Proposed Revision of the Equivalent Axial Load Method for LRFD Steel and Composite Beam-Column Design

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ABSTRACT

After studying the interaction equations that form the basis of the equivalent axial load method for selecting a trial section for a beam-column, the authors propose a series of revisions to the design aids and procedures in the current edition of the LRFD manual.[3] The revisions affect both steel and composite beam-columns. The suggested modifications will permit designers to select a better trial section, thereby reducing the computation time for the design of beam-columns.

INTRODUCTION

The design of beam-column is an iterative procedure. To begin the process, the designer chooses a trial section and then checks the appropriate equation to verify that the capacity of the member is just adequate for the axial load and moment it must support. If the analysis indicates that the capacity of the member is fully or almost fully utilized, the design is complete. On the other hand, if the analysis reveals that either the capacity of the member is exceeded or that the section is substantially understressed (and therefore uneconomical because excess material is employed), a new section is selected and the computations repeated. Typically several trial sections must be investigated before the most economical section is established.

To expedite the process, the designer must initially select a section whose design strength, when fully mobilized, is equal to or slightly less than the required strength. One method, originally developed for allowable stress design by Burgett,[2] that permits the designer to select an efficient W, M or S-shaped section (the sections most commonly used in buildings) is to convert the design moment to an equivalent increment of axial load. The equivalent axial load is then added to the actual axial load to produce a hypothetical effective axial load for which the column must be sized. In other words, for the purpose of selecting a trial section, the more complex beam-column design is converted to the simple problem of designing an axially loaded column. Once the effective axial force is established, the designer can use the column tables in the AISC Manual to select the initial trial section.

The equivalent axial load method can be used in either the *Allowable Stress Design* (ASD) method or in the *Load and Resistance Factor Design* (LRFD) method. In Part 3 of the ASD Manual[1] and in Part 2 of the LRFD Manual,[3] the equivalent load method is described and a Table B supplied which lists m-coefficients that convert a design moment about the strong-axis to an equivalent axial force. If the design moment produces strong-axis bending, the equivalent axial force is evaluated by multiplying the design moment by m only. On the other hand, if the design moment produces weak-axis bending, the equivalent axial force is given by the product of m, the design moment, and a coefficient U. The coefficient U is tabulated in the column tables following Table B.

For both the ASD and the LRFD methods this conversion can be expressed in general terms as:

For ASD $\qquad P_{eff} = P + mM_x + mUM_y \qquad$ (1a)

For LRFD $\qquad P_{ueff} = P_u + mM_{ux} + mUM_{uy} \qquad$ (1b)

where

P_u, P = factored and unfactored axial forces

M_{ux}, M_x = factored and unfactored moments acting about the strong axis

M_{uy}, M_y = factored and unfactored moments acting about the weak axis, respectively.

Although the equivalent load method produces reasonable trial sections in the ASD method, experience with the m-coefficients in Table B of the LRFD Manual indicates that the equivalent axial force computed with Eq. 1b is excessively large. As a result, when the terms in the interaction equation are evaluated using the properties of the trial section, designers will find that the member is significantly understressed and they will have to select a new trial sec-

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tion (with a smaller cross-sectional area) and repeat the computations, which are lengthy. Frequently, the designer must investigate three or four trial sections before arriving at the most economical one. The excessively conservative nature of the m-coefficients in Table B of the current LRFD Manual was also observed by Smith,[4] who recommends that they be reduced by a factor of 0.65 for steel with $F_y = 36$ ksi and by 0.75 for steel with $F_y = 50$ ksi. Smith also notes that at the bottom of Table B, the statement in the footnote which indicates the m values have to be modified by the ratio of $C_m/0.85$ is not applicable and can be eliminated. A recent study by Uang and Wattar[5] derives values of the m-coefficients that compare closely with those recommended by Smith. In addition Uang and Wattar also establish values of U that appear to be superior to those currently tabulated in the column design tables between pages 2-16 and 2-32 in the LRFD Manual.[3] Although no single set of coefficients can ensure that the first trial section will be the most economical, designers who use the values of m and U recommended by Uang and Wattar will typically find that the resulting trial section is very close to the optimum section. Accordingly, the authors recommend that the current m-coefficients in Table B and the U values in the column tables in the current LRFD manual be replaced by the values given in this paper.

In the second half of this paper the authors extend the equivalent axial load method to the design of composite beam-columns carrying axial load and moment. The current manual does not provide an efficient method for the design of these types of members. The discussion of this subject is limited to columns constructed of W shapes encased in reinforced concrete and reinforced with grade 60 reinforcing steel. Typical cross sections of composite columns are shown in Part 4 of the LRFD Manual.

To aid the designer, a table of m-coefficients for composite columns is presented with the recommendation that it too be incorporated into future editions of the LRFD Manual. Following a discussion of the interaction equations upon which the new coefficients for both steel and composite columns are based, example problems covering the design of both steel and composite columns are worked to illustrate the advantages of the coefficients and procedures in this paper compared to those in the LRFD Manual.

EQUIVALENT AXIAL LOAD METHOD

Allowable Stress Design (ASD)

In ASD, a beam-column section has to satisfy the following stability (interaction) formula when $f_a/F_a > 0.15$:

$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{\left(1 - \frac{f_a}{F'_{ex}}\right) F_{bx}} + \frac{C_{my} f_{by}}{\left(1 - \frac{f_a}{F'_{ey}}\right) F_{by}} \leq 1.0 \qquad (2)$$

The above formula can be converted into the following equivalent axial load form:[2]

$$P + mM_x + mUM_y \leq P_{eff} \qquad (3)$$

where

$$m = C_m \left(\frac{A}{S_x}\right) \left(\frac{F_a}{F_{bx}}\right) \left(\frac{149{,}000Ar^2}{149{,}000Ar^2 - P(KL)^2}\right) \qquad (4a)$$

$$U = \frac{C_{mx} S_x F_{bx} (1 - f_a/F'_{ex})}{C_{my} S_y F_{by} (1 - f_a/F'_{ey})} \qquad (4b)$$

For each structural section listed in the manual, the m value can be calculated for arbitrary values of effective length KL and unbraced length L_b. We note from Eq. 4a that m is directly proportional to C_m, the factor in the amplification factor that accounts for the effect of moment gradient. Assuming a compact section an approximate value of the U factor in Eq. 4b is established by the AISC using the simplifications indicated below:

$$U = \frac{C_{mx} S_x F_{bx} (1 - f_a/F'_{ex})}{C_{my} S_y F_{by} (1 - f_a/F'_{ey})} \approx (1) \left(\frac{S_x}{S_y}\right) \left(\frac{0.66F_y}{0.75F_y}\right) = 0.88 \left(\frac{S_x}{S_y}\right) \qquad (5)$$

Load and Resistance Factor Design (LRFD)

The following interaction formulae have to be satisfied for both steel and composite beam-column design:

$$\text{for } \frac{P_u}{\phi P_n} > 0.2 \qquad \frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0 \qquad (6a)$$

$$\text{for } \frac{P_u}{\phi P_n} \leq 0.2 \qquad \frac{P_u}{2\phi P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0 \qquad (6b)$$

where M_u is calculated by multiplying moments from a first order analysis by amplification factors B_1 and B_2 to include the P-Delta effects:

$$M_u = B_1 M_{nt} + B_2 M_{lt} \qquad (7)$$

$$B_1 = \frac{C_m}{1 - \frac{P_u}{P_{ex}}} \geq 1 \qquad (8a)$$

$$B_2 = \frac{1}{1 - \frac{\Sigma P_u}{\Sigma P_e}} \qquad (8b)$$

Equations 6a and 6b can be converted into the equivalent axial load form as follows:

$$\text{for } \frac{P_u}{\phi P_n} > 0.2$$

$$P_u + mM_{ux} + mUM_{uy} \leq P_{ueff} \qquad (9a)$$

$$\text{for } \frac{P_u}{\phi P_n} \leq 0.2$$

$$\frac{P_u}{2} + \frac{9}{8}mM_{ux} + \frac{9}{8}mUM_{uy} \leq P_{ueff} \tag{9b}$$

where

$$m = \frac{8\phi P_n}{9\phi_b M_{nx}} \tag{10a}$$

$$U = \frac{M_{nx}}{M_{ny}} \tag{10b}$$

To establish a design aid, the values of m are computed by Eq. 10a for all W, M and S shapes listed in the LRFD Manual between pages 2-15 and 2-32. In this computation, following the same format as in the LRFD Manual, P_n is evaluated in intervals of one foot for effective length values of KL between 10 and 22 ft using the column equations in Chapter E of the LRFD Manual. M_n is evaluated by the appropriate equations for moment in Chapter F considering the possibility of lateral torsional buckling, and the unbraced length L_b of the compression flange is conservatively assumed equal to the effective length of the column. To facilitate these computations, the authors use an AISC database which lists the properties of each steel section. After the values of m for all members of a given depth are computed, an average value of m is calculated and recorded in Table 1 of the paper, which is equivalent to Table B in the manual.

Following a similar procedure, Eq. 10b is used to evaluate U for each section in the column tables. These values are recorded in Table 2. In the computation of U, the possibility of lateral torsional buckling is considered when evaluating the nominal flexural strength M_{nx} with respect to strong-axis bending. On the other hand, since lateral torsional buckling does not influence the weak-axis nominal flexural strength, M_{ny} may be computed as

$$M_{ny} = Z_y F_y \tag{11}$$

where Z_y is the plastic modulus with respect to the weak-axis. A comparison of U values in Table 2 with those in the column tables of the LRFD Manual shows that the Table 2 values may run as much as 50 to 60 percent larger than those in the manual. Moreover, as can be seen from Eq. 10b, U values decrease as the unbraced length increases because the moment capacity M_{nx} of longer members is controlled by lateral torsional buckling.

Note from Eq. 10a that m is not related to C_m when the value of C_m is not close to one, the amplification factor for B_1 for most columns is likely to equal one. In other words, the role of C_m in LRFD is not as significant as it is in ASD.

RECOMMENDED REVISED PROCEDURE FOR SELECTING A TRIAL STEEL BEAM-COLUMN

1. For a given value of KL, select a first approximate value of m from the second row of Table 1. If moment about the weak axis is involved, let $U = 2.0$.
2. Solve $P_{ueff} = P_u + mM_{ux} + mUM_{uy}$.
3. Calculate P_u/P_{ueff}. If $P_u/P_{ueff} \leq 0.2$, modify the equivalent axial load as:

$$P_{ueff} = \frac{P_u}{2} + \frac{9}{8}mM_{ux} + \frac{9}{8}mUM_{uy}.$$

4. From the column load tables, select a section to support P_{ueff}.
5. Based on the depth of the section selected in step 4, select a revised value of m and U from Tables 1 and 2, respectively. Use the coefficients in the lower part of Table 1 titled "subsequent approximation."
6. With the new values selected, recompute P_{ueff}.
7. Repeat steps 4 and 5 until the values of m and U stabilize.
8. Check the section obtained in step 7 with the appropriate interaction formula (H1-1a) or (H1-1b) in the Manual[3] (also given as Eqs. 6 and 6b in this paper).

Several examples are used to illustrate the above procedure using the values of m and U in this paper. Example 1 is solved first by using m and U values tabulated in the current LRFD Manual and then by the revised m and U values contained in this paper.

EXAMPLE 1

Select the lightest W12 shape for the member in a braced frame shown in Fig. 1. The axial load and moment are calculated from a first order analysis using factored loads. Use A36 steel.

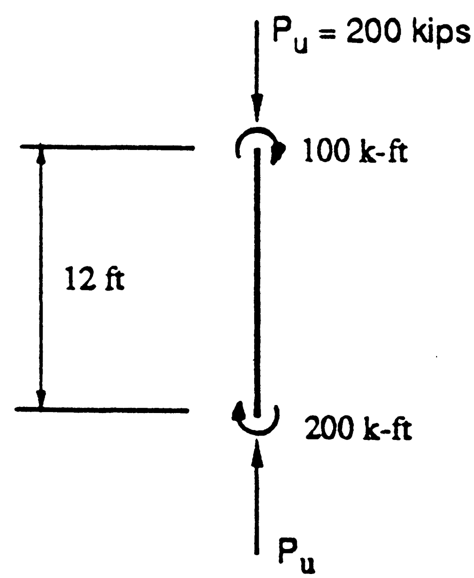

Figure 1

Solution 1:

Using the m coefficient from Table B in the LRFD Manual.

1. $C_m = 0.6 - 0.4(M_1/M_2) = 0.6 - 0.4(100/200) = 0.4$
   For $KL = 12$ ft and W12 shape, the m value from Table B (p. 2-10, LRFD Manual) is:
   $m = 2.5(C_m/0.85) = 2.5(0.4/0.85) = 1.18$
   (Note the effect of C_m on m here.)
2. $P_{ueff} = P_u + mM_{ux} = 200 + (200 \times 1.18) = 436$ kips
   Select W12×58($\phi_c P_n = 437$ kips).
3. From Table B, a second trial m value is computed as
   $m = 2.5(C_m/0.85) = 2.5(0.4/0.85) = 1.18$
   Since the value of m stabilizes, check the W12×58 per Formula (H1-1).
4. Calculate amplification factor B_1. (B_2 is zero for braced frame.)

$$\left(\frac{Kl}{r}\right)_x = \frac{12 \times 12}{5.28} = 27.3$$

$$\lambda_{cx} = \frac{(Kl/r)_x}{\pi}\sqrt{\frac{F_y}{E}} = \frac{27.3}{\pi}\sqrt{\frac{36}{29000}} = 0.306$$

$$P_{ex} = A_g\left(\frac{F_y}{\lambda_{cx}^2}\right) = 17.0\left(\frac{36}{0.306^2}\right) = 6536 \text{ kips}$$

$$B_1 = \frac{C_m}{1-\dfrac{P_u}{P_{ex}}} = \frac{0.4}{1-\dfrac{200}{6536}} = 0.413 < 1.0 \quad \text{use } B_1 = 1.0$$

5. From Step 2

   $\phi_c P_n = 437$ kips
   $[L_p = 10.5 \text{ ft}] < [L_b = 12.0 \text{ ft}] < [L_r = 38.4 \text{ ft}]$
   $BF = 2.91$, $\phi_b M_b = 233$ kip-ft
   $C_b = 1.75 + 1.05(M_1/M_2) + 0.3(M_1/M_2)^2$
   $= 1.75 + 1.05(100/200) + 0.3(100/200)^2 = 2.35 > 2.3$
   Use $C_b = 2.3$.
   $\phi_b M_n = C_b[\phi_b M_p - BF(L_b - L_p)]$
   $= 2.3[233 - 2.91(12 - 10.5)] = 526 \text{ kip-ft} > \phi_b M_p$
   Use $\phi_b M_n = \phi_b M_p = 233$ kip-ft.

$$\frac{P_u}{\phi_c P_n} = \frac{200}{437} = 0.458 > 0.2, \text{ therefore use Formula (H1-la).}$$

$$\frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}}\right) = 0.458 + \frac{8}{9}\left(\frac{200}{233}\right) = 1.220 > 1 \quad \textbf{N.G.}$$

6. Try larger section W12×72.
   $\phi_c P_n = 574$ kips
   $[L_p = 12.7 \text{ ft}] > [L_b = 12 \text{ ft}]$
   $\phi_b M_n = \phi_b M_p = 292$ kip-ft

$$\frac{P_u}{\phi_c P_n} = \frac{200}{574} = 0.348 > 0.2$$

$$\frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}}\right) = 0.348 + \frac{8}{9}\left(\frac{200}{292}\right) = 0.957 < 1 \quad \textbf{O.K.}$$

Use W12×72.

Solution 2:

Using m-coefficients in Table 1 of this paper.

1. For $KL = 12$ ft and W12, $m = 1.7$
2. $P_{ueff} = P_u + mM_{ux} = 200 + 1.7 \times 200 = 540$ kips
3. $\dfrac{P_u}{P_{ueff}} = \dfrac{200}{540} > 0.2$, no need to modify formula.
4. From column table, try W12×72. ($\phi_c P_n = 574$ kips.) Note this shape is the correct one. The check of the interaction formula is the same as in Solution 1.

EXAMPLE 2

Example 2 demonstrates the equivalent axial load method when moment is large relative to the axial force (i.e., $P_u/\phi P_n < 0.2$).

$P_u = 50$ kips
$M_{ntx} = 300$ kip-ft
$M_{ltx} = 0$ kip-ft (braced frame)
$Kl_x = Kl_y = L_b = 10$ ft
$C_m = 1.0$
$F_y = 36$ ksi

Solution:

1. From Table 1 select a first trial value of $m = 2.0$.
2. $P_{ueff} = mM_{ux} = 50 + 2.0 \times 300 = 650$ kips

   $\dfrac{P_u}{P_{ueff}} = \dfrac{50}{650} = 0.077 < 0.2,$

   so use Eq. 9b to recompute P_{ueff}:

   $$P_{ueff} = \frac{50}{2} + \frac{9}{8} \times 2.0 \times 300 = 700 \text{ kips}$$

3. From column load tables select W12×87
   ($\phi_c P_n = 723$ kips)
4. From Table 1, subsequent approximate value of $m = 1.7$

   $$P_{ueff} = \frac{50}{2} + \frac{9}{8} \times 1.7 \times 300 = 599 \text{ kips}$$

5. From column load tables select W12×79
   ($\phi_c P_n = 654$ kips)
6. For W12×79, $m = 1.7$. The m value stabilizes.
7. Check W12×79

   $$\frac{P_u}{\phi P_n} = \frac{50}{654} = 0.08$$

   $\phi_b M_{nx} = 321$ kip-ft (from beam chart)

Table 1. Revised Table of m for Steel Beam-Column														
Values of m														
F_y	36 ksi							50 ksi						
KL (ft)	10	12	14	16	18	20	22 and over	10	12	14	16	18	20	22 and over
1st Approximation														
All Shapes	2.0	1.9	1.8	1.7	1.6	1.5	1.3	1.9	1.8	1.7	1.6	1.4	1.3	1.2
Subsequent Approximation														
S4,5,6	1.3	1.0	0.8	0.7	0.6	0.5	0.5	1.1	0.9	0.8	0.7	0.6	0.5	0.5
W,M4	3.1	2.3	1.7	1.4	1.1	1.0	0.8	2.4	1.8	1.4	1.1	1.0	0.9	0.8
W,M5	3.2	2.7	2.1	1.7	1.4	1.2	1.0	2.8	2.2	1.7	1.4	1.1	1.0	0.9
W,M6	2.8	2.5	2.1	1.8	1.5	1.3	1.1	2.5	2.2	1.8	1.5	1.3	1.2	1.1
W8	2.5	2.3	2.2	2.0	1.8	1.6	1.4	2.4	2.2	2.0	1.7	1.5	1.3	1.2
W10	2.1	2.0	1.9	1.8	1.7	1.6	1.4	2.0	1.9	1.8	1.7	1.5	1.4	1.3
W12	1.7	1.7	1.6	1.5	1.5	1.4	1.3	1.7	1.6	1.5	1.5	1.4	1.3	1.2
W14	1.5	1.5	1.4	1.4	1.3	1.3	1.2	1.5	1.4	1.4	1.3	1.3	1.2	1.2

$$\frac{P_u}{2\phi P_n} + \frac{M_{ux}}{\phi_b M_{nx}} = \frac{50}{2(654)} + \frac{300}{321} = 0.97 \leq 1.0 \quad \textbf{O.K.}$$

Use W12×79.

VALUES OF m AND U FOR COMPOSITE BEAM-COLUMN

The same interaction Eqs. 6a and 6b used for steel beam-column design also control the design of composite beam-columns. The formulas for m and U (Eq. 10) are the same as those for steel member. The only difference in evaluating m and U for the composite column is that the nominal flexural strength is independent of the unsupported length, i.e., lateral-torsional buckling is not a concern for composite beam-columns.

Based on a statistical analysis of the composite column sections listed in the LRFD Manual, the recommended m values are listed in Table 1. The tabular value is for $f'_c = 5$ ksi. When f'_c is other than 5 ksi, the tabular value of m should be multiplied by a factor $\sqrt{f'_c/5}$. The U value is not available in Part 4 of the LRFD Manual. The first approximated U value can be taken as 1.6. The subsequent value for each section can be calculated easily by Eq. 10b from the design flexural strengths $\phi_b M_{nx}$ and $\phi_b M_{ny}$ tabulated in the column tables for composite sections in Part 4 of the LRFD Manual. The value of U typically ranges between 1.4 to 1.7.

RECOMMENDED DESIGN PROCEDURE FOR COMPOSITE BEAM-COLUMN

1. With a known value of KL, select a first approximate value of m from Table 3. If moment about the weak axis is involved, let first approximate value of $U = 1.6$.
2. Solve $P_{ueff} = P_u + mM_{ux} + mUM_{uy}$
3. Calculate P_u/P_{ueff}. If $P_u/P_{ueff} \leq 0.2$, modify equivalent axial load as:

$$P_{ueff} = \frac{P_u}{2} + \frac{9}{8}mM_{ux} + \frac{9}{8}mUM_{uy}.$$

4. From the composite column load tables, select a section to support P_{ueff}.
5. Based on the section selected in step 4, select a subsequent approximate value of m from Table 3. The value of $U = \dfrac{\phi_b M_{nx}}{\phi_b M_{ny}}$.
6. With the values selected, solve for P_{ueff}.
7. Repeat steps 4 and 5 until values of m and U stabilize.
8. Check the section obtained in step 7 with the appropriate interaction formula (H1-1a) or (H1-1b) in the Manual (also given as Eqs. 6a and 6b in this paper).

Examples 3 and 4 in Part 4 of LRFD Manual are resolved using the equivalent axial load method.

EXAMPLE 3

Design a composite encased W shape column to resist a factored axial load of 200 kips and a factored moment about the X-X axis of 240 kip-ft. The unsupported length of the column is 12 ft, $F_y = 50$ ksi, $f'_c = 3.5$ ksi and $C_m = 1.0$. The loads were obtained by the first order elastic analysis and there is no lateral translation of column ends.

Solution:

1. For $KL = 12$ ft, $F_y = 50$ ksi, $f'_c = 3.5$ ksi, from Table 3 select a first trial value of $m = 2.6\sqrt{3.5/5} = 2.2$.

Table 2.
Revised U Values for Steel Beam-Column

	F_y=36 ksi	F_y=50 ksi
W14×730	2.03	2.03
W14×665	2.02	2.02
W14×605	2.02	2.01
W14×550	2.02	2.01
W14×500	2.01	2.00
W14×455	1.99	1.99
W14×426	2.00	1.99
W14×398	1.99	1.98
W14×370	1.98	1.97
W14×342	1.98	1.97
W14×311	1.97	1.96
W14×283	1.97	1.95
W14×257	1.97	1.95
W14×233	1.96	1.94
W14×211	1.95	1.93
W14×193	1.96	1.93
W14×176	1.94	1.92
W14×159	1.94	1.92
W14×145	1.93	1.90
W14×132	2.03	1.99
W14×120	2.04	1.99
W14×109	2.02	1.97
W14×99	2.02	1.95
W14×90	2.02	1.94
W14×82	2.85	2.68
W14×74	2.82	2.62
W14×68	2.80	2.56
W14×61	2.74	2.44
W14×53	3.20	2.70
W14×48	3.12	2.56
W14×43	2.97	2.37
W12×336	2.18	2.17
W12×305	2.18	2.16
W12×279	2.16	2.15
W12×252	2.16	2.14
W12×230	2.15	2.13
W12×210	2.16	2.13
W12×190	2.14	2.11
W12×170	2.14	2.11
W12×152	2.15	2.11
W12×136	2.13	2.09
W12×120	2.12	2.07
W12×106	2.12	2.06
W12×96	2.10	2.04
W12×87	2.10	2.02
W12×79	2.09	2.01
W12×72	2.08	1.98

	F_y=36 ksi	F_y=50 ksi
W12×65	2.06	1.95
W12×58	2.41	2.22
W12×53	2.39	2.16
W12×50	2.85	2.51
W12×45	2.79	2.37
W12×40	2.69	2.22
W10×112	2.06	2.02
W10×100	2.06	2.01
W10×88	2.04	1.99
W10×77	2.03	1.96
W10×68	2.01	1.93
W10×60	2.00	1.90
W10×54	1.97	1.87
W10×49	1.96	1.83
W10×45	2.37	2.17
W10×39	2.31	2.04
W10×33	2.23	1.87
W8×67	2.03	1.96
W8×58	2.00	1.93
W8×48	1.97	1.87
W8×40	1.93	1.80
W8×35	1.89	1.74
W8×31	1.85	1.65
W8×28	2.17	1.87
W8×24	2.07	1.71
W6×25	2.07	1.98
W6×20	2.03	1.91
W6×15	1.98	1.75
W6×16	2.84	2.50
W6×12	2.62	2.13
W6×9	2.24	1.72
W5×19	1.84	1.72
W5×16	1.79	1.63
W4×13	1.89	1.77
M6×20	1.97	1.77
M5×18.9	1.96	1.85
M4×13	1.98	1.88
S6×17.25	3.86	3.57
S6×12.5	3.70	3.24
S5×14.75	3.49	3.29
S5×10	3.38	3.01
S4×9.5	3.10	2.89
S4×7.7	3.03	2.77
S3×7.5	2.57	2.44
S3×5.7	2.55	2.36

2. $P_{ueff} = P_u + mM_{ux} = 200 + 2.2 \times 240 = 728$ kips
3. For $f'_c = 3.5$ ksi, composite column load tables will lead to a W8 shape with 16-in.×16-in. encasement. From Table 3 with a W8, the revised value of $m = 3.5\sqrt{3.5/5} = 2.93$.

   $P_{ueff} = P_u + mM_{ux} = 200 + 2.93 \times 240 = 903$ kips
4. From composite column load tables, try W8×40 ($\phi P_n = 920$ kips)
5. Check W8×40

From composite column load tables, $\dfrac{P_{ex}(KL_x)^2}{10^4} = 78.5$

$$\text{so } P_{ex} = \frac{78.5 \times 10^4}{(12)^2} = 5{,}451 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - \dfrac{P_u}{P_{ex}}} = \frac{1.0}{1 - \dfrac{200}{5{,}451}} = 1.04 \geq 1.0$$

Table 3.
Table of m for Composite Beam-Column

Values of m														
F_y	36 ksi							50 ksi						
KL (ft)	10	12	14	16	18	20	22 and over	10	12	14	16	18	20	22 and over
1st Approximation														
All Shapes	3.1	3.0	3.0	2.9	2.8	2.7	2.6	2.7	2.6	2.6	2.5	2.4	2.4	2.2
Subsequent Approximation														
W8	4.2	4.1	3.9	3.8	3.6	3.5	3.3	3.6	3.5	3.4	3.3	3.1	3.0	2.8
W10	3.4	3.4	3.3	3.2	3.1	3.0	2.9	3.0	2.9	2.9	2.8	2.7	2.6	2.4
W12	2.5	2.4	2.4	2.3	2.3	2.2	2.2	2.2	2.2	2.1	2.1	2.0	2.0	1.9
W14	2.2	2.2	2.2	2.1	2.1	2.0	2.0	2.0	2.0	1.9	1.9	1.8	1.8	1.7

* Values of m are for f_c'=5 ksi. When f_c' is other than 5 ksi, multiply the tubular values of m by $\sqrt{f_c'/5}$.

$$M_{ux} = B_1 M_{ntx} = 1.04 \times 240 = 249.6 \text{ kip-ft}$$

$$\frac{P_u}{\phi P_n} = \frac{200}{920} = 0.217 > 0.2$$

$$\frac{P_u}{\phi P_n} + \frac{8M_{ux}}{9\phi_b M_{nx}} = 0.217 + \frac{8}{9}\left(\frac{249.6}{251}\right) = 1.10 > 1.0 \quad \textbf{N.G.}$$

6. Try next larger section W8×48.
   $\phi P_n = 1{,}010$ kips, $\phi_b M_n = 291$ kip-ft

$$\frac{P_{ex}(KL_x)^2}{10^4} = 89.3, \text{ so } P_{ex} = \frac{89.3 \times 10^4}{(12)^2} = 6{,}201 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - \frac{P_u}{P_{ex}}} = \frac{1.0}{1 - \frac{200}{6{,}201}} = 1.03 \geq 1.0$$

$$M_{ux} = B_1 M_{ntx} = 1.03 \times 240 = 247.2 \text{ kip-ft}$$

$$\frac{P_u}{\phi P_n} = \frac{200}{1{,}010} = 0.198 < 0.2$$

$$\frac{P_u}{2\phi P_n} + \frac{M_{ux}}{\phi_b M_{nx}} = \frac{0.198}{2} + \frac{247.2}{291} = 0.95 < 1.0 \quad \textbf{O.K.}$$

Use 16-in.×16-in. column with W8×48 of F_y = 50 ksi, f_c' = 3.5 ksi, 4-#7 Gr. 60 longitudinal bars and #3 Gr. 60 ties spaced at 10 in.

EXAMPLE 4

Design a composite encased W shape column to resist a factored axial load of 1,100 kips and factored moment of 200 kip-ft. Use 50-ksi structural steel and 5-ksi concrete. The unsupported column length is 11 ft and C_m = 0.85. Assume that sidesway is prevented.

Solution:

1. For KL = 11 ft, F_y = 50 ksi, select a first trial value of m = 2.65 from Table 3 by interpolation.
2. $P_{ueff} = P_u + mM_{ux} = 1{,}100 + 2.65 \times 200 = 1{,}630$ kips
3. For f_c' = 5 ksi, using composite column load tables, try a W10×77 shape with 18-in.×18-in. encasement. From Table 1, the revised value of m = 2.95.
   $P_{ueff} = P_u + mM_{ux} = 1{,}100 + 2.95 \times 200 = 1{,}690$ kips
4. From composite column load tables, try W10×77.
   (ϕP_n = 1,720 kips)
5. Check W10×77

$$\frac{P_{ex}(KL_x)^2}{10^4} = 178, \text{ so } P_{ex} = \frac{178 \times 10^4}{(11)^2} = 14{,}710 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - \frac{P_u}{P_{ex}}} = \frac{0.85}{1 - \frac{1{,}100}{14{,}710}} = 0.92 \geq 1.0, \text{ use } B_1 = 1.0$$

$$M_{ux} = B_1 M_{ntx} = 1.0 \times 200 = 200 \text{ kip-ft}$$
$$\phi_b M_{nx} = 555 \text{ kip-ft}$$

$$\frac{P_u}{\phi P_n} = \frac{1{,}100}{1{,}720} = 0.640 > 0.2$$

$$\frac{P_u}{\phi P_n} + \frac{8M_{ux}}{9\phi_b M_{nx}} = 0.640 + \frac{8}{9}\left(\frac{200}{555}\right) = 0.96 < 1.0 \quad \textbf{O.K.}$$

Use 18-in.×18-in. column with W10×77 of F_y = 50 ksi, f_c' = 5 ksi, 4-#8 Gr. 60 longitudinal bars and #3 Gr. 60 ties spaced at 12 in. (Note that the procedure currently used in the LRFD Manual converges slowly.)

EXAMPLE 5

Example 5 demonstrates the procedure to design a composite column with biaxial bending.

P_u = 300 kips
M_{ntx} = 240 kip-ft
M_{ltx} = 0 (braced frame)
M_{nty} = 60 kip-ft
M_{lty} = 0 (braced frame)
$KL_x = KL_y = L_b$ = 14 ft
C_m = 0.85
F_y = 36 ksi
f'_c = 5 ksi

Solution:

1. For KL = 14 ft, F_y = 36 ksi, f'_c = 5 ksi, from Table 3 select a first trial value of m = 3.0. Let U = 1.6.
2. $P_{ueff} = P_u + mM_{ux} + mUM_{uy} = 300 + 3.0 \times 240 + 3.0 \times 1.6 \times 60 = 1{,}308$ kips
3. From composite column load tables select W10×68 with 18-in.×18-in. encasement. (ϕP_n = 1,360 kips.) For a W10×68, the revised value of m = 3.3.

$$U = \frac{\phi_b M_{nx}}{\phi_b M_{ny}} = \frac{378}{256} = 1.48$$

$P_{ueff} = P_u + mM_{ux} + mUM_{uy} = 300 + 3.3 \times 240 + 3.3 \times 1.48 \times 60 = 1{,}385$ kips

4. Check W10×68

$$\frac{P_{ex}(KL_x)^2}{10^4} = \frac{P_{ey}(KL_y)^2}{10^4} = 163$$

$$\text{So } P_{ex} = P_{ey} = \frac{163 \times 10^4}{(14)^2} = 8{,}316 \text{ kips}$$

$$B_{1x} = B_{1y} = \frac{0.85}{1 - \frac{300}{8{,}316}} = 0.88 \leq 1.0. \text{ Use } B_{1x} = B_{1y} = 1.$$

$\phi_b M_{nx}$ = 378 kip-ft, $\phi_b M_{ny}$ = 256 kip-ft

$$\frac{P_u}{\phi P_n} = \frac{300}{1{,}360} = 0.22 > 0.2$$

$$\frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.22 + \frac{8}{9}\left(\frac{240}{378} + \frac{60}{256}\right) = 0.99 < 1.0 \quad \textbf{O.K.}$$

Use 18-in.×18-in. column with W10×68 of F_y = 36 ksi, f'_c = 5 ksi, 4-#8 Gr. 60 longitudinal bars and #3 Gr. 60 ties spaced at 12 in.

CONCLUSIONS

Revised m and U tables for the equivalent axial load method of steel beam-column design using LRFD have been presented. The values proposed are significantly smaller than those listed in the current LRFD Manual. The LRFD Manual suggests that the tabular m value be scaled by $C_m/0.85$; however, since this procedure may lead to an inefficient section, it is recommended that the value of m not be modified by the C_m factor.

The current LRFD Manual does not provide an efficient procedure for composite beam-column design. The equivalent axial load method for steel columns is extended to composite beam-column design and a Table 3 containing m-coefficients is presented. Values of U can be calculated easily from the design flexural strengths tabulated in Part 4 of the LRFD Manual. Since a variety of beam-column examples show that m and U factors in this paper lead to an improved trial section and reduce design time, we recommend that the AISC replace the factors in the current design table with the ones listed in this paper.

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