied first and the above assumptions are employed, the lateral load analysis is a linear analysis for which superposition is valid.

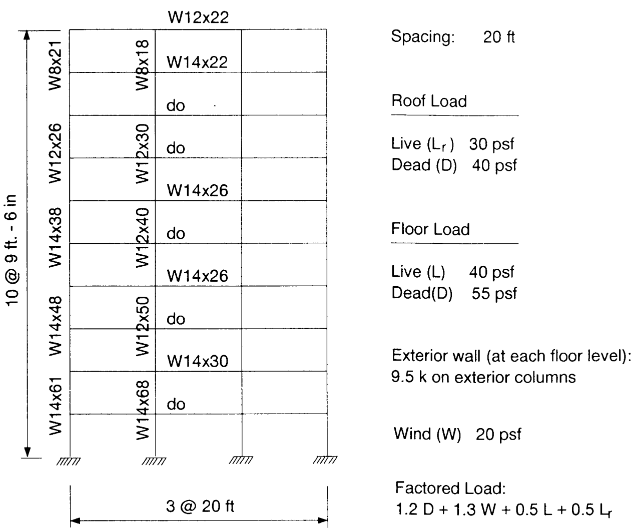


Fig. 6a. Ten-story planar frame example.

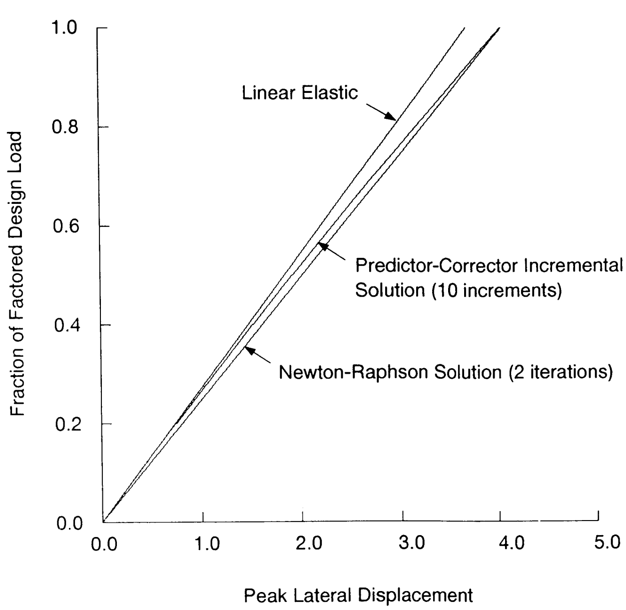


Fig. 6b. Comparison of 10-step incremental and one-increment Newton-Raphson iterative solution techniques for the ten-story planar frame.

Even when the above simplifying conditions are satisfied, a separate nonlinear analysis must be performed for each of the factored levels of gravity load considered in the design. Also, if any patterned gravity loads need to be considered in the overall system analysis, a separate nonlinear analysis would be required for the combinations with these loadings. As an example, the designer might decide to check the following load combinations:

$$\begin{aligned} &1.4D \\ &1.2D + 1.6L + 0.5L_r \\ &1.2D + 0.5L + 0.5L_r + 1.3W \\ &1.2D + 0.5L + 1.5E \end{aligned} \tag{5}$$

In this scenario, the second-order amplification associated with gravity load is different and a separate nonlinear analysis would be required for all four of these combinations.

However, if wind and earthquake loads from multiple directions must be considered, the satisfaction of the conditions mentioned in the above paragraph allows for efficient analysis of lateral load effects from each of the directions. A particular factored gravity load combination can be applied in an initial step, and iterations can be carried out to achieve an equilibrium solution for this step. Then, as a second step, all the appropriate lateral load cases may be applied starting from the gravity loaded state of the structure. As a result of the simplifying assumptions discussed above: (1) the global behavior is linear during these lateral load increments, and (2) the second-order elastic structure stiffness matrix determined at the end of the gravity load portion of the analysis may be used directly to solve for the response to all the lateral loads. No iterations are required in the lateral load step, and in fact, since the structural behavior is incrementally linear, all the lateral loads which must be combined with this particular gravity load combination can be handled as a multiple right-hand side vector in the global equation solution. The second-order structural stiffness only needs to be decomposed once, and then back-substitution can be performed for all the lateral load cases to be combined with this particular

* This assumption implies that the additional P-δ effects in the frame members due to changes in the member axial forces associated with lateral loading are small. The story P-Δ effect is not changed due to variations in the member axial forces associated with lateral loading. Therefore, the P-Δ effects are still represented exactly.

gravity loading case. In short, this procedure is a generalization of procedures suggested by a number of researchers in the last several years.[15,16,24,25]

Solution Complexity

An engineer who wishes to analyze the nonlinear response of a general structure is typically faced with multiple decisions regarding numerical parameters such as increment size, convergence tolerance, global solution techniques, etc. These parameters often have subtle implications depending on the analysis program that is being used and they are often subject to misinterpretation by engineers who do not use the program regularly. In summary, it can be said that a substantial amount of knowledge and numerical expertise is required to utilize a general nonlinear finite element analysis package.

For comprehensive matrix procedures to be applied optimally in design practice, the engineer should not need to be distracted by details of how to perform the nonlinear analysis. The engineer must have an understanding of the analysis process, but decisions about the detailed procedural aspects of the analysis should not be required when at all possible. This is a definite advantage of the one-increment Newton-Raphson procedure discussed in the previous section. The only analysis parameters which must be specified for the Newton-Raphson algorithm are the tolerances used to test for convergence, and the maximum number of iterations allowed, which is essentially a check for divergence. For practical analysis of building frames, a convergence tolerance of 0.001 is sufficient for use with ordinary convergence tests[19] and a maximum number of iterations of 10 is reasonable. In can be concluded that the one-increment Newton-Raphson solution procedure employed with these simple parameters is nearly as straightforward and simple to perform as linear elastic analysis.

Preliminary Analysis

For preliminary analysis of the structural system, the assumed member sizes may not be appropriate for the design. Therefore, if second-order effects are accounted for rigorously (as is done by using a second-order elastic matrix analysis), these effects may be unrealistically large or small. It is thus better simply to estimate the second-order amplification at preliminary stages. This is more appropriate as well from a standpoint of computational efficiency. A comprehensive matrix analysis approach is strictly appropriate only as the member sizes approach the final design solution in a series of analysis/design iterations. Approaches for obtaining estimates of second-order effects include the use of the LRFD Equation H1-5 for B_2 with an expected or "target" design value for Δ_{oh} as well as other methods for obtaining "target" system amplification factors.[5,26,27]

APPLICATION OF SECOND-ORDER ELASTIC MATRIX ANALYSIS IN DESIGN

If second-order elastic analysis is to be used optimally in design, the engineer must carefully consider a number of aspects related to the modeling, analysis, and design procedures. These aspects include: (1) calculation of maximum second-order elastic moments within the members, (2) calculation of member effective length factors, (3) efficient modeling of gravity columns, i.e., leaned columns, in the analysis of the lateral framing system, (4) performance of live load reduction with forces obtained from second-order elastic analysis, and (5) specification of initial imperfections, specifically story out-of-plumbness. Another aspect which the authors believe is essential for the effective use of comprehensive analysis/design procedures is (6) the availability of advanced graphical user-interfaces for depiction of the structural behavior and the analysis/design process. The first two topics above are primarily member analysis/design considerations, whereas the next three topics are more concerned with aspects of the overall structural system. The last topic is associated with software development. All of these aspects are discussed below.

ASPECTS AT THE MEMBER LEVEL

Calculation of Maximum Second-Order Elastic Moments Within Members

Even in members which are loaded only by end moments, the maximum second-order elastic moment within the span of the member (defined as M_u in Chapter H of the LRFD Specification) can be greater than the member end moments due to P-δ effects. This behavior is illustrated by Fig. 7. As the axial force becomes large or the member becomes more slender, it is more likely that M_u does not occur at the end of the member. However, in many practical situations, the maximum second-order elastic moment does occur at the member end (this is evidenced by the LRFD B_1 parameter often being equal to 1.0).

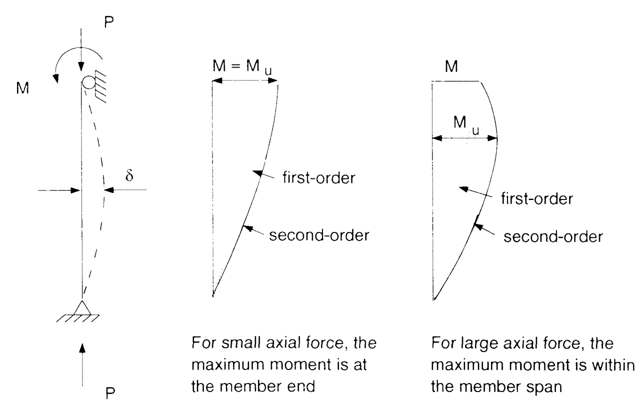


Fig. 7. Effect of axial force on the location of the maximum second-order elastic moment M_u.

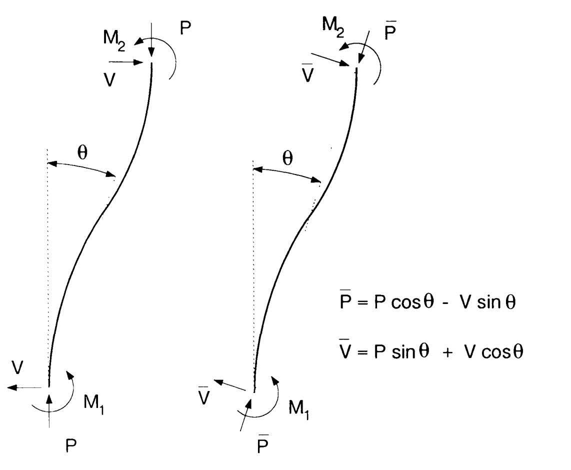
Although the member stiffness as calculated by the matrix approaches may represent the P-δ effects adequately (see the discussion in the second section of this paper), one problem of these approaches is that the analysis provides the member forces only at the element ends. Therefore, additional calculations are necessary to determine the location and magnitude of M_u. These calculations may be easily appended to the structural analysis and do not need to be performed by hand.

There are a number of ways to compute this maximum moment. For example, these values can be obtained directly from the fundamental differential equation solution for an isolated member. That is, M_u can be obtained on the same basis as the matrix terms of the stability function approach discussed previously. The reader is referred to Refs. 4 and 6 for details. Essentially, if a one-increment Newton-Raphson solution procedure is employed, the second-order matrix analysis gives the end forces on any particular member as shown in Fig. 8a. Then, a free-body diagram may be constructed for each isolated member using these end forces and any loads applied directly to the member. If the applied forces are transformed to local axes oriented with the member chord (Fig. 8b), there is no difference between the behavior of this isolated member and the behavior of a simply supported beam-column with the same applied forces.

Another procedure that can be used to determine M_u, which is consistent with the finite element approach, is to use cubic Hermitian interpolation for the member transverse displacements in calculation of the P-δ moments within the span. However, the cubic Hermitian interpolation functions only approximate the true transverse displacements associated with the P-δ moments along the member length.

For practical design, the authors recommend the following simple and approximate approach for calculation of M_u within end-loaded members:

1. Obtain the member end forces from the second-order elastic analysis and consider the isolated member with the end forces oriented with the chord. This is shown in Fig. 8b for the case in which only end forces are applied.
2. Approximate equations can be derived, as shown by Chen and Lui,[6] which give an adequate estimate of the maximum second-order bending moment along the span for particular loading cases on an isolated member. It turns out that, for members loaded only by end forces, these approximate equations are the same as the equations given in the LRFD Specification for the B_1 factor (but using $K = 1$ for calculation of P_e instead of K less than one). Therefore, for these types of members, the end moments from the analysis may be amplified using the B_1 equation to obtain the maximum second-order elastic moment for design.*



(a) Element end forces obtained from the analysis

(b) Element end forces oriented with the rotated element chord

Fig. 8. Beam-column end forces.

If one element is employed for second-order analysis of transversely loaded members, the governing differential equation solution should be used to calculate member internal forces. These solutions may be programmed for second-order elastic analysis on a case-by-case basis.

Effective Length Factors

For buildings which are not restrained against sidesway, the LRFD Specification requires the calculation and use of effective length factors greater than unity when computing the nominal axial resistance of a member P_n. This is the case even when second-order elastic analysis is used to determine the load effects.[3,4,28,29] If K is taken as unity, as proposed for conventional P-Δ analysis/design, the present AISC interaction equations, both in Allowable Stress Design[30] and in LRFD, are unconservative if the member is relatively slender and has a relatively high axial load.[3,4,17,28,29] Among the different forms of equations considered during the development of the LRFD equations for beam-column design, equations which include a K factor were reported to give the best fit to "exact" solutions of the beam-column strength for a wide range of cases.[3,4,29]

Figures 9a and 9b illustrate the unconservative nature of assuming K equal to one for the LRFD design of a simple portal frame. In Fig. 9a. the LRFD curves for $K = 1.0$ and 2.0 are compared to the "exact" solutions for the beam-column strength reported by Kanchanalai.[31] The exact solutions for strong- and weak-axis bending action are shown by the solid lines in the figure, and the specification curves are illustrated by the dotted lines. The exact solutions are based on W8×31 columns, a yield stress of $F_y = 36$ ksi, an

* The B_1 equation with $K = 1$ is valid only for cases in which $P < P_e$. If $P > P_e$, the use of the analytical solution[4,6] is recommended.

elastic modulus of E = 29,000 ksi, a peak compressive residual stress of $0.3F_y$, and initially perfect geometry. Figure 9b compares the specification curves to the "exact" strengths when the effects of initial geometric imperfections are included. These curves are generated by adjusting the curves from Fig. 9a as explained in Refs. 3 and 4. For the frame shown in the figure, this adjustment accounts approximately for the effect of an initial out-of-plumbness of $L/750$.

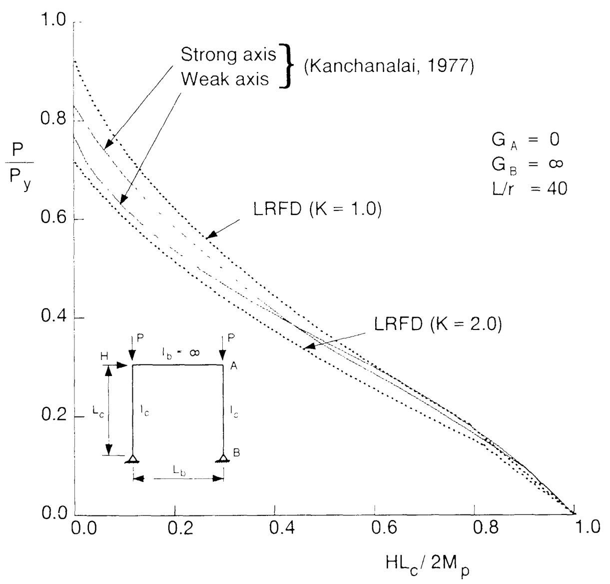


Fig. 9a. General effect of using $K = 1$ with the LRFD beam-column interaction equations—frame with initially perfect geometry.

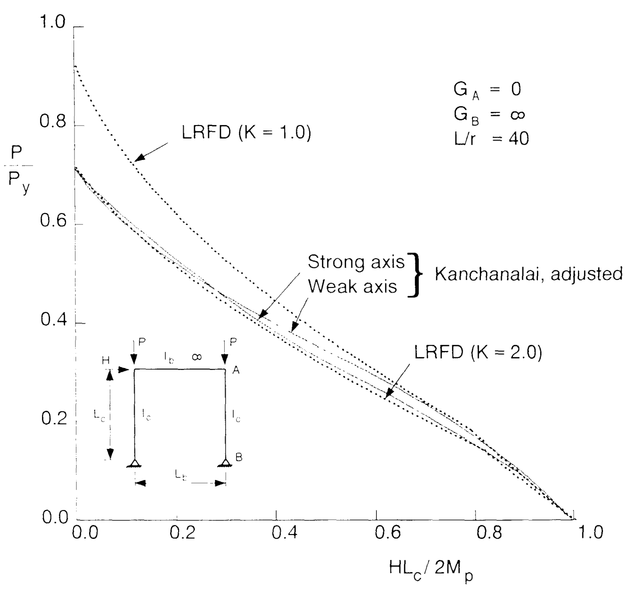


Fig. 9b. General effect of using $K = 1$ with the LRFD beam-column interaction equations—comparison of specification equations with strengths including imperfection effects.

Direct second-order elastic analysis eliminates the need for moment amplification formulas, some of which involve a K factor. However, it is important to emphasize that generally speaking, there is no type of second-order elastic analysis which eliminates the need to consider the effective length factor in the determination of axial resistance for design by the current ASD and LRFD provisions. Current developments in the Canadian Limit States Design Specifications have resulted in an approach that does not involve a K factor in the beam-column design expressions.[2] With regard to this issue in conventional P-Δ analysis, coupled with Allowable Stress Design, the SSRC Guide[28] states:

> When the magnitude of the first term [the axial term of the interaction equations] is less than some critical value, classical (bifurcation) buckling of the frame does not govern and the effective length of the column need not be used. This critical value can be conservatively estimated as f_a/F_a = 0.85, where F_a is evaluated using an effective length.

It is not certain what the limit on $P_u/\phi P_n$ should be for the use of this philosophy with the LRFD provisions. The design summaries in this paper therefore presume that for unbraced frames, effective length factors generally greater than one are to be used in the calculation of P_n.

ASPECTS AT THE SYSTEM LEVEL

Modeling of Gravity Columns

In current practice, when first-order analysis is employed, it is common to model the lateral system of the structure without including the beams and columns that constitute the gravity framing. The gravity loads included in the analysis are only those which come from the floor areas tributary to the columns of the lateral system as well as from partitions, walls, and cladding supported by the lateral system. In many buildings, a significant percentage of the gravity load is not carried by the lateral framing system.

If it is desired to perform a second-order elastic analysis of the structural system, the destabilizing effects of the gravity columns must be included. The loads in the gravity columns cannot simply be lumped with the loads applied to the lateral system columns, since in general, this results in an over-conservative design of the lateral system columns.[32] Wilson[15] describes a useful procedure by which the P-Δ effects of the full gravity loads supported at each floor level may be considered in a matrix analysis approach. Other techniques similar to Wilson's have been discussed in Refs. 5, 14, 16, and 17. Wilson's procedure requires that the floors be modeled as rigid diaphragms and that the floor degrees of freedom be placed at the centroid of the floor. Furthermore, the total P-Δ effect at each story is approximated by assuming that the support of the floor gravity loads is distributed uniformly over the floor slab. This can result in some error in the representation of P-Δ effects on the torsional response of the building. For example, if the lateral resisting system is composed of a central core and framing on the exterior wall of the building with long floor spans between the central core and the exterior wall, then much of the support for the gravity load in the floor will be along the perimeter of the frame. In this case, Wilson's assumption can be somewhat unconservative. Fortunately, most tall buildings are relatively stiff in torsion, and the P-Δ torsional effect is small.

An alternative procedure is described here which the authors believe to be more accurate, more direct to implement and execute in second-order matrix analysis programs, and which does not require the above assumptions. The proposed procedure is derived directly from the P-Δ effect on an individual column. These P-Δ forces are illustrated in Fig. 10. The relationship between these forces and the lateral translations at the top and bottom of a column can be written in matrix form as

$$\begin{Bmatrix} f_{xi} \\ f_{yi} \\ f_{xj} \\ f_{yj} \end{Bmatrix} = P/L \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_{xi} \\ u_{yi} \\ u_{xj} \\ u_{yj} \end{Bmatrix} \tag{6}$$

where P is the axial force in the column (negative for compression), and L is the column height. This matrix is actually the geometric stiffness of a three-dimensional truss element. There is a small approximation in the modeling of the gravity columns in this way. The actual columns are often continuous from floor to floor. Therefore, if the sway of the column is not the same in adjacent stories, some bending is induced in the column. However, the effect of these moments in the gravity columns is not expected to be large.

The full P-Δ contribution from a gravity column at any story level can be represented in a global matrix solution by simply assembling the stiffness matrix of Equation 6 for that column into the global structure stiffness. While a finite element must be included in the model for each gravity column, definition of its elastic properties is optional, as is its contribution to the structure's elastic stiffness. If a rigid floor constraint is employed, these element stiffnesses would be transformed to the master node of each floor. Constraint procedures which perform this operation are discussed in a number of references including Ref. 33. In this case, the constraints associated with the gravity column nodes are the same as those for the nodes of the lateral framing system, with the exception that a rotational degree of freedom normal to the plane of the floor does not have to be modeled at a gravity column node. The rigid floor assumption allows the description of the in-plane displacements of the floor simply as two in-plane displacements and one rotation at the floor master node. The out-of-plane degrees of freedom at the nodes belonging to the lateral framing system are independent of these floor in-plane displacements. It is not necessary to model the out-of-plane degrees of freedom at the gravity column nodes in the suggested procedure. If the matrix analysis program requires a constant number of degrees of freedom at each nodal point, this may be accommodated by fixing the out-of-plane degrees of freedom and the rotational degree of freedom normal to the plane of the floor at the gravity nodes. Also, the beams of the gravity framing system do not need to be modeled if the rigid floor constraint is employed.

If the rigid floor assumption is not employed, then the gravity column nodal degrees of freedom are included directly in the global structure stiffness. In this case, the model of the floor system must include some connectivity to all of the gravity column nodes.

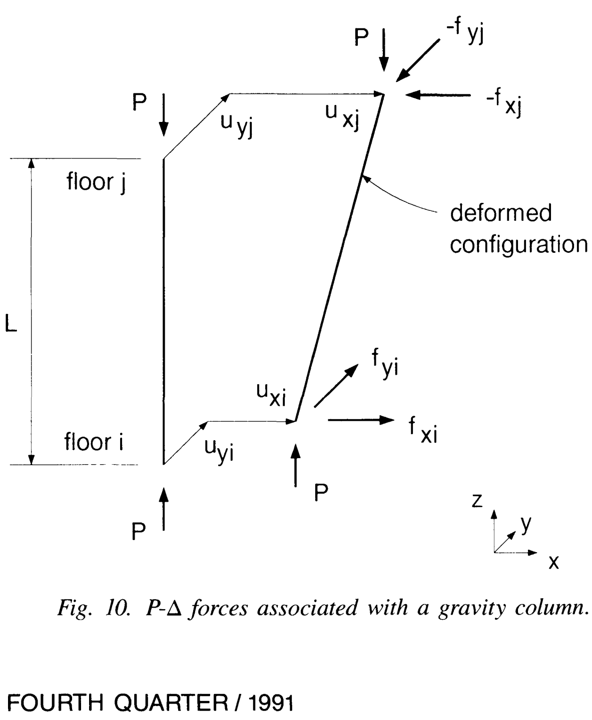


Fig. 10. P-Δ forces associated with a gravity column.

Live Load Reduction

Incorporating live load reduction into a second-order elastic analysis/design procedure poses some unique problems. In current practice, if linear elastic analysis is employed, it is common to apply the full factored live load in the analysis. The full factored live load forces in the structural members are thus obtained. Then, it is typical to apply live load reduction to these forces on a member by member basis, considering the influence area for each particular member. Finally, these reduced live load member forces are used for the design.

Two problems occur when attempting to apply this live load reduction procedure with second-order elastic analysis. The first problem relates to the fact that superposition of the results for separate factored load effects is not valid. As discussed previously, in general a separate second-order elastic analysis must be performed for each different combination of factored loads. The main problem here is that, since all the load types are applied together in any of the second-order analyses and superposition is not valid, it is generally not possible to determine the portion of the second-order elastic forces in a particular member resulting from the live load part of the loading.

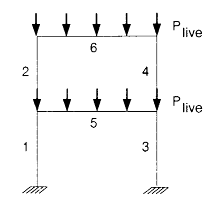
One possible method of alleviating this problem is first to perform a linear elastic analysis for the full nominal live load (using a load factor of 1.0). Member live load reduction percentages are then computed by code provisions, based on the influence areas of the different members in the structure. Using these percentages, the engineer computes for each member the amount of live load force which may be subtracted to achieve the member live load reductions permitted by the particular building code being used. These forces are referred to here as the "live load reduction forces." The second-order analyses for all appropriate load combinations are then performed. After this, for each load combination, the "live load reduction forces" are multipled by the live load factor for that load combination, and these resulting forces are subtracted from the member second-order analysis forces. These final forces are the ones used for design. The suggested procedure is summarized in Fig. 11.

This approach should be conservative since the second-order effects of the full unreduced live load are included in the analysis but not in the reduction of the member live load forces. Alternatively, a more involved approach would be to obtain "live load reduction forces" based on a number of second-order elastic analyses, one for each factor by which the live load is multiplied in the design load combinations.

In complex structural systems, the influence area for a particular member may not be straightforward to estimate. Therefore, other methods of handling live load reduction may be desirable. For example, a simpler procedure would be to apply the reduction directly to the live loads used in the analysis. The results from the second-order elastic analyses could then be used directly for design. However, in this approach, some members would be designed for more than the permitted reduced live load member forces. For example, the fully reduced live load in a beam member may transfer greater loads to the adjacent columns than need to be designed for according to the maximum live load reduction for the columns.

It should be emphasized that live load reduction accounts for the aspect that the maximum live load effects in the different members of a frame do not occur at the same point in time. This leads to the second problem associated with the use of live-load reduction with second-order elastic analysis: the live loads causing the second-order effects in a frame structure are assumed to be applied at the same point in time. The first procedure suggested above is conservative in that, for calculation of second-order effects, the full unreduced live load is used. However, one might question if some lesser value of live load might not be appropriate for assessment/calculation of the overall stability effects in the structure.

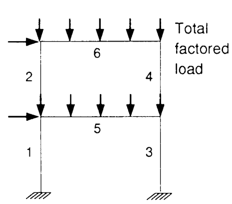
1. Apply the nominal live load (Load factor of 1.0). Perform a linear elastic analysis.



2. Compute the member live load reduction forces (LLRF) based upon code provisions.

Member #	Nominal live load force	Live load reduction factor	Live load reduction force
1	30	0.3	9
2	20	0.2	4
⋮			

3. Perform second-order elastic analyses for the desired factored load combinations.



4. For each load combination, multiply the LLRF by the combination' s live load factor and subtract from the member forces to obtain the design forces.

Member #	Live load reduction force	Live load factor	Analysis force (all loads)	Design force
1	9	0.5	50	45.5
2	4	0.5	40	38.0
⋮				

Fig. 11. Live-load reduction procedure.

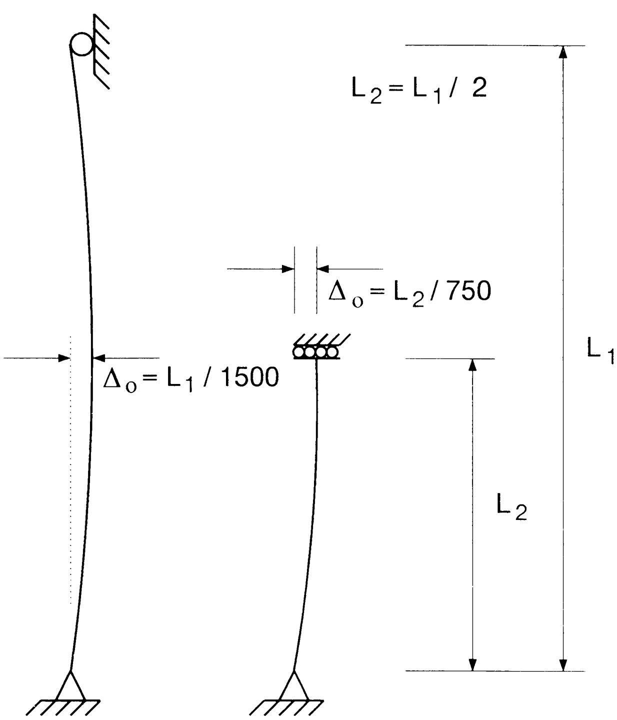


Fig. 12. Identical columns with initial imperfections.

Minimum Out-of-Plumbness for Gravity Loaded Frames

It is not necessary, nor is it practical, to model initial member out-of-straightness effects in a second-order elastic analysis. This is because member out-of-straightness effects are accommodated implicitly in the AISC-LRFD formulas for column axial resistance. Specifically, the AISC-LRFD column strength equations are essentially the same as column strength curve 2 of the Fourth Edition SSRC Guide,[28] which was developed based on an out-of-straightness criterion of $L/1500$.

Also, the effects of column out-of-plumbness are accounted for, to a certain extent, in the AISC-LFRD column strength equations. This aspect is illustrated by the two columns in Fig. 12, which behave identically in both the elastic and inelastic ranges of loading. The simply supported column has an initial out-of-straightness of $L_1/1500$ whereas the corresponding sway column has an initial out-of-plumbness of $L_2/750$.

Since the AISC erection tolerance for columns is $L/500$, it could be stated that the maximum possible out-of-plumbness of the columns is not accounted for in the AISC-LRFD column strength formulas. Furthermore, the effect of member out-of-plumbness on the force distribution in the structural system can be accounted for rigorously only if the out-of-plumbness is considered directly in the analysis. For example, to obtain the forces associated with out-of-plumbness in a braced or shear-wall core of a tall building system, the out-of-plumbness effects might be considered directly in the analysis. However, in most cases, these types of forces may be estimated conservatively by simple hand calculations, and they are often quite small compared to the forces associated with the lateral loads on the system.

Also, one would never expect all the columns of a building to be at the maximum permitted out-of-plumbness and all leaning in the same direction. Beaulieu and Adams[34] made an extensive study of out-of-plumbness and erection tolerances and their effect on frame stability and statistically derived the following "effective" value of Δ_i for stories of gravity loaded frames:

$$\Delta_i = \frac{0.006L}{n^{0.445}} \tag{7}$$

where L is the story height and n is the number of columns in the story. When n becomes larger than 12, the value of Δ_i obtained from this equation is less than $L/500$ (the AISC erection tolerance for columns), and when n is larger than 29, Δ_i is less than $L/750$.

Based on the above observations, the authors conclude that out-of-plumbness effects may often be neglected in the second-order elastic analysis of a structural system. However, the decision about whether they should be considered or not must be left to the judgment of the engineer. If these effects are considered, the frame might be analyzed for an out-of-plumbness specified as given by Equation 7 for two orthogonal directions in each story. The specification of out-of-plumbness may be easily automated using the computer.

INTERFACE BETWEEN THE COMPUTER AND THE USER

While second-order elastic analysis offers many benefits, the large amount of data which is generated from the multiple analyses requires careful management. Even the use of the LRFD B_1/B_2 approach generally requires more analyses and more careful management of data than do the equivalent analysis/design procedures of the ASD.[30] An interactive computer graphics interface greatly aids in this data manipulation. In particular, the creation and management of the structural geometry, loading, member properties, analysis parameters, analysis results, and design results will be expedited significantly if they are coupled with a fully developed graphical interface. Preparation and management of the results from different load combinations especially will be facilitated. The authors have had the opportunity to work with several current systems in research and in industry which serve as examples of the potential of sophisticated graphical interfaces.[18,23] There are several other emerging packages which provide similar interfaces. While the design procedure outlined in the next section may be more complex than procedures that are used commonly in practice, it is expected that the complexity will be offset by the added power and capacity of the current and new generations of computers and by the use of an effective interactive computer graphics interface. This environment will then allow the broad scope of analysis and design capabilities that make second-order elastic analysis so appealing. In fact, with an appropriate user interface, the comprehensive approaches are in certain respects simpler and more intuitive than conventional approaches.

RECOMMENDED DESIGN PROCEDURE

Based on the concepts outlined in the previous sections, it is possible to utilize comprehensive second-order elastic analysis approaches in the design of a lateral framing system according to the AISC LRFD Specification. Below are two recommended step-by-step analysis/design approaches which achieve this goal. The purpose of outlining these approaches is not to specify a restrictive procedure that should be followed if comprehensive second-order elastic analysis is to be used, but to outline an appropriate general framework for analysis and design using such methods. The first approach is a general procedure which does not employ any of the simplifying assumptions needed to apply superposition in a limited sense. The second procedure takes advantage of the simplifying assumptions previously discussed such that superposition can be employed for all the lateral load cases which are combined with a particular set of factored gravity loads.

Procedure 1

1. Perform one or more preliminary analysis/design steps. Some or all of these steps might be computerized. As discussed previously, approximate calculation of second-order effects, or use of "target" amplification factors, is considered to be more appropriate than use of a comprehensive matrix analysis procedure at these preliminary stages.
2. If desired, specify a minimum initial out-of-plumbness in two orthogonal directions for each story of the frame.
3. Analyze the structure for each factored combination of gravity and gravity plus lateral loads. Use a one-increment Newton-Raphson procedure for the analysis. A separate analysis is needed for all the different distributions and magnitudes of loading considered: for wind load in different directions, for full and patterned gravity loads, etc. Use the factored unreduced live load in these analyses.
4. Perform a linear elastic analysis of the nominal live load effects (load factor of 1.0), calculate member live load reduction percentages, and determine the member live load reduction forces as discussed previously.
5. For each load combination considered in the analysis/design iterations:
   a. Perform live load reduction for each member in the framework by subtracting the factored live load reduction forces obtained in Step 4 from the member load effects obtained in the analysis.
   b. For beam-columns, obtain the maximum second-order elastic moments, M_{ux} and M_{uy}, along the length of the member based on the end moments and axial force from the analysis and any local loading on the isolated member. For end-loaded members, determine these moments based on these loads and the B_1 amplification factor from Chapter H of the LRFD Specification (as discussed previously).
   c. Check each beam-column in the lateral framing system against the specification provisions. Use effective length factors that are greater than or equal to one for the calculation of the nominal axial resistance P_n unless the beam-column is "braced" by another portion of the structural system (as per Chapter C of the AISC LRFD Specification).
   d. Check the beams of the lateral system against the specification provisions. Obtain the moments within the span of the beams by statics given the end moments from the analysis and any local loading applied to the beam. These local loads may be reduced by member live load reduction percentages.
   e. Check the gravity columns using an effective length factor of unity for the calculation of P_n. Often, the forces for these members may be obtained by calculations separate from the matrix analysis.
6. Resize the members as required.

Of course, in Step 5, local loading effects often cannot be considered effectively in the overall analysis of the structure (this is true for any type of analysis, linear or second-order elastic). The forces computed for a particular member in the second-order elastic analysis may be used directly for design unless extra local effects need to be taken into account.

Procedure 2

The second procedure is exactly the same as the first with the exception of the performance of the matrix analyses in Step 3. For Step 3 of the second procedure, utilize the assumptions necessary to justify the use of superposition for all lateral load cases added to a particular set of factored gravity loads. For each of these sets of gravity load combined with multiple cases of lateral load, perform the analysis in two steps as discussed previously in the paper: first apply the factored gravity loads, and then apply all the corresponding lateral load cases as a linear increment from this factored gravity load state. Superimpose the gravity and lateral loads for use in Step 5.

CONCLUSIONS

The matrix analysis procedures outlined in this paper account comprehensively for second-order effects and are thus suitable for use in design of both simple structural systems as well as for systems with characteristics such as irregular geometry, leaned columns, and flexible floor diaphragms. These methods account for P-δ effects accurately in any situations where they may be significant. When formulated as outlined, the comprehensive matrix analysis procedures require about the same computational effort as conventional P-Δ type methods. Furthermore, use of these types of procedures reduces the number of tedious hand calculations and analysis decisions that must be made. For example, direct second-order elastic analysis eliminates the need for setting up separate no-translation and lateral-translation analysis cases. The designer is freed to concentrate on the physical behavior and the performance of the structure at hand.

The comprehensive procedures outlined here require some additional computation compared to linear elastic approaches. Because complications arise with the use of superposition, they require management of results from several analyses to account properly for load combinations. Nevertheless, continuing advances in hardware coupled with development of interactive computer graphics analysis/design systems will allow this technology to be applied effectively in practical design. At no time should any computer analysis, design algorithm, or data management be a "black box." The engineer must always be aware of the assumptions and capabilities of the software system being used. However, through the use of sophisticated interactive computer environments, integrated management and design of complex structural models may be performed efficiently and intuitively.

This paper has addressed the ways in which several critical design issues relate to the use of second-order elastic analysis. Further work is recommended in the following areas:

1. Assessment of the need for and possible improvement of the analysis technology to account better for effects of member lateral-torsional stability on the overall limit-states behavior of structural systems.
2. Investigation of possible LRFD beam-column design recommendations which alleviate the need for calculation of effective length factors. The use of the effective length concept seems necessary to formulate beam-column interaction equations which are generally applicable for a wide range of frame types and for any range of axial force versus moment in the beam-column members.[3,4] Yet, for complex structural systems, the calculation of effective length is a challenging and time-consuming part of the design effort which, in the context of the current specifications, is not alleviated by simply accounting for second-order elastic amplification of the load effects. Liew, et al.[3,4] have shown that if (1) limits are placed on the maximum allowed LRFD B_2 amplification factor and (2) a minimum out-of-plumbness is modeled in the analysis, the LRFD interaction curves with $K = 1$ used to determine the axial strength P_n correlate well with "exact" strength curves for some types of unbraced frame subassemblies. The current Canadian Standard[2] places a maximum limit of 1.4 on B_2, requires that a minimum lateral force of $0.005P$ be applied for gravity load analysis (this is related to out-of-plumb effects), and employs the actual column length in the beam-column deslgn calculatlons.
3. Assessment of the proper use of live load reduction with second-order elastic analysis of the structural system. The method proposed in this paper is relatively straightforward. It allows the engineer to use judgment in the computation of the reduction factors throughout the building in a manner consistent with current design practice. This is an important feature since the calculation of influence areas requires judgment on the part of the engineer. However, the implications of this as well as other live load reduction approaches should be investigated more thoroughly with respect to the designs obtained as well as the design effort required.
4. Implementation and use of a wide variety of possible second-order elastic analysis/design methods in a practical design setting. To the author's knowledge, the appropriate commercial software for accomplishing the comprehensive procedures discussed here does not currently exist. Several comprehensive analysis/design procedures have been outlined in this paper not only to summarize their scope and characteristics, but also to promote discussion of their desirable and undesirable features.

Second-order elastic matrix analysis, when coupled with integrated graphical analysis/design systems, allows the engineer to exercise greater freedom in structural design. Only by continuing to attack these coupled analysis and design issues will more advanced approaches be able to assist in providing both efficient and cost-effective design solutions.

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