DISCUSSION

A Practical P-Delta Analysis Method for Type FR and PR Frames

Paper by E. M. LUI
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Discussion by **Phillip S. Carskaddan**

For the engineer who has never been totally comfortable with the destabilizing effects of compressive forces on an elastic frame, this paper is just the ticket. Starting with the well-known differential equation of a beam-column, Lui deftly moves away from the calculus by a subtle algebraic redefinition of terms that reduces the problem to a traditional first-order structural analysis problem. Requiring at most a few iterations, it basically makes the computation of secondary moments a bookkeeping exercise. It is good potent stuff.

Lui's method involves the application of pseudo forces that, along with the real forces, create the final moments, which include the secondary moments. There are two types of pseudo forces—member ($P\delta$) and frame ($P\Delta$). Lui shows that his method gives exact $P\delta$ and $P\Delta$ results for single member cases.

When adding pseudo forces to real forces, an analyst must keep clear the distinction between the two. For the $P\Delta$ pseudo forces, Lui's Fig. 5 demonstrates that they are essentially balanced by the inclination of the column loads $Pcos\Theta$. Actually, these $P\Delta$ forces do increase the base shear above the real value, but they are needed to generate the $P\Delta$ moments. Statically it would be better to add pseudo moments that would not artificially increase the base shear above the real value.

The $P\delta$ forces also artificially modify the base shear from the real value. The imbalance of the $P\delta$ forces has no effect on a single member, and probably has little effect on the low-rise frames that Lui presents. For a high-rise frame, however, these $P\delta$ forces could accumulate. For example, it does not seem reasonable that the $P\delta$ force of a member on story 40 should exert a moment on story 1 of the frame.

As a correction to the imbalance of the $P\delta$ forces, it is suggested that additional pseudo forces be added to balance the $P\delta$ forces applied along a member length. These additional pseudo forces would act at the member ends in an opposite direction to the $P\delta$ forces and would statically balance them. They would be algebraically added to the $P\Delta$ pseudo forces. For example, consider Lui's Figure 10: The $P\delta$ distributed triangular loads on the columns should be balanced by opposite acting concentrated forces at the column tops and bottoms. These balancing forces should provide moment and force equilibrium with the $P\delta$ forces on each column.

I look forward to applying Lui's method to frames.

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Addendum/Closure by **Eric M. Lui**

The author is grateful to Carskaddan for his interest in the paper and for his valuable comments. One major objective of the paper is to provide engineers and designers with a practical engineering approach to account for second-order effects in structural design. By expressing the $P\delta$ and $P\Delta$ effects in terms of quantities well-known to structural engineers, viz. moments and shears, the author believes that engineers can gain better insight into these destabilizing forces on members and frames. The work was prompted by the adoption and publication of the AISC-LRFD specification in which second-order elastic analysis is recommended for frame design. With the proposed method, an analyst can obtain secondary moments by a first-order analysis technique. This is a technique which all structural engineers are familiar with and apply on a routine basis.

Carskaddan is correct in stating that the application of the pseudo $P\delta$ forces should not upset the equilibrium state of the member. In fact, member equilibrium is implicitly enforced if one applies both pseudo $P\delta$ and pseudo $P\Delta$ forces to the member. The pseudo $P\delta$ force is obtained by scaling the moment diagram down by the factor P/EI and the pseudo $P\Delta$ force (pseudo shear) is obtained by multiplying P by the end slope of the member. Suppose member AB is subjected to a pseudo $P\delta$ force of PM/EI where M is the distribution of bending moment along the member, the cumulative effect of this force is

$$\int_A^B \frac{PM}{EI}\,dx \tag{a}$$

For small displacements, the substitution

$$\frac{M}{EI} = -\frac{dy^2}{dx^2} \tag{b}$$

where y is the lateral displacement of the member can be made and so we can write

$$\int_A^B \frac{PM}{EI}\,dx = -\int_A^B P\,\frac{dy^2}{dx^2}\,dx \qquad \text{(c)}$$

From calculus,

$$-\int_A^B P\,\frac{dy^2}{dx^2}\,dx = -P\left(\frac{dy}{dx}\right)_B = P\left(\frac{dy}{dx}\right)_A \qquad \text{(d)}$$

By comparing Equations (c) and (d), we obtain

$$\int_A^B \frac{PM}{EI}\,dx = -P\left(\frac{dy}{dx}\right)_B + P\left(\frac{dy}{dx}\right)_A \qquad \text{(e)}$$

Equation (e) implies that the cumulative effect of the pseudo $P\delta$ force is statically equivalent to the pseudo $P\Delta$ forces. Therefore, by applying a pair of pseudo $P\Delta$ forces at the ends of the member in conjunction with the pseudo $P\delta$ forces which act along the length of the member, one automatically enforces member equilibrium. Figures 5(b) and 5(c) thus represent a set of forces in equilibrium.

The fact that the pseudo $P\delta$ force is balanced by the pseudo $P\Delta$ force is apparent for the cantilever beam-column shown in Fig. 9 of the paper. Note that no additional base shear is induced at the support. As for the beam-column shown in Fig. 7 of the paper, it seems at first glance that additional support reactions will be induced due to the application of the pseudo $P\delta$ force. However, if one also applies pseudo shears at the member ends, these 'pseudo' reactions will be canceled. This is shown in Fig. A for the second cycle of analysis. Note that no additional reactions are induced at the supports since the 'pseudo' reactions are balanced by the pseudo shears.

Although member equilibrium is always satisfied, joint equilibrium may be violated if pseudo shears from adjacent members meeting at the joint do not balance. To ensure that joint equilibrium is satisfied, a pseudo joint load calculated according to Eq. 10 must be applied at the joint. For the frame examples shown in the paper, only the pseudo joint loads are shown, pseudo shears which act at the ends of the members are deliberately omitted to avoid obscuring the figures. Although these pseudo shears were not shown, they were being considered in the calculations. A computer program incorporating the pseudo load method for second-order elastic frame analysis is available. Interested parties can obtain a listing of the program free of charge by writing to the author at the Department of Civil and Enviornmental Engineering, 220 Hinds Hall, Syracuse University, Syracuse, NY 13244-1190.

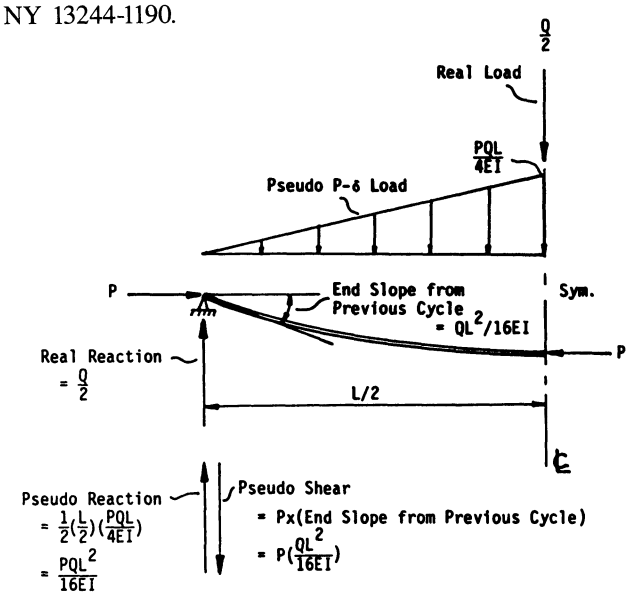

Fig. A. Member equilibrium at 2nd cycle of beam column shown in Figure 7.