

The In-Plane Stability of a Frame Containing Pin-Based Stepped Columns

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This paper presents five graphs and a simple formula from which the in-plane buckling load of a pin-based stepped column, forming part of a rigid-jointed frame, can be calculated. These analytical aids were derived from a parametric study of the problem using a proven finite-element stability analysis programme. They account for the rotational and translational restraints within the frame system, and enable the designer to readily determine the most critical disposition of the crane loads. The method is illustrated by an example.

INTRODUCTION

Many industrial buildings contain stepped columns in order to support both the roof and the travelling crane servicing the work area. The earliest examples consisted of two column segments, the shorter one directly under the crane girder and the longer one, close by, that continued up to the roof. The segments were laced or battened together as a composite unit (Fig. 1). The bases of the composite columns were usually fully fixed and the roof truss was simply supported on top of the longer segment. The stepped column was in effect a free-standing cantilever to which standard values of effective length factors, in the plane of the frame, could be applied with reasonable accuracy.

In recent times, the practice has been to make a single column unit by welding web and flange plates together and forming a step at the level of the crane girder. The upper section of the column is then rigidly connected to the roof beam so as to form a structural frame. The bases may again be fully fixed, but this is no longer essential because frame action provides resistance to in-plane translation or sway, consequently, pinned bases are often adequate. The simpler base and foundation details can lead to significant economies, even allowing for an increase in steel used in the frame.

Given the use of a pin-based rigid-jointed frame incorporating stepped columns, the designer needs to know the effective lengths of each segment. Current simplified methods do not produce satisfactory values. If a stability analysis program is available, the buckling loads can be calculated and then the effective lengths. However, such programs are not

as readily available as they should be, so some other method is required.

This paper presents a simple method for calculating the buckling load of a framed stepped column, taking into account the rotational restraint from the roof beam and the translational restraint offered by the lesser-loaded crane column.

The designer is, therefore, able to check whether the unsymmetrical case of crane loading is more critical than the symmetrical case. The method is illustrated by a numerical example.

BACKGROUND

The complexity of the problem has meant that most solutions depend on simplifying assumptions that limit the scope of application. Typical of such methods are Lay's solution¹ based on Livesley and Chandler's stability functions,² and the transcendental equations of Anderson and Woodward.³

In the former, Lay ignores the translational or in-plane sway restraint available from the lighter-loaded column to the more heavily loaded column, a regular load case as the crane works along one side of the building. He argues that the flexibility of the upper portion of the structure would counter this effect and so only a small amount of conservatism is involved by neglecting the translational restraint. However, the author has found from practical cases that the degree of conservatism can be at least 20 percent, which is sufficient to warrant a refinement of the method in order to achieve economy in column design.

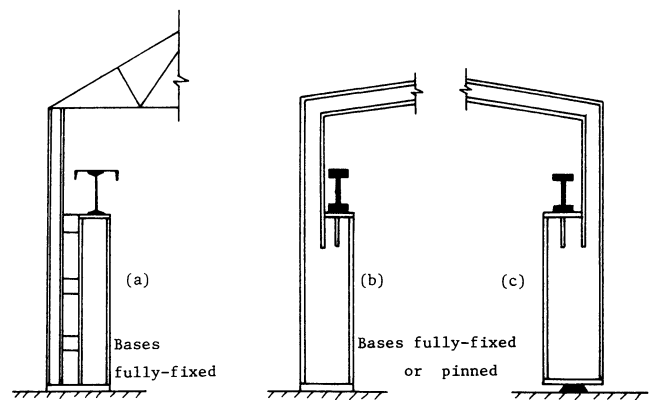


Fig. 1. Stepped column construction, old and new.

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In Anderson and Woodward's paper they present a number of transcendental equations for the stepped columns shown in Fig. 2. In reality, none of these cases occurs in practice when the upper column becomes part of a rigid-jointed frame. However, particular framing arrangements may approximate any one of the five cases, which raises the question of how close is the solution of the approximate system to the true solution?

Another useful paper, by William E. Moore,⁴ deals with double shaft crane columns of the type shown in Fig. 1(a) as well as the bayonet type in (b). However, the roof system is a conventional truss and the analysis depends on two assumptions that do not apply to the rigid frame arrangements in Figs. 1(b) and (c).

The limitations of these methods have been resolved by the development of stability analysis computer programs using stability functions or finite elements (Fig. 3). The complete generality of these programs enables the correct solution for the whole structure to be obtained, including the effects of unequal crane loads. But to the designer, there is still an important limitation; such programs are not generally available.

Consequently, there was scope for the development of a designer-oriented method for calculating the buckling load of a framed stepped column which in turn allows the effective lengths of the component parts to be determined. The author has, using a finite-element program, produced a set of graphs that provides for the direct evaluation of the buckling load, without iterative solutions, transcendental equations and other limitations.

THE IDEALIZED CRANE COLUMN

The complexity of the stability analysis of a crane column, forming part of a rigid-jointed frame, can be gauged from Fig. 4 which shows that nine parameters are required to define the geometry, the member properties and the loads.

In order to assess the feasibility of producing direct-solution graphs for so many variables, it was necessary to examine the most important factors affecting the buckling capacity of a framed stepped column. They were:

1. the buckling strength of the isolated column, Fig. 5(a)

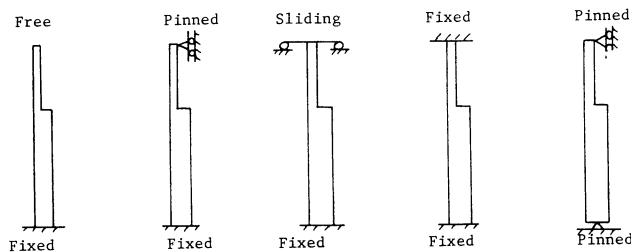


Fig. 2. The stepped columns analysed by Anderson and Woodward.

2. the support received from the flexural or rotational stiffness of the roof beam, Fig. 5(b), and
3. the translational or sway support received from the lighter-loaded column, Fig. 5(c).

Provided each of these factors could be easily evaluated, then the composite buckling load could be evaluated directly from the following simple formula:

$$P_{tot,cr} = \eta \frac{\pi^2 EI_L}{4L_L^2} \times C_R \times C_T \quad (1)$$

in which $\eta \pi^2 EI_L / 4L_L^2$ defines the buckling load of the isolated stepped column, C_R measures the effect of the rotational restraint and C_T accounts for the translational restraint.

If the translational restraint is ignored, as proposed by Lay, or if the columns are indeed equally loaded, then the translational restraint is zero and C_T equals unity.

If the roof does not provide effective frame action then the rotational restraint may be taken as zero and C_R equals zero. That is, the structure has become a mechanism.

All cases dealt with by Anderson and Woodward are covered by appropriate values of C_R and C_T .

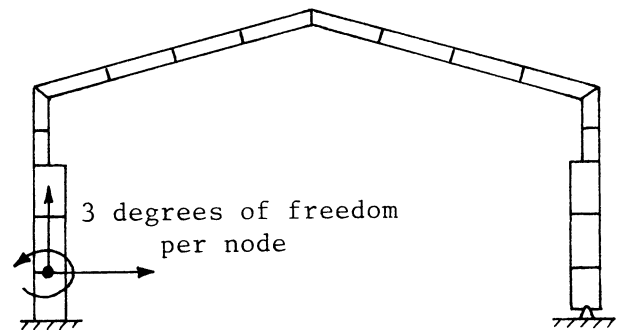


Fig. 3. Typical finite element model.

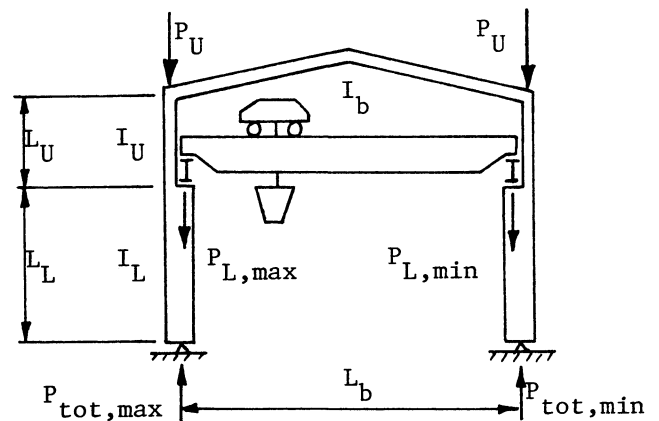


Fig. 4. Parameters required to define a framed crane column.

PIN-BASED STEPPED COLUMNS

When the component structures, shown in Fig. 5, were analysed by the finite element program and the results examined, two significant simplifications occurred: (1) factors η and C_R were relatively insensitive to changes in the load ratio P_L/P_U and (2) the amount of translational restraint was only slightly affected by changes in the geometry and stiffness of the upper portion of the frames. Consequently, a single graph was possible for η and a small set of graphs for C_R . Further, the results for C_T could be expressed by the following simple formula:

$$C_T = 1.0 + 0.77 \log R \quad (2)$$

where $R = P_{tot,max}/P_{tot,min}$. Figure 6 shows graphs of η and C_R .

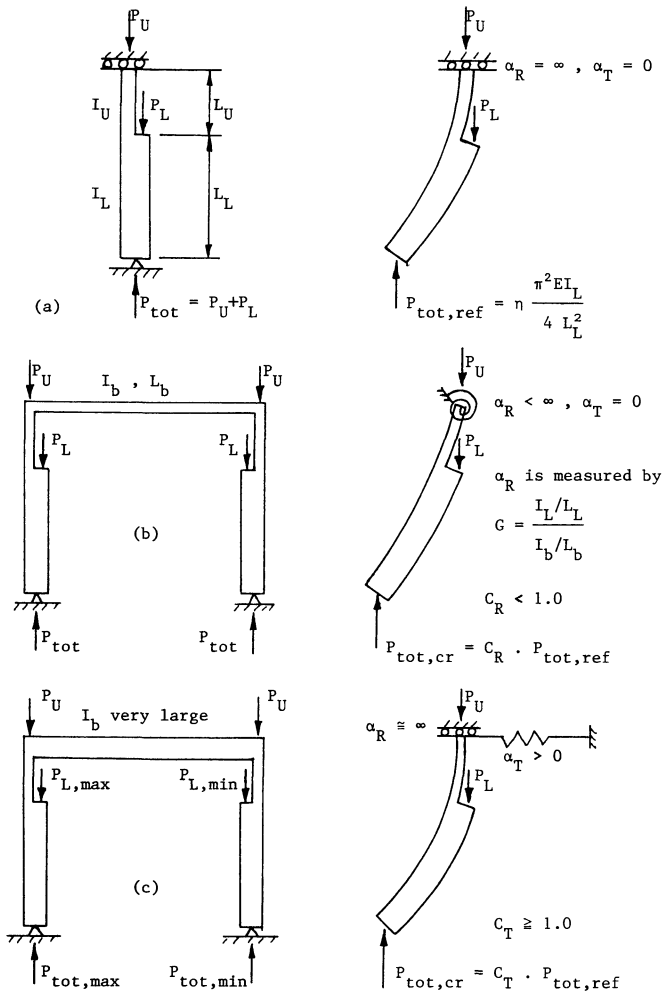


Fig. 5. Structural models for evaluating η , C_R and C_T

NUMERICAL EXAMPLE

Calculate the buckling load of the heavier-loaded column ABC shown in Fig. 7, and calculate the effective lengths and slenderness ratios of its parts.

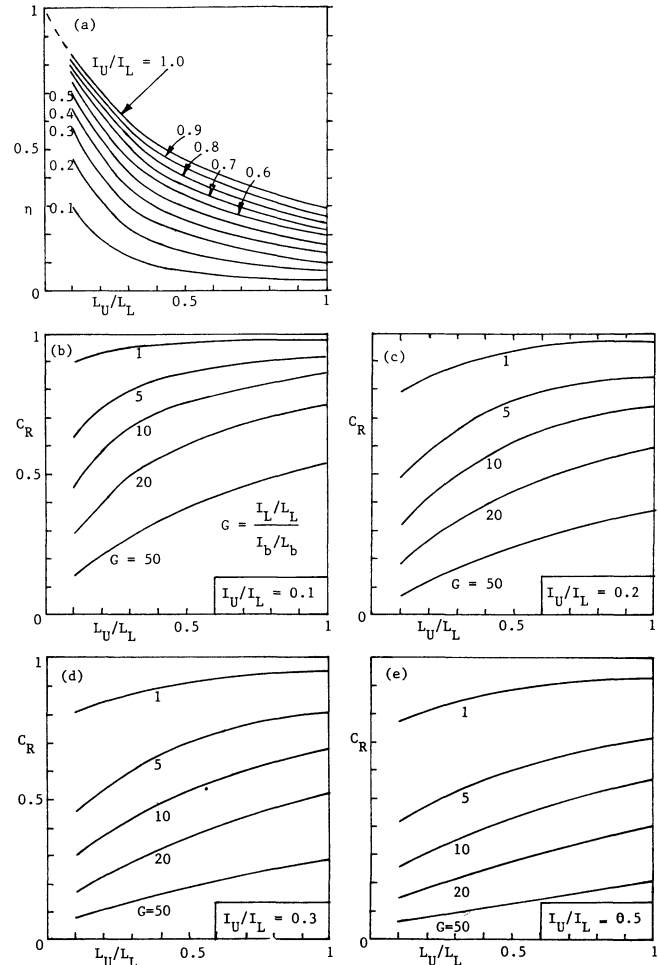


Fig. 6. Graphs of factors η and C_R for use in equation (1).

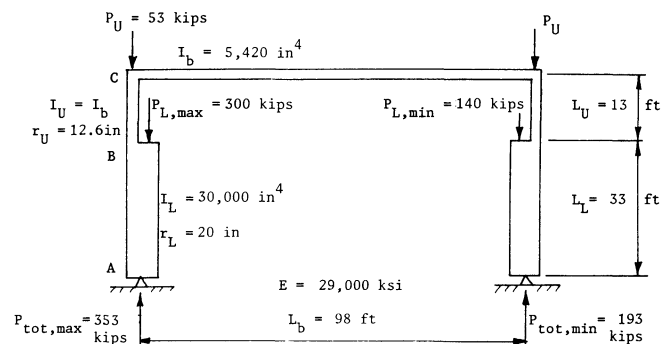


Figure 7

Solution:

$$\frac{I_U}{I_L} = \frac{5,420}{30,000} = 0.18 \text{ use } 0.2$$

$$\frac{L_U}{L_L} = \frac{13}{33} = 0.4$$

$$G = \frac{I_L/L_L}{I_b/L_b} = \frac{30,000/33}{5,420/98} = 16.4$$

$$R = \frac{P_{tot,max}}{P_{tot,min}} = \frac{353}{193} = 1.8$$

from Fig. 6(a), $\eta \cong 0.15$

from Fig. 6(d) and (e), $C_R \cong 0.45$

from Eq. (2), $C_T = 1.2$

$$P_{tot,ref} = \frac{\pi^2 \times 29,000 \times 30,000}{4 \times 33^2 \times 12^2} = 13,690 \text{ kips}$$

then by Eq. (1)

$$P_{tot,cr} = \eta \times C_R \times C_T \times P_{tot,ref} = 0.15 \times 0.45 \times 1.2 \times 13,690 = 1,110 \text{ kips (computer check } 1,300 \text{ kips)}$$

For the lower segment AB, $P_{cr,L}$ equals $P_{tot,cr} = \frac{\pi^2 EI_L}{\ell_{AB}^2}$

whence its effective length is:

$$\ell_{AB} = \pi \sqrt{\frac{29,000 \times 30,000}{1,110}} = 2,780 \text{ in.} = 231.7 \text{ ft}$$

and its effective length factor is $k_{AB} = \frac{231.7}{33} = 7$

the slenderness ratio is $(\ell/r)_{AB} = \frac{2,780}{20} = 139$

For the upper segment BC, the corresponding results are:

$$P_{cr,U} = P_{tot,cr} \times \frac{P_U}{P_{tot,max}} = 1,110 \times \frac{53}{353} = 167 \text{ kips}$$

$$\ell_{BC} = \pi \sqrt{\frac{29,000 \times 5,420}{167}} = 3,048 \text{ in.} = 254 \text{ ft}$$

$$k_{BC} = \frac{254}{13} = 19.5 \quad (\ell/r)_{BC} = \frac{3,048}{12.6} = 242$$

The slenderness ratios of 139 and 242 are large, which suggests that the in-plane stiffnesses should be increased, but they would have to be compared to the corresponding values about the minor axis.

OUT-OF-PLANE STABILITY

The design of a stepped column is not complete until the slenderness ratios in the out-of-plane or longitudinal direc-

tion have also been evaluated. A conventional steel factory building is usually braced in that direction, which means that the columns are sway-prevented about their minor axes. Also, there is usually a lack of frame action in the longitudinal direction because only light-weight girts and/or simply supported structural elements are used along the side of the building. Consequently, each stepped column becomes a continuous tiered column braced about its minor axis. The author has presented a technique, based on linearised stability functions, for the calculation of effective length factors for such a structure.⁵

PIN-BASED UNIFORM COLUMNS

In the case when I_U equals I_L the columns become uniform and the crane girders are usually supported by brackets at an intermediate location on the column defined by L_L . This is a common form of construction for small industrial buildings using light crane loads, and has been the subject of a separate investigation and paper by the author.⁶

CONCLUSION

In this paper on stepped columns, reference has been made to: (a) three methods of analysis that have important limitations, (b) the generality of finite-element methods, and (c) a direct-solution method based on a set of graphs and two simple equations.

Ultimately, it is the suitability of any method to the particular design circumstances that will determine its usefulness. On the basis of the following criteria:

1. readily available
2. quick and easy to use
3. reliable answers
4. generality of application
5. cost-effectiveness in design time

The direct-solution method, presented here, should be of significant benefit to the designer.

Similar information for fixed-base stepped columns has been accepted for publication in the *Journal of Constructional Steel Research*.

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NOMENCLATURE

| | |
|--------|--|
| C_R | a factor measuring rotational restraint |
| C_T | a factor measuring translational restraint |
| E | modulus of elasticity |
| G | a stiffness ratio |
| I_b | second moment of area of roof beam |
| I_L | second moment of area of lower portion of stepped column |
| I_U | second moment of area of upper portion of stepped column |
| k | effective length factor |
| ℓ | effective length |

| | |
|---------------|--|
| ℓ/r | slenderness ratio |
| L_b | span of roof beam |
| L_L | length of lower portion of stepped column |
| L_U | length of upper portion of stepped column |
| $P_{cr,L}$ | buckling load of the lower portion of a stepped column |
| $P_{cr,U}$ | buckling load of the upper portion of a stepped column |
| P_L | a symmetrical crane load at the column step |
| $P_{L,max}$ | the larger crane load of an unsymmetrical case |
| $P_{L,min}$ | the lesser crane load of an unsymmetrical case |
| $P_{tot,cr}$ | the critical buckling load at base of a stepped column |
| $P_{tot,max}$ | the larger total load of a stepped column |
| $P_{tot,min}$ | the lesser total load of a stepped column |
| $P_{tot,ref}$ | a reference buckling load |
| P_U | the roof load on the upper portion of a stepped column |
| r | radius of gyration |
| R | ratio of the total loads on the stepped columns |
| α_R | rotational restraint |
| α_T | translational restraint |
| η | a buckling load parameter |