Notes on Bracing Design

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It is a common practice among engineers to design a bracing system to utilize maximum allowable stresses in all its members. This is done to arrive at economical or optimal design. This approach, however, is consistent with the basic principles of economical design only in the case of statically determinate systems or in some specific cases of indeterminate systems.

The ambiguities associated with statical indeterminacy are caused by the fact that the stress distribution depends on the design parameters. These parameters are unknown at the start of the design process. They will have to be assumed before stresses can be evaluated. This, in a natural way, leads to a trial-and-error approach which is the only way to design statically indeterminate system.

The above-mentioned characteristics of behavior and design of indeterminate system is commonly known. To arrive at an economical design, however, the basic principles of optimization also have to be known. Unfortunately, this knowledge is confined to a narrow group of specialists. Lacking this specific knowledge, some engineers may still be tempted to look for an economical design by utilizing maximum allowable stresses, in excess of what is possible from the theoretical viewpoint.

In view of this, it seems worthwhile to present an example of a statically indeterminate multiple bay bracing system frequently encountered in engineering practice. It will be pointed out that, in general, utilization of maximum allowable stresses has to be compromised if basic principles of structural analysis are to be obeyed. We will also show how to select the geometry to fully stress the members of the bracing system.

ANALYSIS OF A BRACING SYSTEM

Consider a wall framing system consisting of M columns, M-1 struts, and I braced bays of different sizes. The engineer must design this framing to handle a given horizontal shear and to satisfy an allowable stress criterion. Assume in our case that it includes a tension bracing system in which only tension diagonals are active. The horizontal load will be distributed between the I Bays in proportion to their stiffnesses in accordance with principles of analysis (stiffness k is equal to the inverse of the deflection of a braced bay due to unit load). This proportion is not known, however, until the sectional properties of the diagonals are known (column and strut sizes are assumed large enough to affect final analysis in a very insignificant way). Our goal might seem obvious: Select the cross sectional areas (A_1 and A_2) of the diagonals to achieve their minimum weight by stressing them to maximum allowable level. We will assume they are made of ASTM A36 steel having an allowable tensile stress F_T of 21600 PSI.

Let us formulate our analysis in terms of the appropriate equations. Note that the general equation defining horizontal displacement Δ of a single braced bay I, due to unit horizontal force, is:

$$\Delta_I = \frac{N_I^2 L_I}{EA_I} = \frac{\dfrac{1}{cos^2\ \alpha_r}\ \dfrac{h}{sin\ \alpha_I}}{EA_I} \qquad (1)$$

where α_I, A_I for I=1,2, and h have been shown in Fig. 1. N_I and L_I are the force in the diagonal and its length, respectively.

Consequently, the stiffness of the single bay is:

$$K_I = EA_I \frac{cos^2\ \alpha_I\ sin\ \alpha_I}{h} \qquad (2)$$

Thus the total shear force H is distributed to the two braced bays as follows:

$$H_I = H \frac{A_I\ cos^2\ \alpha_I\ sin\ \alpha_I}{\sum_{I=1}^{2} A_I\ cos^2\ \alpha_I\ sin\ \alpha_I} \qquad (3)$$

On the other hand, if the allowable stressses are to be completely utilized, the following must be true:

$$H_I = F\ A_I\ cos\ \alpha_I \qquad (4)$$

Substitution of this relationship in Eq. 3 yields:

$$F = H \frac{cos\ \alpha_I\ sin\ \alpha_I}{\sum_{I=1}^{2} A_I\ cos^2\ \alpha_I\ sin\ \alpha_I} \qquad (5)$$

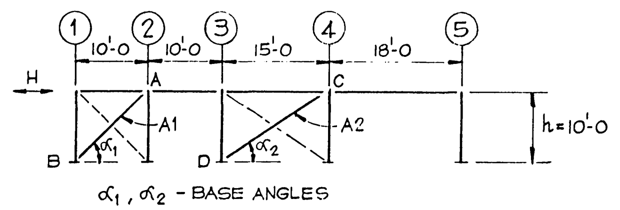

Figure 1

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Table 1.

Member	l(in.)	N	$\Delta = \frac{N^2 l}{AE}$(in.)	$K = \frac{1}{\Delta}$
AB	169.7	$\sqrt{2}$	$\frac{339.4}{EA1}$	$\frac{EA1}{339.4}$
CD	216.3	1.20	$\frac{312.4}{EA2}$	$\frac{EA2}{312.4}$

Table 2.

Member	l(in.)	N	$\Delta = \frac{N^2 l}{AE}$(in.)	$K = \frac{1}{\Delta}$
AB	144.2	1.80	$\frac{468.7}{EA1}$	$\frac{EA1}{468.7}$
CD	216.3	1.20	$\frac{312.4}{EA2}$	$\frac{EA2}{312.4}$

which should hold for I=1 and I=2. Since the denominator in Eq. 5 is the same for I=1 and I=2, one can conclude that the two equations can be satisfied only if angles α_1 and α_2 are equal or complementary. In general, the solution to the two equations does not exist, and allowable stresses cannot be utilized to maximum in both diagonals simultaneously.

EXAMPLES

To illustrate points proven here, let us consider the two cases shown in Figs. 1 and 2. Assume $H = 100\ K$. In the first example, base angles α_1 and α_2 are not equal or complementary. Brace force N, used in the deflection analysis, is calculated by applying a unit horizontal force at the top of each braced bay and by using the method of sections. Table 1 shows the resulting stiffnesses k.

Assume for instance a 40 percent and 60 percent shear in bays 1–2 and 3–4.

As required by allowable stress condition:

$$A_1 = \frac{.40H}{F\ cos\ \alpha_1} = \frac{.40 \times 100}{21.6 \times cos\ 45°} = 2.62\ \text{in.}^2$$

$$A_2 = \frac{.60H}{F\ cos\ \alpha_2} = \frac{.60 \times 100}{21.6 \times cos\ 33.7°} = 3.34\ \text{in.}^2$$

$$\frac{A_1}{A_2} = \frac{2.62}{3.34} = \frac{1}{1.27}$$

as required by the stiffness condition:

$$\frac{K_1}{K_2} = \frac{312.4\ A_1}{339.4\ A_2} = \frac{40}{60} \rightarrow \frac{A_1}{A_2} = \frac{1}{1.38}$$

Obviously, the two conditions are not satisfied simultaneously. Since the stiffness ratio requirement must be met to agree with the assumed shear distribution, the allowable

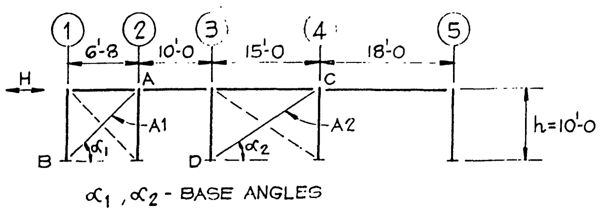

Figure 2

stress condition must be compromised—one diagonal will have to be understressed:

$$A_2 = 2.62 \times 1.38 = 3.62\ \text{in.}^2 > 3.34\ \text{in.}^2$$

Table 2 presents data for Case 2 in which the stiffness condition is satisfied and the full tensile capacity of diagonals is reached. In this case, angles α_1 and α_2 are complementary. This occurs when bays 1–2 and 3–4 are 6 ft-8 in. and 15 ft, respectively. Assume $A_1/A_2 = 2$.

The calculations are as follows:

$$\frac{K_1}{K_2} = \frac{A_1\ 312.4}{A_2\ 468.7} = \frac{A_1}{1.5\ A_2}$$

$$\frac{K_1}{K_2} = \frac{2}{1.5} \rightarrow H_1 = 57.1\ \text{kip},\ H_2 = 42.9\ \text{kip}$$

The diagonal areas required to satisfy the allowable stress condition are:

$$A_1 = \frac{57.1 \times 1.8}{21.6} = 4.76\ \text{in.}^2$$

$$A_2 = \frac{42.9 \times 1.2}{21.6} = 2.38\ \text{in.}^2$$

$$\frac{A_1}{A_2} = 2$$

which agrees with our assumption and shear distribution criterion.

CONCLUSION

In general, bracing design in a wall framing containing multiple braced bays of different sizes will result in some diagonals being understressed. Full capacity of members can be achieved only by assuming braced bays with geometry consisting of either equal or complementary base angles.

The extent that allowable stresses can be utilized and how this is related to optimal design of the system shown in Case 1 is a broader issue that cannot be addressed without use of optimization theory. Some guidance, however, to approaching this problem can be obtained from Eqs. 2 through 5.

Although our considerations here refer to bracing system, the conclusion reached is more general and relates to most statically indeterminate structural systems for which pertinent equations would have to be considerably more complex.