# Electronic Spreadsheet Tools for Semi-Rigid Frames

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This paper describes a personal computer software tool for automating the analysis computations for semi-rigid steel frames. The computations are based partly on a design procedure proposed earlier by the author and partly on approximate modeling techniques developed for flexibly-connected building frames. The computations are implemented in an electronic spreadsheet providing a fully interactive design medium that can be readily accessed by office practitioners.

## INTRODUCTION

The behavior of semi-rigid frames under gravity and wind loadings has received much attention over the past decade, and simplified models for this behavior have been proposed that make it possible to predict the design forces in the members and connections using manual computations.[1] While these computations are simple enough to be automated on programmable calculators for individual framing members, a major portion of the design task is the organization of the calculations into a format that promotes design of the entire building frame. In applying these computations to regular building frames, one starts at the top of the frame, analyzes the beams and columns in that story, and then carries the computations down throughout the height of the frame to the ground. Because of the "bootstrapping" nature of the procedure and because of the rectilinear topography of a building frame, this analysis problem is ideally suited for solution by electronic spreadsheet.

This paper presents such a spreadsheet solution for behavior of semi-rigid unbraced frames. In addition, because drift of flexibly-connected frames is a critical issue, a spreadsheet for predicting the drift of such frames is also presented that clearly identifies the contributions from the separate sources of beam flexure, column flexure, column axial, and connection flexibility. With such tools for modeling flexibly-connected building frames, a designer can obtain a better understanding of how to apportion the steel in his building to provide the most cost effective system for both strength and stiffness by direct interaction with the spreadsheet.

## ASSUMPTIONS AND LIMITATIONS

The application presented here is a "bare bones" solution, intended to demonstrate how the analysis procedure is readily implemented in electronic spreadsheets. It is restricted to regular two-dimensional unbraced building frames that have three or more bays and up to nine stories in height. The sizes of members can vary from story to story. The frame is assumed to be symmetrical with all interior bays having the same girder spans, symmetrical girder loading, and same girder sizes. All interior columns are assumed to be the same size. The same connection size will be used throughout the entire frame. Wind loading is uniform over the height of the frame.

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## FORCE ANALYSIS OF BUILDING FRAMES USING ELECTRONIC SPREADSHEET

The analysis of a building frame for the design forces in the members and connections is broken down into six steps: input of frame data; Type 2 gravity analysis; portal wind analysis; selection of member sizes; specification of connection strength and stiffness; and gravity re-analysis accounting for connection stiffness.

### Step 1—Input of Frame Data

The initial definition of the frame topography need only consist of the numbers of stories and bays, the spans of interior and exterior girders, the story heights, and the transverse spacing of wind bents. Figure 1A shows the form used to organize this input data into spreadsheet cells that will be used in subsequent computations. In this figure, as well as

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A. External Display

FRAME GEOMETRY

                                                        Level    h (ft)

Number of Stories=          3 stories                     3        14.0
Number of Bays=             3 bays                        2        14.0
Exterior Bay Span=      39.00 FEET                        1        14.0
Interior Bay Span=      39.00 FEET
Width of Building=      30.00 FEET

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B. Underlying Cell Formulas

FRAME GEOMETRY

                                                Level           h (ft)

Number of Stories=     Ns    stories          (n3=Ns)          (h3=h2)
Number of Bays=        Nb    bays            (n2=n3-1)         (h2=h1)
Exterior Bay Span=     Le    FEET            (n1=n2-1)           h1
Interior Bay Span=     Li    FEET
Width of Building=     B     FEET

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In General:

nn = nn+1 - 1                     Story number

hn = hn-1                         Story height
```

*Fig. 1. Input form for defining frame geometry.*

in all spreadsheet figures, the following notation is used. The top portion (i.e., "Figure 1A") shows the actual spreadsheet. The lower portion of the figure (i.e., "Figure 1B") shows a conceptual version of the cell formulas that automate the computations for representative cells in each given range. Note in Fig. 1B that if all the stories in the building have the same height, the user need only type in a value for the height of the bottom story, and the cell formula automatically "fills in" the data for the stories above.

### Step 2—Type 2 Gravity Analysis

The first analysis step of the procedure described in reference 1 is a standard Type 2 analysis for gravity loads. Here the designer performs manual computations for the shears and moments in a typical interior bay and a typical exterior bay of his frame. He must manually calculate the values of moment at midspan of the girder if it were fixed at both ends, and the vertical reactions at the ends. If dead loads due to curtain walls are to be included, he will calculate the force to be carried by exterior columns at each story level.

Normally, the designer will calculate the vertical loads transferred into the columns at each level on a tributary area basis and accumulate these loads down the height of the frame to obtain the axial loads in columns due to gravity. This task is a trivial bookkeeping job for an electronic spreadsheet. As illustrated in Fig. 2A, all the user of the spreadsheet needs to provide are the results for a typical interior bay and a typical exterior bay; the spreadsheet automatically generates the axial loads in all columns in the frame using the cell formulas shown in Fig. 2B.

### Step 3—Portal Analysis for Wind Loads

Because the frame geometry has already been entered into the cells shown in Fig. 1A, the only additional data needed for the wind analysis is the intensity of the wind pressure on the face of the building. In this application, this pressure is assumed to be uniform over the height of the building.

Normally, the designer will compute panel point loads to be applied at each story on a tributary wall area basis to start the portal analysis. Then he will compute the shears in columns by assuming that an interior column will carry twice as much shear as an exterior column. By assuming inflection points at mid-lengths of columns and girders, he can readily use statics to compute the moments in columns, then the moments in girders, followed by the shears in girders, and finally the axial loads in the columns. If this data is organized into the spreadsheet shown in Fig. 3A, these computations result in the cell formulas shown in Fig. 3B.

### Step 4—Selection of Member Sizes

The results of Steps 2 and 3 now give the design forces for members for a standard Type 2 frame, and are organized into the summary spreadsheet shown in Fig. 4. These axial loads and moments in the members can now be used to select section sizes that provide resistance levels that satisfy the AISC Specification.[2,3]

A. External Display

TYPE 2 GRAVITY ANALYSIS

| | | | | | |
|---|---|---|---|---|---|
| Ext Bay Mf= | 356.5 | FT-KIPS | Int Bay Mf= | 356.5 | FT-KIPS |
| Ext Bay Mss= | 713.0 | FT-KIPS | Int Bay Mss= | 713.0 | FT-KIPS |
| ExtBayReact= | 73.1 | KIPS | IntBayReact= | 73.1 | KIPS |
| SpandrelLoad= | 12.6 | KIPS | | | |

Accumulated Column Axial Loads:

| level | Pe (k) | Pi (k) |
|---|---|---|
| 3 | 86 | 146 |
| 2 | 171 | 293 |
| 1 | 257 | 439 |

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B. Underlying Cell Formulas

TYPE 2 GRAVITY ANALYSIS

| | | | | | |
|---|---|---|---|---|---|
| Ext Bay Mf= | (Mfe=Mfi) | FT-KIPS | Int Bay Mf= | Mfi | FT-KIPS |
| Ext Bay Mss= | (Msse=Mssi) | FT-KIPS | Int Bay Mss= | Mssi | FT-KIPS |
| ExtBayReact= | (Re=Ri) | KIPS | IntBayReact= | Ri | KIPS |
| SpandrelLoad= | Pe | KIPS | | | |

Accumulated Column Axial Loads:

| level | Pe (k) | Pi (k) |
|---|---|---|
| 3 | $(Pe_3=Pe_4+Re+Pe)$ | $(Pi_3=Pi_4+Ri+Re)$ |
| 2 | $(Pe_2=Pe_3+Re+Pe)$ | $(Pi_2=Pi_3+Ri+Re)$ |
| 1 | $(Pe_1=Pe_2+Re+Pe)$ | $(Pi_1=Pi_2+Ri+Re)$ |

============================================================

In General, for three-bay frames:

$Pe_n = Pe_{n+1}+Re+Pe$ Axial load in exterior column due to gravity

$Pi_n = Pi_{n+1}+Ri+Re$ Axial load in interior column due to gravity

*Fig. 2. Form for performing type 2 gravity analysis.*

A. External Display

PORTAL ANALYSIS FOR RECTANGULAR FRAME WITH UNEQUAL BAY WIDTHS

Lateral pressure= 33.00 PSF

| N | h (ft | P (k) | Vh (k) | Vc (k) | Mwi (ft-k) | Mwe (ft-k) | Mw (ft-k) | Vgi (k) | Vge (k) | Pi (k) | Pe (k) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 14 | 6.93 | 6.93 | 1.16 | 16.17 | 8.09 | 8.09 | 0.41 | 0.41 | 0.00 | 0.41 |
| 2 | 14 | 13.86 | 20.79 | 3.47 | 48.51 | 24.26 | 32.34 | 1.66 | 1.66 | 0.00 | 2.07 |
| 1 | 14 | 13.86 | 34.65 | 5.77 | 80.85 | 40.43 | 64.68 | 3.32 | 3.32 | 0.00 | 5.39 |

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B. Underlying Cell Formulas

PORTAL ANALYSIS FOR RECTANGULAR FRAME WITH UNEQUAL BAY WIDTHS

Lateral pressure= w PSF

| n | h (ft) | P (k) | Vh (k) | Vc (k) | Mwi (ft-k) | Mwe (ft-k) | Mw (ft-k) | Vgi (k) | Vge (k) | Pi (k) | Pe (k) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 1 | | | | | | | | | | | |

================================================================

Where:

| | |
|---|---|
| $P_n = w*B*(h_{n+1} + h_n)/(2*1000)$ | Panel point load, kips |
| $Vh_n = Vh_{n+1} + P_n$ | Total story shear, kips |
| $Vc_n = Vh_n/(2*Nb)$ | Shear in exterior column, kips |
| $Mwi_n = Vc_n*h_n$ | Moment in interior column,kips |
| $Mwe_n = Vc_n*h_n/2$ | Moment in exterior column, ft-k |
| $Mw_n = Mwe_{n+1} + Mwe_n$ | Moment in all girders, ft-k |
| $Vgi_n = 2*Mw_n/Li$ | Shear in interior girder, kips |
| $Vge_n = 2*Mw_n/Le$ | Shear in exterior girder, kips |
| $Pi_n = abs(Vgi_n - Vge_n) + Pi_{n+1}$ | Axial load in interior column, kips |
| $Pe_n = Vge_n + Pe_{n+1}$ | Axial load in exterior column, kips |

*Fig. 3. Spreadsheet for portal analysis*

## Step 5—Specification of Connection Strength and Stiffness

In order to continue this design as a semi-rigid frame, the designer must evaluate the stiffness of his selected connection. Any number of approaches could be taken in such an evaluation, but the one used here is that proposed in reference 1 for either flange plate connections or top and seat angle connections. Specifically, for this application, it is assumed that the same size connection will be used throughout the entire frame (it is a straightforward extension to generalize the given application to handle a different connection size for each girder, if desired).

For flange plate connections, one uses the initial tangent stiffness of the flange plates as the connection stiffness and the yield moment as its capacity. The input data for this connection type is shown in spreadsheet form in Fig. 5A, and the stiffness and capacity formulas are shown in Fig. 5B.

For top and seat angle connections, one uses a representative stiffness given as half of the initial tangent stiffness obtained from the polynomial moment-rotation curve for this connection. (It should be noted here that the formula for $k_i$ given in the Appendix of reference 1 was misprinted as the reciprocal of the correct value.) The input data for this connection type is shown in Fig. 6A, and the stiffness formula is shown in Fig. 6B. Note that, while reference 1 suggests that it is not necessary to prescribe a capacity for connections that have continuously nonlinear moment-rotation curves, the option is provided in Fig. 6A in the event the designer feels the polynomial curve may not be applicable above a given value of moment. If no capacity value is input here, the later computations will skip capacity checks.

## Step 6—Gravity Re-Analysis Accounting for Connection Stiffness

The last analysis step in the strength computations for semi-rigid frames is to account for the effects of connection restraint on the gravity-loaded frame. In order to start this re-analysis, the designer needs to input the moments of inertia of all columns and beams that he has selected in his current frame design. On the first iteration, these member sizes are those obtained from a standard Type 2 design. On subsequent iterations, these member sizes are obtained from the previous cycle.

In order to perform these computations, the designer computes a connection flexibility parameter, "$a$," for each girder, and a relative flexibility factor, "$g$," for each exterior girder. The values of "$a$" and "$g$" allow computation of the flexible end moment coefficients, "$C_i$" and "$C_e$," which in turn

SUMMARY OF DESIGN FORCES IN MEMBERS

| | N | P,grav (kips) | M,grav (ft-kips) | P,wind (kips) | M,wind (ft-kips) |
|---|---|---|---|---|---|
| E C | 3 | 85.73 | 0.00 | 0.41 | 8.09 |
| X O | 2 | 171.45 | 0.00 | 2.07 | 24.26 |
| T L | 1 | 257.18 | 0.00 | 5.39 | 40.43 |
| I C | 3 | 146.25 | 0.00 | 0.00 | 16.17 |
| N O | 2 | 292.50 | 0.00 | 0.00 | 48.51 |
| T L | 1 | 438.75 | 0.00 | 0.00 | 80.85 |
| E G | 3 | | 713.00 | | 8.09 |
| X I | 2 | | 713.00 | | 32.34 |
| T R | 1 | | 713.00 | | 64.68 |
| I G | 3 | | 713.00 | | 8.09 |
| N I | 2 | | 713.00 | | 32.34 |
| T R | 1 | | 713.00 | | 64.68 |

*Fig. 4. Spreadsheet for force analysis results.*

```
A. External View

Flange Plate Connection Details

      b=     7.00 inches          k=      2.63E+06  in-k/rad
      d=    33.00 inches          Mp=        173.3  ft-k
      t=     0.25 inch
      l=    10.50 inches
     Fy=    36.00 ksi

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B. Underlying Cell Formulas

Flange Plate Connection Details

      b=    b   inches            k=      (kFP)  in-k/rad
      d=    d   inches            Mp=     (MpFP) ft-k
      t=    t   inch
      l=    l   inches
     Fy=    Fy  ksi

=================================================================

Where:
```

$k_{FP} = 29000*b*t*d^2/(2l)$ Initial stiffness of flange plates, in-k/rad

$Mp_{FP} = b*d*t*Fy/12$ Yield moment of flange plates, ft-k

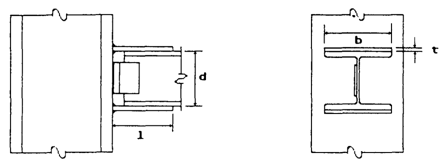


*Fig. 5. Form for flange plate connection details.*

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A. External Display

   Top and Seat Angle Connections

      g=     2.5 inches           ki=      1931366.  in-k/rad
     tf=     0.5 inch             k=        965683.  in-k/rad
      l=       7 inches           Mp=             0  ft-k
     tc=     0.5 inch
      f=    0.75 inch
      d=      33 inches

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B. Underlying Cell Formulas

   Top and Seat Angle Connections

      g=    g    inches           ki=        (kiT&S)
     tf=    tf   inch             k=         (kT&S)
      l=    l    inches           Mp=          0
     tc=    tc   inch
      f=    f    inch
      d=    d    inches

=================================================================

Where:
```

$ki_{T\&S} = 1/(.000022324 tf^{-1.128} tc^{-0.4145} l^{-0.6491} (g-f/2)^{1.35}$ initial slope

$k = ki_{T\&S}/2$ Representative stiffness, in-k/rad

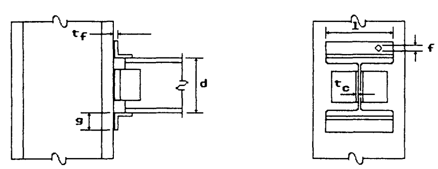


*Fig. 6. Form for top and seat angle connection details.*

give the values of flexible end moments, "$M_i$" and "$M_e$," as a fraction of the fully fixed end moments, "$M_f$":

$$a = EI_g/kl$$

$$g = \frac{\Sigma I_g/l}{\Sigma I_c/h} \quad \begin{array}{l}\text{(for exterior girders only)}\\ \text{(for interior girder, use } g = 0)\end{array}$$

$$C_i = 1 + 2a - \frac{g}{3(g + 1 + 6a)}$$

$$C_e = 1 + 2a + \frac{2g(1 + 3a)}{3(1 + 6a)}$$

$$M_i = M_f/C_i$$

$$M_e = M_f/C_e$$

Then he distributes these flexible end moments to the columns on a relative stiffness basis.

This step of the proposed procedure can become error prone and tedious, especially for larger frames and when more than one cycle of iteration is desired. Again, however, the nature of the computational procedure readily lends itself to automation in electronic spreadsheets. The only input required from the designer is the value of the moment of inertia of each member he has selected in the previous design step. Here, again, as in the case of specifying the story heights in Step 1 above, if the designer types in the first story values first, the spreadsheet automatically "fills in" the values for stories above. The spreadsheet is shown in Fig. 7A, and the cell formulas are shown in Fig. 7B.

Iterative Re-Analysis. Once the re-analysis for gravity loads has been automated in an electronic spreadsheet as shown in Fig. 7, it is a simple task to refer to the updated summary of design forces (Fig. 4B), modify the selections of member sizes where necessary, and type in the new moments of inertia for the modified members. The spreadsheets automatically update the analysis results. When the analysis results no longer change, then the "optimum" design has been reached and the frame has been sized for strength.

## Drift Computations for Semi-Rigid Building Frames

For building frames having regular geometry, it has been shown that the sway due to lateral loads can be accurately predicted using approximate models of the frame. One simple model that has been shown to predict top story sway of flexibly-connected building frames with less than 5 percent error[5] is an equivalent cantilever beam that has both shear rigidity, *GA*, and bending rigidity, *EI*. The *GA* and *EI* terms are explicitly evaluated as formulas in terms of the aggregate properties of the beams, columns, and connections at each story. Thus the equivalent cantilever beam is non-prismatic, and its lateral deflection under load is most conveniently calculated using numerical integration approaches. Thus two steps are taken to predict the drift of a frame: conversion to the approximate cantilever beam and numerical integration to obtain beam deflections.

A. External Display

GRAVITY RE-ANALYSIS BY MODIFIED TYPE 2 METHOD

| | n | GIRD | Ig (in^4) | l (in) | COL | Ic (in^4) | h (in) | k (in-k) |
|---|---|---|---|---|---|---|---|---|
| E | 3 | | 6710 | 468 | | 291 | 168 | 2631750 |
| X | 2 | | 6710 | 468 | | 428 | 168 | 2631750 |
| T | 1 | | 6710 | 468 | | 640 | 168 | 2631750 |
| I | 3 | | 6710 | 468 | | 428 | 168 | 2631750 |
| N | 2 | | 6710 | 468 | | 640 | 168 | 2631750 |
| T | 1 | | 6710 | 468 | | 999 | 168 | 2631750 |

| Mp (ft-k) | a | g | Ci | Ce | Mi (ft-k) | Me (ft-k) | Mcl (ft-k) |
|---|---|---|---|---|---|---|---|
| 173.25 | 0.157990 | 8.277381 | 1.046147 | 5.491540 | 173.25 | 64.91803 | 593.9159 |
| 173.25 | 0.157990 | 3.350094 | 1.105204 | 3.005949 | 173.25 | 118.5981 | 567.0759 |
| 173.25 | 0.157990 | 2.255353 | 1.137124 | 2.453703 | 173.25 | 145.2906 | 553.7296 |
| 173.25 | 0.157990 | na | 1.315980 | 1.315980 | 173.25 | 173.25 | 539.75 |
| 173.25 | 0.157990 | na | 1.315980 | 1.315980 | 173.25 | 173.25 | 539.75 |
| 173.25 | 0.157990 | na | 1.315980 | 1.315980 | 173.25 | 173.25 | 539.75 |

| ** Mg (ft-k) | dP (k) | ** Pc (k) | Mca (ft-k) | Mcb (ft-k) | ** Mc (ft-k) |
|---|---|---|---|---|---|
| 593.9159 | 2.777742 | 82.94725 | 0 | 64.91803 | 64.91803 |
| 567.0759 | 4.179072 | 167.2709 | 48.00007 | 70.59805 | 70.59805 |
| 553.7296 | 4.895980 | 252.2790 | 58.22507 | 87.06553 | 87.06553 |
| 539.7500 | na | 149.0277 | 0 | 0 | 0 |
| 539.7500 | na | 296.6790 | 0 | 0 | 0 |
| 539.7500 | na | 443.6459 | 0 | 0 | 0 |

B. Underlying Cell Formulas

```
  n Ig l Ic h k Mp  a  g  Ci  Ce  Mi  Me  Mcl  Mg  dP  Pc Mca  Mcb  Mc
E 3
X 2
T 1

I 3
N 2
T 1
=====================================================================
Where, for exterior bay entries (EXT):
     Ig_n = Ig_{n-1}                       Moment of inertia of girder, in^4
     l_n = 12*Le                           Length of girder, in
     Ic_n = Ic_{n-1}                       Moment of inertia of column, in^4
     h_n = 12*h_n                          Length of column, in
     k_n = k_FP or k_T&S                   Connection stiffness, in-k/rad
     Mp = Mp_FP or Mp_T&S                  Connection capacity, ft-k
     a_n = E*Ig_n/(k_n*l_n)                Connection flexibility parameter
     g_n = (Ig_Ns/l_Ns)/(Ic_Ns/h_Ns)        for n=Ns  Relative flexibility
        = (Ig_n/l_n)/(Ic_n/h_n + Ic_{n+1}/l_{n+1}) for n>Ns
     Ci_n = 1+2a_n-g_n/(3(g_n+1+6a_n))      Interior flexible end moment coeff
     Ce_n = 1+2a_n+2g_n(1+3a_n)/(3(1+6a_n))  Exterior flexible end moment coeff
     Mi_n = Mi_n,EXT = if(Mp_n>0,min(Mfe/Ci_n,Mp_n),Mfe/Ci_n)  Interior M_flexible
     Me_n = if(Mp_n>0,min(Mfe/Ce_n,Mp_n),Mfe/Ce_n)             Exterior M_flexible
     Mcl_n = Msse-(Mi_n+Me_n)/2            Centerline moment, simpy supported
     Mg_n = max(Mi_n,Me_n,Mcl_n)           Girder design moment, ft-k
     dP_n = dP_n,EXT = 12*(Mi_n-Me_n)/l_n  Column axial load increment, kips
     Pc_n = Pe_n-dPe_n                     Net column axial load, kips
     Mca_n = Me_n*(Ic_{n+1}/h_{n+1})/(Ic_{n+1}/h_{n+1} + Ic_n/hc_n)  Moment to column above
     Mcb_n = Me_n*(Ic_n/h_n)/(Ic_{n+1}/h_{n+1} + Ic_n/hc_n)       Moment to column below
     Mc_n = max(Mcb_n,Mca_{n-1})           Column design moment, ft-k
For interior bay entries (INT), formulas are the same as for exterior bays,
except for:
     l_n = Li*12
     g_n = not applicable
     Ci_n = 1+2a_n
     Ce_n = 1+2a_n
     Mi_n = if(Mp_n>0,min(Mfi/Ci_n,Mp_n),Mfi/Ci_n)
     Me_n = Me_n,INT = if(Mp_n>0,min(Mfi/Ce_n,Mp_n),Mfi/Ce_n)
     dP_n = not applicable
     Pc_n = Pi_n + dP_n,EXT
     Mca_n = (Me_n,INT - Mi_n,EXT)*(Ic_{n+1}/h_{n+1})/(Ic_{n+1}/h_{n+1} + Ic_n/hc_n)
     Mcb_n = (Me_n,INT - Mi_n,EXT)*(Ic_n/h_n)/(Ic_{n+1}/h_{n+1} + Ic_n/hc_n)
```

*Fig. 7. Spreadsheet for gravity analysis of semi-rigid frame.*

### Step 1—Conversion of Actual Frame Properties to Equivalent Non-Prismatic Cantilever Beam

The formulas for converting a regular building frame into an equivalent cantilever beam depend upon the kinematic assumptions for mapping the actual structure to the equivalent beam, but generally, most approximate models result in similar formulas. The formulas used herein were based upon the principle of superposition and the assumption that the joints in any given floor remain in the same plane under all sway modes (this is analogous to the "plane sections remain plane" assumption in beam theory). The derivations of these formulas are given in the Appendix.

Four sources of sway are to be considered in the proposed model: bending of the columns, axial deformations of the columns, bending of the girders, and relative rotations of the semi-rigid connections. Accordingly, the following beam rigidity formulas are to be used to compute the properties of the equivalent beam.

Shear rigidity in the equivalent beam due to bending of the columns:

$$GA_c = \Sigma\frac{12EI_c}{h^2} = \frac{12E[2I_{ce} + (N_b - 1)I_{ci}]}{h^2}$$

where

$GA_c$ = the shear rigidity due to column bending
$I_{ce}$ = the moment of inertia of exterior columns
$I_{ci}$ = the moment of inertia of interior columns
$N_b$ = the number of bays, $> 2$
$h$ = the story height, and
$E$ = Young's modulus of elasticity

Bending rigidity in the equivalent beam due to axial deformation of the columns:

$$EI = E\,\Sigma A_c X_c^2 = 2E[A_e((N_b - 2)L_i/2 + L_e)^2 + A_i L_i^2(N_b)(N_b - 1)(N_b - 2)/24]$$

where

$EI$ = the flexural rigidity due to columns axial deformations
$A_e$ = the cross sectional area of exterior columns
$A_i$ = the cross sectional area of interior columns
$L_e$ = the span of exterior bays, and
$L_i$ = the span of interior bays

Shear rigidity in the equivalent beam due to bending of the girders:

$$GA_g = (12E/h)\,\Sigma I_g/l = (12E/h)[2I_{ge}/L_e + (N_b - 2)I_{gi}/L_i]$$

where

$GA_g$ = the shear rigidity due to girder flexure
$I_{ge}$ = the moment of inertia of exterior girders, and
$I_{gi}$ = the moment of inertia of interior girders

Shear rigidity in the equivalent beam due to flexibly-connected girders:

$$GA_{flex} = (12E/h)\,\Sigma(I_g/l)/[1 + 6EI_g/kl] = (12E/h)\{2(I_{ge}/L_e)/[1 + 6EI_{ge}/k_eL_e] + (N_b{}^{-2})(I_{gi}/L_i)/[1 + 6EI_{gi}/k_iL_i]\}$$

where

$GA_{flex}$ = the shear rigidity due to flexure of girders connected with semi-rigid connections
$k_e$ = the stiffness of connections of exterior girders, and
$k_i$ = the stiffness of connections of interior girders

These four formulas can be readily used with the data in the spreadsheets above to automatically generate a spreadsheet that computes the properties of the equivalent beam at each story level, as shown in Fig. 8. Note, however, that if column axial deformations are to be included, the designer must type in the column cross sectional areas into the cells, "$A_e$" and "$A_i$" (columns 3 and 4 of the spreadsheet shown in Fig. 8). As in earlier spreadsheets, he can speed up data entry by starting at the bottom of the frame and allow the automatic "fill in" of repetitive values.

### Step 2—Numerical Computations for Determining Sway

The spreadsheet shown in Fig. 8 now contains a complete description of a non-prismatic cantilever beam. The loading on this beam is obtained directly from the portal analysis of Fig. 3. All that remains is the computation of the deflections of this beam for the four sources of flexibility. For this application, a Newmark integration scheme[4] is ideally suited, because it is very accurate for beams loaded with concentrated forces, and it uses integration formulas that are convenient for automation in a spreadsheet. In the following integration formulas, the subscript "$n$" refers to the story level.

#### Case A—Flexural Beams

For beams having only flexural rigidity, $EI$, the Newmark integration scheme starts with a tabulation of the concentrated loads at the panel points, $P_n$. The scheme then proceeds in the following steps:

Create an imaginary panel beyond each end of the beam for generalizing the computations.

Compute panel shears: Assign "zero" shear in the imaginary panel beyond the free end. Working from the free end to the base, compute the shear in each panel as the shear in the panel above plus the panel point load at the level above, $V_n = V_{n+1} + P_n$.

Compute the panel point moments: Assign "zero" moment at the free end of the beam. Working from the free end to the base, for each panel point, the moment equals the moment at the panel point above plus the product of the shear and the panel height of the panel above, $M_n = M_{n+1} + V_n h_n$.

Compute curvatures: The curvature on each side of each panel point is simply the moment at the panel point divided by the $EI$ of the adjacent panel, $\phi_{n-} = M_n/EI_{n-}$ and $\phi_{n+} = M_n/EI_{n+}$. Note that this results in discontinuities of curvature at panel points where $EI$ is discontinuous, that is, at stories where column sizes change.

Compute equivalent concentrated angles: Use the single panel integration formula for computing concentrated angles at each end of each panel: $\theta_{n-} = h_n/6[2\phi_{n-} + \phi_{(n-1)+}]$ and $\theta_{n+} = H_n/6[2\phi_{n+} + \phi_{(n+1)-}]$.

Compute panel chord slopes: Assign "zero" slope in the imaginary panel below the base of the cantilever. Working from the base, for each panel, the chord slope equals the slope in the panel below plus the sum of the two equivalent concentrated angles at the intervening panel point, $y'_n = y'_{n-1} + \theta_{n-} + \theta_{n+}$.

Compute panel point deflections: Assign "zero" at the panel point at the base of the cantilever. Working from the base, for each panel point, the deflection equals the deflection at the panel point below plus the product of the chord slope in the panel and the panel length, $y_n = y_{n-1} + y'_n h_n$.

The spreadsheet implementation of deflection computations for flexural beams is shown in Fig. 9.

## Case B—Shear Beams

For beams having only shear rigidity, the Newmark integration scheme starts with a tabulation of the concentrated loads at the panel points, $P_n$. The scheme then proceeds in the following steps:

Create an imaginary panel beyond each end of the beam for generalizing the computation formulas.

Compute panel shears: Assign "zero" shear in the imaginary panel beyond the free end. Working from the free end to the base, compute the shear in each panel as the shear in the panel above plus the panel point load at the level above, $V_n = V_{n+1} + P_n$.

Compute panel chord slopes: Assign "zero" slope in the imaginary panel below the base of the cantilever. Working from the base, for each panel, the chord slope equals the shear in the panel divided by the shear rigidity of the panel, $y'_n = V_n/GA_n$.

```
A. External Display

   EQUIVALENT BEAM CALCULATIONS

      N      h        Ae        Ai     Ice       Ici       Ige       Igi
            (ft)    (in^2)    (in^2)  (in^4)    (in^4)    (in^4)    (in^4)

      3      14      8.85      12.6    291       428       6710      6710
      2      14      12.6      17.9    428       640       6710      6710
      1      14      17.9      29.1    640       999       6710      6710
_______________________________________________________________________//
       ke        ki        EI       GAc        GAg       GAflex
     (in-k)    (in-k)   (ft^2-k)    (k)        (k)        (k)

     2631750   2631750   2.0E+09 17730.44 89097.98 45739.57
     2631750   2631750   2.9E+09 26336.73 89097.98 45739.57
     2631750   2631750   4.2E+09 40417.51 89097.98 45739.57
+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
B. Underlying Cell Formulas

   EQUIVALENT BEAM CALCULATIONS

      n   h Ae Ai Ice Ici Ige Igi EI GAc GAg GAflex
      3
      2
      1
=====================================================

Where:

EIn = 2E(Aen(Le + (Nb-2)Li/2)^2 + Ain*Li^2*(Nb)(Nb-1)(Nb-2)/24)*

GAcn = 12E(2Icen + (Nb-1)Icin)/(12*hn)^2

GAgn = 12E(2Igen/Le + (Nb-2)Igin/Li)/(144*hn)

GAflexn = 12E((2Igen/Le)/(1+6an) + (Nb-2)(Igin/Li)/(1+6an))/(144*hn)


 * The summation term for interior columns in a frame with Nb bays is
 obtained as follows:

 For Nb, odd:
     Σ Ac*xc^2 = A Σ xc^2                       for c=1,Nb-1
               = 2A ((L/2)^2 + (3L/2)^2 + (5L/2)^2 + ... ((1+2i)L/2)^2)
               = 2A*L^2/4 Σ (1+2i)^2            for i=0,(Nb-3)/2
       Let j = i+1
       then
     Σ Ac*xc^2 = 2A*L^2/4 Σ (2j-1)^2            for j=1,(Nb-1)/2
               = 2A*L^2/4 [Σ 4j^2 - Σ 4j + Σ 1]  for j=1,(Nb-1)/2
               = 2A*L^2 [Σ j^2 - Σ j + 1/4 Σ 1]  for j=1,(Nb-1)/2

       Noting that Σ k^2 = n(n+1)(2n+1)/6       for k=1,n
                   Σ k  = n(n+1)/2              for k=1,n
                   Σ 1  = n                     for k=1,n
       then
     Σ Ac*xc^2 = 2A*L^2 [(Nb-1)(Nb+1)(Nb)/24-(Nb-1)(Nb+1)/8+(Nb-1)/8]
     Σ Ac*xc^2 = 2A*L^2 (Nb-1)/24[(Nb+1)(Nb)-3(Nb+1)+3]
     Σ Ac*xc^2 = 2A*L^2 (Nb-1)(Nb)(Nb-2)/24

 For Nb, even:

     Σ Ac*xc^2 = 2A(L^2 + (2L)^2 + (3L)^2 + ... + (iL)^2)
               = 2A*L^2 Σ i^2                   for i=1,(Nb-2)/2
               = 2A*L^2 (Nb-2)(Nb)(Nb-1)/24     Same as for Nb,odd!
```

*Fig. 8. Spreadsheet for computing properties of equivalent beam.*

```
A. External Display

n  h      EI       P        V        M        y''       Ls         y'          y
  (ft)(ft^2-k)(kips)  (kips)(ft-k)   (1/ft)                                   (in)


3 14   2.0E+09   6.93                0 0.00E+00 1.11E-07              1.26E-03
                          6.93                                3.50E-06
                                       4.77E-08 2.23E-07
2 14   2.9E+09 13.86             97.02 3.35E-08 4.69E-07              6.71E-04
                         20.79                                2.81E-06
                                       1.34E-07 7.04E-07
1 14   4.2E+09 13.86            388.08 9.25E-08 9.17E-07              1.99E-04
                         34.65                                1.19E-06
                                       2.08E-07 1.19E-06
0  0         0      0           873.18 0.00E+00 0.00E+00              0.00E+00
                         34.65                                0.00E+00

+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
B. Underlying Cell Formulas

n  h      EI       P        V        M        y''       Ls         y'          y
3
2
1

===============================================================================

Vn = Vn+1 + Pn                         Shear in story n, kips

Mn = Mn+1 + Vnhn                       Moment at story n, ft-k

y''n- = Mn/EIn-                        Curvature just below story n, 1/ft

y''n+ = Mn/EIn+                        Curvature just above story n, 1/ft

Lsn- = hn/6*(2y''n- + y''(n-1)+)       Equivalent concentrated angle below...

Lsn+ = hn/6*(2y''n+ + y''(n+1)-)       Equivalent concentrated angle above...

y'n = y'n-1 + Lsn- + Lsn+              Chord slope in story n

yn = yn-1 + y'nhn*12                   Deflection at story n, in
```

*Fig. 9. Spreadsheet for deflection of flexural beam.*

Compute panel point deflections: Assign "zero" at the panel point at the base of the cantilever. Working from the base, for each panel point, the deflection equals the deflection at the panel point below plus the product of the chord slope in the panel and the panel length, $y_n = y_{n-1} + y'_n h_n$.

The spreadsheet implementation of deflection computations for shear beams is shown in Fig. 10.

These two spreadsheets are now available for computing the drift of the building for the four sources of flexibility: column flexure, column axial, girder flexure, and connection flexibility. The four components are readily combined to give the total sway of the building in the spreadsheet shown in Fig. 11. Also, most commercially available spreadsheet packages provide in-line plotting facilities, so that deformed shapes and drift-component bar graphs can be used to review the results. Such tools will be demonstrated in the following example.

A. External Display

| n | h (ft) | GA (kips) | P (kips) | V | y' | y (in) |
|---|---|---|---|---|---|---|
| 3 | 14 | 17730.44 | 6.93 | | | 3.42E-01 |
| | | | | 6.93 | 3.91E-04 | |
| 2 | 14 | 26336.73 | 13.86 | | | 2.77E-01 |
| | | | | 20.79 | 7.89E-04 | |
| 1 | 14 | 40417.51 | 13.86 | | | 1.44E-01 |
| | | | | 34.65 | 8.57E-04 | |
| 0 | 0 | 0 | 0 | | | 0.00E+00 |
| | | | | 34.65 | 0.00E+00 | |

+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

B. Underlying Cell Formulas

| n | h | GA | P | V | y' | y |
|---|---|---|---|---|---|---|
| 3 | | | | | | |
| 2 | | | | | | |
| 1 | | | | | | |

=============================================================

Where:

$V_n = V_{n+1} + P_n$ Shear in story n, kips

$y'_n = V_n/GA_n$ Chord slope in story n

$y_n = y_{n-1} + y'_n h_n * 12$ Deflection at story n, in.

*Fig. 10. Spreadsheet for deflection of shear beam.*

A. External Display

RESULTS OF DRIFT ANALYSIS

| Elev (ft) | y_EI (in) | y_GAc (in) | y_GAg (in) | y_flex (in) | y_conn (in) | y (in) | Bldg Drift Index | Inter Story Drift |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | 1/ | 1/ |
| 42 | 0.00126 | 0.34232 | 0.11760 | 0.22908 | 0.11148 | 0.69025 | 730 | 1603 |
| 28 | 0.00067 | 0.27664 | 0.10454 | 0.20363 | 0.09909 | 0.58548 | 574 | 676 |
| 14 | 0.00020 | 0.14403 | 0.06533 | 0.12727 | 0.06193 | 0.33683 | 499 | 499 |
| 0 | | | | | 0 | 0 | | |

=============================================================

B. Underlying Cell Formulas

Where:

$Elev_n = Elev_{n-1} + h_n$ Elevation of story n, ft

$y\_conn_n = y\_flex_n - y\_GAg_n$ Deflection due to semi-rigid connections, in

$y_n = y\_EI_n + y\_GAc_n + y\_GAg_n + y\_conn_n$ Total deflections at story n, in

$BldgDriftIndex_n = y_n/Elev_n$ Absolute building drift at story n

$InterStoryDrift_n = (y_n - y_{n-1})/(h_n - h_{n-1})$ Relative building drift...

*Fig. 11. Spreadsheet for building drift results.*

## EXAMPLE PROBLEM

To demonstrate how a designer can use the above tools in the design of semi-rigid frames, the following example steps through the design of a three-story, three-bay building frame. For this example, the design conforms to the 8th edition of the AISC Specification.[2] For brevity of presentation, the design calculations have been omitted. One can equally well use these tools for designing according to the LRFD Specification.[3]

The example shown here has been solved using a general version of the above spreadsheet approach using LOTUS-123, version 2*. A specialized menu has been developed for making the maneuvering among the various spreadsheets straightforward. To display this menu, the user simply holds in the "Alt" key while pressing the letter "m" (for menu). The available items are:

Input Gravity Wind Results FlangePlates
T&SAngles MType2 Drift

For typical designs, one will execute these menu items from left-to-right, and skip either "FlangePlates" or "T&SAngles."

Executing the "Input" menu item displays the form shown in Fig. 12 for entering the data that define the frame geome-

(**boldface** fields are to be filled in by user)

FRAME GEOMETRY

| | | | Level | h(ft) |
|---|---|---|---|---|
| Number of Stories= | **0** | stories | 0 | **0.0** |
| Number of Bays= | **0** | bays | | **0.0** |
| Exterior Bay Span= | **0.00** | FEET | | **0.0** |
| Interior Bay Span= | **0.00** | FEET | | **0.0** |
| Width of Building= | **0.00** | FEET | | **0.0** |
| | | | | **0.0** |
| | | | | **0.0** |
| | | | | **0.0** |
| | | | | **0.0** |
| | | | | **0.0** |

*Fig. 12. Initial form for input of frame geometry.*

(**boldface** fields are to be filled in by user)

TYPE 2 GRAVITY ANALYSIS

| | | | | | |
|---|---|---|---|---|---|
| Ext Bay Mf= | **0.0** | FT-KIPS | Int Bay Mf= | **0.0** | FT-KIPS |
| Ext Bay Mss= | **0.0** | FT-KIPS | Int Bay Mss= | **0.0** | FT-KIPS |
| ExtBayReact= | **0.0** | KIPS | IntBayReact= | **0.0** | KIPS |
| SpandrelLoad= | **0.0** | KIPS | | | |

Accumulated Column Axial Loads:

| level | Pe (k) | Pi (k) |
|---|---|---|
| 3 | 0 | 0 |
| 2 | 0 | 0 |
| 1 | 0 | 0 |

*Fig. 13. Initial form for input of gravity load effects.*

try. All input actions are simple data entry operations, except for the entry of the number of stories. When the user enters the number of stories, the spreadsheet automatically generates the story numbers. The completed form for this problem has already been shown in Fig. 1A.

Executing the "Gravity" menu item will allow the user to input the gravity load effects on girders using the form shown in Fig. 13. The subsequent semi-rigid analysis tools will need some basic beam results such as simple beam moments, fixed-end beam moments, and vertical loads acting at support points. As the user types in vertical load data, the spreadsheet automatically accumulates the gravity loads carried into each column stack in the table below the input area. The results of the Type 2 analysis are shown in Fig. 2A.

Executing the "Wind" menu item will initially display the form shown in Fig. 14 for performing a portal analysis for the wind loading. At this point, the only information that has not yet been provided is the intensity of the wind loading, the "Lateral pressure." As soon as the user enters a value for the lateral pressure, the spreadsheet performs the portal analysis and displays the results in the lower portion of the screen.

At this point the analysis results can be used to perform a design of all members and connections in the frame. The user can obtain a summary of the forces due to gravity and due to wind by executing the "Results" menu item. The summary for this problem is shown in Fig. 4. Based upon the values of axial force and moment in this table, the user selects member sizes that satisfy the Specification strength provisions. Also, he uses simple beam reactions from the Type 2 gravity analysis along with the end moments on girders due to wind to design his connections.

(**boldface** fields are to be filled in by user)

PORTAL ANALYSIS FOR RECTANGULAR FRAME WITH UNEQUAL BAY WIDTHS

Lateral pressure= **0.00** PSF

| N | h (ft) | P (k) | Vh (k) | Vc (k) | Mwi (ft-k) | Mwe (ft-k) | Mw (ft-k) | Vgi (k) | Vge (k) | Pi (k) | Pe (k) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 14 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2 | 14 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1 | 14 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

*Fig. 14. Spreadsheet for portal analysis.*

SUMMARY OF DESIGN FORCES IN MEMBERS

| | | N | P,grav (kips) | M,grav (ft-kips) | P,wind (kips) | M,wind (ft-kips) |
|---|---|---|---|---|---|---|
| E | C | 3 | 82.95 | 64.92 | 0.41 | 8.09 |
| X | O | 2 | 167.27 | 70.60 | 2.07 | 24.26 |
| T | L | 1 | 252.28 | 87.07 | 5.39 | 40.43 |
| I | C | 3 | 149.03 | 0.00 | 0.00 | 16.17 |
| N | O | 2 | 296.68 | 0.00 | 0.00 | 48.51 |
| T | L | 1 | 443.65 | 0.00 | 0.00 | 80.85 |
| E | G | 3 | | 593.92 | | 8.09 |
| X | I | 2 | | 567.08 | | 32.34 |
| T | R | 1 | | 553.73 | | 64.68 |
| I | G | 3 | | 539.75 | | 8.09 |
| N | I | 2 | | 539.75 | | 32.34 |
| T | R | 1 | | 539.75 | | 64.68 |

*Fig. 15. Input form for flange plate connection details.*

The next step is to refine the gravity analysis to account for the rotational restraint offered by the semi-rigid connections. First the user must input his connection dimensions either through the "FlangePlates" or the "Top&SeatAngles" connection menu items. This example is using flange plate connections, so the "FlangePlates" menu item will display the form shown in Fig. 15. The user only needs to enter the values of plate width ($b$ = 7 in.) and thickness ($t$ = ¼ in.), the depth of the beam ($d$ = 33 in.), and the distance from the face of the column to the first line of fasteners ($l$). A standard dimension for "$l$" is 1.5 times the plate width ($l$ = 10.5 in.).

To allow completion of the gravity re-analysis for semi-rigid connections, the user executes the "MType2" menu item, which displays the table shown in Fig. 16. Now, he enters the values for the moments of inertia of the girders and columns that he has designed in the preceding step. The spreadsheet automatically performs the computations for redistribution of gravity moments as he enters his data.

Finally, the designer can review the results of the analysis by executing the "Results" menu item again to obtain the new axial forces and moments throughout the frame, as shown in Fig. 17. He modifies the designs of members that were either understrength or uneconomical and enters these changes into the "MType2" table, until no further changes occur. Typically, no more than two cycles of partial re-design are necessary to reach a least-weight frame design.

(**boldface** fields are to be filled in by user)

Flange Plate Connection Details

| | | | | | |
|---|---|---|---|---|---|
| b= | **0.00** | inches | k= | 0.00E+00 | in-kips/radian |
| d= | **0.00** | inches | Mp= | 0.0 | ft-kips |
| t= | **0.00** | inch | | | |
| l= | **0.00** | inches | | | |
| Fy= | **36.00** | ksi | | | |

*Fig. 16. Initial form for input data for analysis of semi-rigid frame.*

(**boldface** fields are to be filled in by user)

GRAVITY RE-ANALYSIS BY MODIFIED TYPE 2 METHOD

| | LEV | GIRD | Ig (in^4) | l (in) | COL | Ic (in^4) | h (in) | k (in-k/rad) |
|---|---|---|---|---|---|---|---|---|
| E | 3 | | **0** | 468 | | **0** | 168 | 2631750 |
| X | 2 | | **0** | 468 | | **0** | 168 | 2631750 |
| T | 1 | | **0** | 468 | | **0** | 168 | 2631750 |
| I | 3 | | **0** | 468 | | **0** | 168 | 2631750 |
| N | 2 | | **0** | 468 | | **0** | 168 | 2631750 |
| T | 1 | | **0** | 468 | | **0** | 168 | 2631750 |

*Fig. 17. Force results after analysis as semi-rigid frame.*

RESULTS OF DRIFT ANALYSIS

| Elev (ft) | y_EI (in) | y_GAc (in) | y_GAg (in) | y_flex (in) | y conn (in) | y (in) | Bldg Drift Index 1/ | Inter Story Drift 1/ |
|---|---|---|---|---|---|---|---|---|
| 42.00 | | | | | 0.000 | 0.00 | | |
| 28.00 | | | | | 0.000 | 0.00 | | |
| 14.00 | | | | | 0.000 | 0.00 | | |
| 0.00 | | | | | 0.000 | 0.00 | | |

*Fig. 18. Initial spreadsheet for performing drift analysis.*

### Drift Assessment of the Frame

At any time after the designer has selected member sizes and connection sizes, he can have the drift computations performed by simply executing the "Drift" menu item. While the computations are in process, the display shown in Fig. 18 shows the numerical values of the individual contributors to building drift. When the computations are complete, the spreadsheet automatically creates a "stacked bar" graph of the story drifts as a function of elevation, as shown in Fig. 19. This graph clearly depicts the fraction of drift caused by column flexure, column axial, girder flexure, and connection flexibility, so that if drift is considered excessive, the designer can immediately ascertain which elements of the frame are most effective candidates for modification. To return to the spreadsheets, the designer presses the "Q" (Quit) key twice. Then the completed drift results, shown in Fig. 20, allow him to review the numerical results in detail. The last two columns show generally accepted measures of drift index. "Bldg Drift Index, /1. . ." is the nondimensionalized value obtained by dividing the elevation by the sway at each story level. "InterStory Index, 1/. . ." is obtained by dividing the story height by the relative drift within the story.

At any time, the designer can use the menu to return to any input forms, modify his design, and have the spreadsheet automatically update the results.

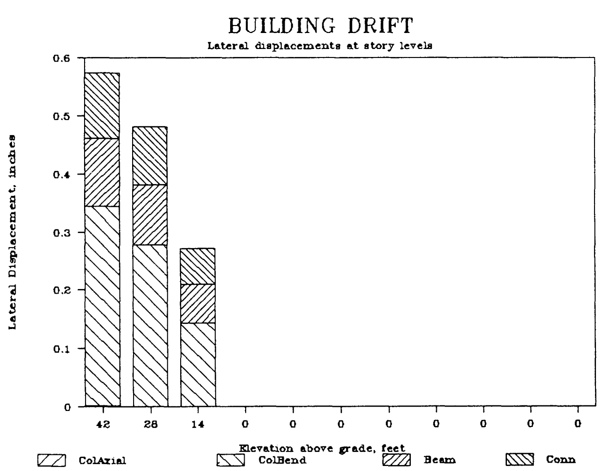


*Fig. 19. Stacked-bar graph of story drifts.*

RESULTS OF DRIFT ANALYSIS

| Elev (ft) | y_EI (in) | y_GAc (in) | y_GAg (in) | y_flex (in) | conn (in) | y (in) | Bldg Drift Index 1/ | Inter Story Drift 1/ |
|---|---|---|---|---|---|---|---|---|
| 42.00 | 0.001 | 0.342 | 0.118 | 0.229 | 0.111 | 0.69 | 730 | 1603 |
| 28.00 | 0.001 | 0.277 | 0.105 | 0.204 | 0.099 | 0.59 | 574 | 676 |
| 14.00 | 0.000 | 0.144 | 0.065 | 0.127 | 0.062 | 0.34 | 499 | 499 |
| 0.00 | | | | | 0.000 | 0.00 | | |

*Fig. 20. Results from drift analysis.*

## SUMMARY AND CONCLUSIONS

This paper has presented a spreadsheet-based solution to the analysis of semi-rigid building frames that facilitates the process of taking advantage of the beneficial effects of connection stiffness on frame strength. The method has been based on extensive research in flexibly-connected steel frames and is derived from simple models of structural behavior. The particular implementation presented uses LOTUS-123, but most commercially available spreadsheet packages can support the required computations. The application presented here is intended for introduction to the approach: there are many possibilities for extensions and customizations.

## REFERENCES

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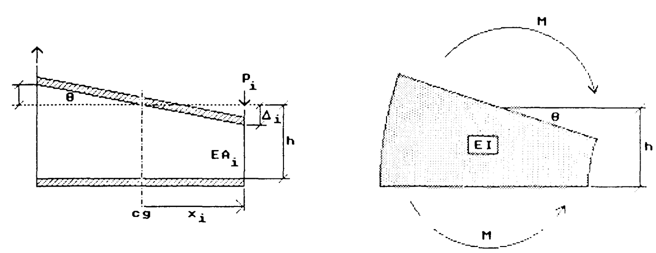
5. Torres-Macias, M. H., "Approximate Analysis of Building Frames," Master of Engineering thesis, Rensselaer Polytechnic Institute, Troy, New York, May 1982.

## APPENDIX
## DERIVATION OF EQUIVALENT BEAM FORMULAS FOR DRIFT CALCULATIONS

### 1. Column Axial Deformations




*A. Actual Story:*

Force-Deformation of Individual Column:

$$\Delta_i = P_i h / A_i E$$

*Kinematics:* $\Delta_i = \theta x_i$

Real Work $= \Sigma P_i \Delta_i / 2$

$= \Sigma(A_i E x_i \theta/h)(x_i \theta)/2$

$= \theta^2/(2h) \times E\ \Sigma\ A_i x_i^2$

*B. Equivalent Beam Segment:*

Force-Deformation: $d\theta/dx = M/EI \rightarrow M = EI\theta/h$

Model Work $= M\theta/2$

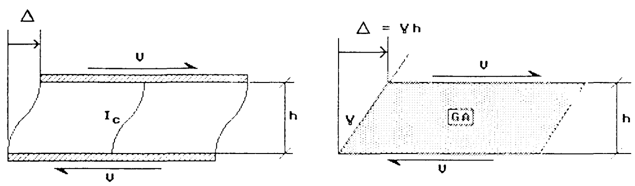
$= (EI\theta/h)(\theta)/2$

$= \theta^2/(2h) \times EI$

*C. Equating Real Work to Model Work:*

$$EI = E\ \Sigma\ A_i x_i^2$$

**2. Column Bending Deformations**




*A. Actual Story:*

Force-Displacement of Individual Column:

$$V = \Sigma(12EI_c/h^3)\Delta$$

Real Work $= V\Delta/2$

$= (\Delta^2/2) \times \Sigma 12EI_c/h^3$

*B. Equivalent Beam Segment:*

Force-Deformation: $V = GA\ \gamma$

Kinematics: $\Delta = \gamma h \rightarrow \gamma = \Delta/h$

Model Work $= V\Delta/2$

$= (GA\Delta/h)(\Delta)/2$

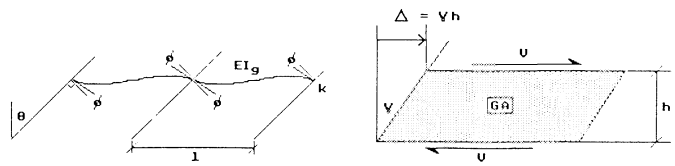
$= (\Delta^2/2) \times GA/h$

*C. Equating Real Work to Model Work:*

$$(\Delta^2/2) \times \Sigma 12EI_c/h^3 = (\Delta^2/2) \times GA/h$$

$$GA = \Sigma 12EI_c/h^2$$

**3. Flexibly-Connected Girder Deformations:**




*A. Actual Story:*

Isolated Connection: $M_g = k\phi \rightarrow \phi = M_g/k$

Flexibly-Connected Girder:

$M_g = (6EI_g/l)(\theta - \phi)$

$M_g = (6EI_g/l)(\theta - M_g/k)$

$M_g[1 + 6EI_g/kl] = (6EI_g/l)\theta$

$$M_g = \frac{(6EI_g/l)}{[1 + (6EI_g/kl)]} \times \theta$$

Actual Work $= \Sigma_{ends} M_g \theta/2$

$= \Sigma_{girders}(2M_g)\theta/2$

$= \theta^2 6\Sigma_{girders} \dfrac{EI_g/l}{[1 + (6EI_g/kl)]}$

*B. Equivalent Beam Segment:*

Kinematics: $\Delta = \theta h$

Force-Deformation: $V = GA\theta$

Model Work $= V\Delta/2 = \theta^2 GAh/2$

*C. Equating Real Work to Model Work:*

$$\theta^2 \times 6\Sigma_{girders} \frac{EI_g/l}{[1 + (6EI_g/kl)]} = \theta^2 \times GAh/2$$

$GA = (12E/h)\ \Sigma\ (I_g/l)/[1 + (6EI_g/kl)]$

**3.1 Contribution from Girder**

If rigidly-connected to Columns, $k$ = infinity, then

$$GA = (12E/h)\ \Sigma\ I_g/l$$