Design Strength of Schifflerized Angle Struts

SESHU MADHAVA RAO ADLURI and MURTY K. S. MADUGULA

ABSTRACT

Latticed triangular-base steel towers have been used as communication structures for a long time. The legs of these towers generally consist of "schifflerized" angles (90° equal leg angles bent to 60°). For ready use of designers, the geometric properties and design axial compressive strengths of schifflerized angles are presented using recommendations based on a recent experimental study.

NOTATION

A = A_g = gross area of cross-section
a = length of unbent portion of schifflerized angle leg
b = length of bent portion of schifflerized angle leg
C_w = warping constant
F_{cr} = critical stress
I_{pc} = polar moment of inertia about centroid
I_{ps} = polar moment of inertia about shear center
I_u = maximum moment of inertia (about u-u axis)
I_v = minimum moment of inertia (about v-v axis)
J = Saint-Venant's torsion constant
r_u = maximum radius of gyration (about u-u axis)
r_v = minimum radius of gyration (about v-v axis)
t = thickness of the leg of schifflerized angle member
$\bar{u}_c$ = distance of the centroid from heel
$\bar{u}_s$ = distance between the centroid and shear center
ϕ_c = resistance factor for axial compression

INTRODUCTION

General

Steel angles find widespread use in steel construction. Of special interest is their application in latticed towers. These towers are generally designed to have a rectangular or triangular base. The triangular-base tower, with one leg member at each of the three corners of an equilateral triangle, is frequently employed in guyed and self-supporting communication towers. Triangular-base towers result in a significant reduction in the weight of the structure when compared to rectangular-base towers. In case of triangular-base towers, in order to have smooth bracing connections, the included angle between the two leg plates of the main leg members should be 60°. This is achieved by "schifflerizing" the hot-rolled equal leg 90° angles. Each leg plate is bent inwards by 15° so that the angle between the leg plate and the axis of symmetry of the section is 30° as shown in Fig. 1 instead of 45°. The process involves either re-rolling or brake-pressing a 90° angle. The finished member is called a schifflerized angle. Such angles are used in U.S.A., Canada, Australia and other countries in preference to solid round bars whenever possible. The schifflerization process increases the moment of inertia of the schifflerized angle about the minor principal axis by approximately 20% to 50% and reduces the maximum moment of inertia by an equivalent amount when compared to regular 90° angle. Properties like area and Saint-Venant's torsion constant remain the same for both the schifflerized angle and the corresponding 90° angle. While the shear center of a 90° angle lies at the intersection of the center lines of the legs, for a schifflerized angle, it does not lie at this intersection point but is shifted to a new location along the axis of symmetry further away from the centroid as shown in Fig. 1. The warping constant of the schifflerized angle is increased to one-and-a-half to two times the value for 90° angle. Consequently, the member is stronger in flexural buckling and is weaker in flexural-

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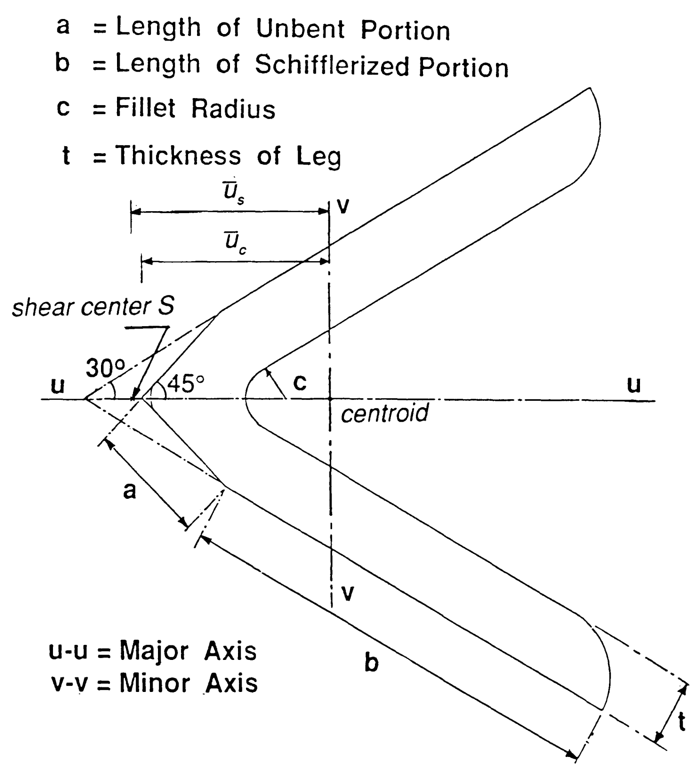

Fig. 1. Typical cross-section of a schifflerized angle.

torsional buckling when compared to the regular 90° angle. The schifflerization process, while bending the legs inward by 15° each, cannot deform the heel (or root) portion of the original regular angle because of its high rigidity. As a result, every 60° angle will have a 90° unchanged root portion of length ranging from 1 in. to 2 in. (25 to 50 mm) depending upon the leg thickness (dimension "a" in Fig. 1). The process also introduces additional residual stresses into the section which add to the existing residual stresses due to the differential cooling of hot-rolled angles. However, studies[1] have shown that the effect of these additional residual stresses is minimal.

Geometric Properties of Schifflerized Angles

Although schifflerized angles are extensively used in triangular-base towers, no published literature is available concerning their properties and behavior. Design engineers dealing with schifflerized angles have thus far depended on knowledge extrapolated from the published literature about the regular 90° angles. For the ready use of designers, the properties of these angles are presented in Table 1. These are based on equations developed in Ref. 2. As is the accepted practice, all the properties (including Saint-Venant's torsion constant) of schifflerized angles have been consistently derived by idealizing the section into segments of rectangles. The derivations are made by assuming that the angle leg plates are thin (i.e., the section is represented by center lines of plates). This does not result in any error for computing the area of cross-section. However, the radii of gyration will be conservative on an average by approximately 0.5% when compared to the values derived using thick plate assumption. The warping constant has been estimated using Goodier's theory.[2] It may be noted that the AISC LRFD manual[3] lists the properties of regular 90° angle sections (except Saint-Venant's torsion constant) computed by idealizing the section into rectangular segments (Saint-Venant's torsion constant was estimated by including the effect of fillet radius).

Design Axial Compressive Strength

Although AISC LRFD manual[3] states that it is virtually impossible to load single angle struts concentrically, it is to be noted that single schifflerized angles are traditionally designed as concentrically loaded struts. A recent study[1] at the University of Windsor showed that AISC LRFD specifications satisfactorily predict the strength of schifflerized angles when the flat width to be used in strength computations is taken as the width of the bent portion ("b" in Fig. 1) of the angle leg. A total of 18 specimens were tested under concentric loading with slenderness ratios ranging between 50 and 100. The test specimens were selected to cover both flexural and flexural-torsional modes of failure. The average value of the ratio of nominal axial compressive strength ($A_g F_{cr}$) computed using the above recommendation and the test failure load was 0.88. The standard deviation of the ratios was 0.056. Using this recommendation, for ready use of designers, the design compressive strengths ($\phi_c A_g F_{cr}$) of these angles as per AISC LRFD specifications,[3] for different lengths are presented in Tables 2 and 3 for nominal yield stresses of 36 ksi and 50 ksi respectively. In calculating the strengths, the local buckling effects are considered as per Appendix B of the AISC LRFD specification. Although the maximum allowable slenderness ratio for compression members is 200, slenderness ratios for schifflerized angles rarely exceed 120 (since they are only used as main legs and not as web members). To include special cases when the slenderness ratio may exceed 120, the design axial compressive strengths have been tabulated for a maximum slenderness ratio of 150. The stepped lines in Tables 2 and 3 show the demarcation between flexural-torsional buckling mode and flexural buckling mode. Member strengths to the left of the stepped lines are governed by flexural-torsional buckling and those to the right are governed by flexural buckling.

ACKNOWLEDGMENTS

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REFERENCES

1. Adluri, Seshu Madhava Rao, *Ultimate Strength of Schifflerized Angles,* Master of Science Thesis, University of Windsor, Windsor, Ontario, Canada, 1990.
2. Adluri, Seshu Madhava Rao and Madugula, Murty K. S., "Geometric Properties of Schifflerized Angles," *Journal of Constructional Steel Research,* (London: Elsevier, in press).
3. American Institute of Steel Construction, Inc., *Load and Resistance Factor Design Manual of Steel Construction,* (Chicago: AISC, 1986).

Table 1.
Geometric Properties of Schifflerized Angles

Size in.	Thickness in.	a in.*	Area sq. in.	$\bar{u}_c$ in.	I_u in.^{4}	r_u in.	I_v in.^{4}	r_v in.	J in.^{4}	$\bar{u}_s$ in.	I_{ps} in.^{4}	I_{pc} in.^{4}	C_w in.^{6}
8×8	1⅛	2	16.7	3.81	98.0	2.42	56.4	1.84	7.06	3.48	357	154	38.5
	1	1¾	15.0	3.77	86.5	2.40	51.7	1.86	5.00	3.50	322	138	27.3
	⅞	1⅝	13.2	3.72	76.6	2.41	46.4	1.87	3.38	3.53	288	123	18.7
	¾	1⅜	11.4	3.68	65.2	2.39	41.0	1.89	2.14	3.54	249	106	11.9
	⅝	1¼	9.61	3.63	55.0	2.39	35.1	1.91	1.25	3.56	212	90.1	7.03
	9/16	1 3/16	8.68	3.60	49.8	2.40	32.0	1.92	0.916	3.57	192	81.8	5.18
	½	1⅛	7.75	3.58	44.6	2.40	28.8	1.93	0.646	3.58	172	73.3	3.68
6×6	1	1¾	11.0	2.91	36.7	1.83	20.1	1.35	3.67	2.58	130	56.8	11.1
	⅞	1⅝	9.73	2.86	32.6	1.83	18.2	1.37	2.48	2.61	117	50.8	7.67
	¾	1⅜	8.44	2.82	27.7	1.81	16.3	1.39	1.58	2.63	102	44.0	4.91
	⅝	1¼	7.11	2.77	23.5	1.82	14.1	1.41	0.926	2.66	87.7	37.5	2.94
	9/16	1 3/16	6.43	2.74	21.3	1.82	12.9	1.42	0.679	2.67	80.0	34.2	2.18
	½	1⅛	5.75	2.72	19.1	1.82	11.6	1.42	0.479	2.68	72.0	30.7	1.56
	7/16	1 3/32	5.06	2.68	16.9	1.83	10.4	1.43	0.323	2.69	64.0	27.3	1.07
	⅜	1 1/16	4.36	2.65	14.7	1.84	9.02	1.44	0.204	2.71	55.7	23.7	0.698
	5/16	1	3.65	2.63	12.4	1.84	7.65	1.45	0.119	2.72	47.0	20.0	0.416
5×5	⅞	1⅝	7.98	2.43	19.0	1.54	9.93	1.12	2.04	2.14	65.3	28.9	4.29
	¾	1⅜	6.94	2.39	16.1	1.52	8.98	1.14	1.30	2.17	57.7	25.1	2.78
	⅝	1¼	5.86	2.34	13.7	1.53	7.80	1.15	0.763	2.20	49.8	21.5	1.68
	½	1⅛	4.75	2.28	11.2	1.53	6.51	1.17	0.396	2.22	41.2	17.7	0.897
	7/16	1 3/32	4.18	2.25	9.93	1.54	5.81	1.18	0.267	2.24	36.7	15.7	0.622
	⅜	1 1/16	3.61	2.22	8.66	1.55	5.07	1.19	0.169	2.25	32.0	13.7	0.408
	5/16	1	3.03	2.20	7.29	1.55	4.32	1.19	0.0986	2.27	27.1	11.6	0.245
4×4	¾	1⅜	5.44	1.96	8.31	1.24	4.25	0.884	1.02	1.69	28.1	12.6	1.36
	⅝	1¼	4.61	1.91	7.09	1.24	3.74	0.901	0.600	1.73	24.6	10.8	0.832
	½	1⅛	3.75	1.86	5.80	1.24	3.16	0.918	0.313	1.76	20.5	8.96	0.453
	7/16	1 3/32	3.31	1.82	5.18	1.25	2.83	0.925	0.211	1.77	18.4	8.01	0.316
	⅜	1 1/16	2.86	1.79	4.54	1.26	2.48	0.932	0.134	1.79	16.2	7.02	0.210
	5/16	1	2.40	1.77	3.83	1.26	2.13	0.941	0.0782	1.80	13.8	5.95	0.128
	¼	15/16	1.94	1.74	3.10	1.27	1.75	0.950	0.0404	1.82	11.2	4.85	0.0701
3½×3½	½	1¼	3.25	1.64	3.93	1.10	2.03	0.791	0.271	1.52	13.5	5.96	0.298
	7/16	1 3/32	2.87	1.61	3.52	1.11	1.83	0.798	0.183	1.53	12.1	5.34	0.209
	⅜	1 1/16	2.48	1.58	3.09	1.12	1.61	0.805	0.116	1.55	10.7	4.70	0.140
	5/16	1	2.09	1.55	2.61	1.12	1.38	0.814	0.0680	1.57	9.1	3.99	0.0858
	¼	15/16	1.69	1.53	2.12	1.12	1.14	0.822	0.0352	1.58	7.48	3.26	0.0476
3×3	½	1¼	2.75	1.43	2.51	0.954	1.21	0.663	0.229	1.28	8.18	3.71	0.182
	7/16	1 3/32	2.43	1.40	2.25	0.962	1.09	0.670	0.155	1.29	7.39	3.34	0.129
	⅜	1 1/16	2.11	1.37	1.98	0.969	0.966	0.677	0.0989	1.31	6.54	2.95	0.0865
	5/16	1	1.78	1.34	1.68	0.972	0.836	0.686	0.0579	1.32	5.63	2.52	0.0537
	¼	15/16	1.44	1.31	1.37	0.975	0.694	0.695	0.0300	1.34	4.65	2.06	0.0302
	3/16	15/16	1.09	1.28	1.07	0.989	0.535	0.700	0.0128	1.35	3.60	1.60	0.0157
2½×2½	½	1¼	2.25	1.22	1.47	0.808	0.644	0.535	0.188	1.02	4.47	2.12	0.0987
	⅜	1 1/16	1.73	1.15	1.18	0.823	0.521	0.548	0.0813	1.05	3.63	1.70	0.0480
	5/16	1	1.46	1.13	0.999	0.826	0.455	0.557	0.0477	1.08	3.15	1.45	0.0303
	¼	15/16	1.19	1.10	0.816	0.829	0.381	0.566	0.0247	1.09	2.62	1.20	0.0173
	3/16	15/16	0.90	1.06	0.640	0.842	0.295	0.572	0.0106	1.11	2.04	0.935	0.00912

* Note: See Fig. 1 for definition of "*a*."

Table 2. Design Strength of Schifflerized Angle Struts (Yield Stress = 36 ksi)

Size in.	Thickness in.	Compressive Strength in Kips for an Effective Length (in ft.) of																						
		2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
8×8	1 1/8	483	478	474	469	463	457	443	427	409	390	370	350	329	309	288	267	247	227	208	189	173	158	145
	1	427	421	417	413	408	402	395	384	368	352	334	316	298	280	261	243	225	207	190	174	158	145	133
	7/8	368	363	359	355	351	346	340	334	326	312	297	281	265	249	233	217	201	186	171	156	142	130	119
	3/4	308	302	298	295	291	287	283	277	272	265	258	245	231	217	204	190	176	163	150	138	126	115	105
	5/8	244	238	234	231	229	225	222	218	214	209	204	198	192	184	173	161	150	139	128	118	107	98	90
	9/16	211	204	201	198	196	193	190	187	184	180	175	170	165	160	154	147	136	126	117	107	98	89	82
	1/2	174	167	164	161	159	157	155	152	150	147	143	140	136	131	127	122	118	113	104	96	88	81	74
6×6	1	321	317	313	303	290	275	258	241	222	204	185	167	149	132	116	103	92	83					
	7/8	280	277	273	268	257	244	230	215	199	182	166	150	135	120	105	93	83	75					
	3/4	238	235	231	228	223	213	201	188	174	161	147	133	120	107	94	84	75	67					
	5/8	193	190	188	185	181	176	170	160	148	137	125	114	103	92	81	72	64	58					
	9/16	170	167	165	162	159	155	151	145	135	125	114	104	94	84	75	66	59	53					
	1/2	147	144	141	139	136	133	130	125	121	112	103	93	85	76	67	60	53	48					
	7/16	122	119	117	115	113	110	107	104	100	96	91	83	75	67	60	53	47	43					
	3/8	96	93	91	89	88	86	84	81	79	76	72	69	65	58	52	46	41	37					
	5/16	67	64	63	61	60	59	58	56	55	53	51	49	47	44	42	39	35	31					
5×5	7/8	233	230	222	210	196	181	165	149	133	117	102	87	75	65	57								
	3/4	199	196	193	183	172	159	146	132	118	104	91	79	68	59	52								
	5/8	164	162	159	155	146	136	125	113	101	90	79	68	59	51	45								
	1/2	127	125	123	120	116	111	102	93	84	74	66	57	49	43	38								
	7/16	107	105	103	101	98	95	90	82	74	66	58	51	44	38	34								
	3/8	87	85	83	82	79	77	74	71	64	58	51	44	38	33	29								
	5/16	66	64	62	61	60	58	56	53	51	48	43	38	33	28	25								
4×4	3/4	158	152	142	131	117	103	89	76	63	52	44	37											
	5/8	132	129	121	112	101	89	78	66	55	46	38	33											
	1/2	104	102	99	92	83	74	64	55	47	39	32	28											
	7/16	90	88	85	81	74	66	57	49	42	35	29	25											
	3/8	74	73	71	68	64	57	50	43	37	30	26	22											
	5/16	58	57	55	54	51	48	42	37	31	26	22	19											
	1/4	41	40	39	38	36	35	33	30	26	21	18	15											
3 1/2×3 1/2	1/2	92	89	82	73	64	55	46	37	30	25	21												
	7/16	79	77	73	65	57	49	41	33	27	22	19												
	3/8	67	65	63	57	50	43	36	29	24	20	17												
	5/16	53	52	50	48	42	36	31	25	20	17	14												
	1/4	39	38	37	35	33	30	25	21	17	14	12												
3×3	1/2	78	72	64	55	45	36	28	22	18														
	7/16	68	64	57	49	41	33	25	20	16														
	3/8	58	56	50	43	36	29	22	18	14														
	5/16	47	46	42	36	30	25	19	15	12														
	1/4	35	34	33	30	25	20	16	13	10														
	3/16	23	22	21	20	19	16	12	10	8														
2 1/2×2 1/2	1/2	62	54	45	36	27	19	15																
	3/8	48	42	35	28	21	16	12																
	5/16	40	36	30	24	19	14	11																
	1/4	31	29	25	20	16	12	9																
	3/16	21	20	19	15	12	9	7																

Table 3.
Design Strength of Schifflerized Angle Struts (Yield Stress = 50 ksi)

Size in.	Thickness in.	Compressive Strength in Kips for an Effective Length (in ft.) of																						
		2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
8 × 8	1 1/8	656	646	638	630	619	607	582	552	520	487	453	419	385	352	319	288	258	231	209	189	173	158	145
	1	576	566	558	551	542	531	518	498	470	440	411	380	350	321	292	264	236	212	191	174	158	145	133
	7/8	493	482	475	468	461	452	442	430	417	391	365	339	312	286	261	236	212	191	172	156	142	130	119
	3/4	406	395	389	383	377	370	361	352	342	330	318	296	273	251	229	208	188	168	152	138	126	115	105
	5/8	315	304	297	292	288	282	276	270	262	254	245	235	225	213	195	177	160	144	130	118	107	98	90
	9/16	260	249	244	239	235	231	227	222	216	210	203	196	188	179	171	160	145	131	118	107	98	89	82
	1/2	207	197	192	188	185	182	179	175	171	166	161	156	150	144	138	132	125	118	107	97	88	81	74
6 × 6	1	437	431	423	405	380	353	323	293	263	233	204	176	152	132	116	103	92	83					
	7/8	380	374	367	358	338	314	289	262	236	209	184	160	138	120	105	93	83	75					
	3/4	320	314	308	301	292	275	253	231	208	185	164	143	123	107	94	84	75	67					
	5/8	257	251	246	241	234	226	215	196	177	159	140	123	106	93	81	72	64	58					
	9/16	224	218	214	209	203	196	189	179	162	145	128	112	97	85	75	66	59	53					
	1/2	190	184	180	176	172	166	160	152	144	130	116	102	88	77	67	53	48						
	7/16	153	148	144	141	137	133	128	123	117	111	102	90	78	68	60	53	47	43					
	3/8	113	108	106	103	101	98	95	91	87	83	79	74	68	59	52	46	41	37					
	5/16	75	72	70	68	67	65	63	61	59	56	54	51	48	45	42	39	35	31					
5 × 5	7/8	318	311	296	275	250	224	197	171	146	122	102	87	75	65	57								
	3/4	270	265	258	241	220	198	175	153	131	110	92	79	68	59	52								
	5/8	220	216	210	203	187	169	150	131	113	96	80	68	59	51	45								
	1/2	167	163	159	154	148	139	123	108	94	80	67	57	49	43	38								
	7/16	139	136	132	128	123	117	109	96	83	71	60	51	44	38	34								
	3/8	110	107	104	101	97	93	88	82	73	62	52	44	38	33	29								
	5/16	77	74	72	70	68	65	62	59	55	51	44	38	33	28	25								
4 × 4	3/4	216	205	186	165	142	119	98	78	63	52	44	37											
	5/8	179	174	159	142	123	104	85	68	55	46	38	33											
	1/2	140	136	130	117	102	86	72	58	47	39	32	28											
	7/16	119	115	111	103	90	77	64	52	42	35	29	25											
	3/8	97	94	91	86	79	67	56	45	37	30	26	22											
	5/16	74	72	69	66	62	57	48	39	32	26	22	19											
	1/4	49	47	45	43	41	39	36	32	26	21	18	15											
3 1/2 × 3 1/2	1/2	124	119	105	91	75	60	47	37	30	25	21												
	7/16	106	102	94	81	67	54	42	33	27	22	19												
	3/8	88	85	81	70	59	48	37	29	24	20	17												
	5/16	69	66	63	59	50	41	32	25	20	17	14												
	1/4	48	46	44	42	39	33	26	21	17	14	12												
3 × 3	1/2	106	94	80	64	49	37	28	22	18														
	7/16	92	84	71	58	44	33	25	20	16														
	3/8	77	73	62	50	39	29	22	18	14														
	5/16	62	59	53	43	34	25	19	15	12														
	1/4	45	43	41	35	28	21	16	13	10														
	3/16	27	26	25	23	21	16	12	10	8														
2 1/2 × 2 1/2	1/2	83	69	53	38	27	19	15																
	3/8	64	54	42	31	21	16	12																
	5/16	53	46	36	27	19	14	11																
	1/4	40	38	30	22	16	12	9																
	3/16	26	25	23	17	12	9	7																