

Lehigh Conference on Plastic Design of Multi-Story Frames—A Summary

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IN AUGUST, 1965, a conference of teachers of structural engineering and practicing structural engineers was held at Lehigh University. The purpose of the conference was to present a comprehensive coverage of new developments in the application of plastic design principles to the design of steel multi-story building frames. The conference consisted of a group of basic lectures and experimental tests performed by members of the Lehigh staff and additional supplemental lectures by outstanding engineers and educators who were attending the conference.

This paper will give an outline of the plastic design procedures proposed with specific detailed coverage on some key new developments. The scope of the conference covered rigid frameworks considered as single plane structures in which sway was resisted either with the aid of diagonal X-bracing or by rigid frame action alone. An essential part of the conference was a comprehensive set of lecture notes and a design aids booklet which will serve as the primary references for this paper.^{1, 2}

PRELIMINARY DESIGN

The preliminary stages of the design of either braced or unbraced frames consist of typical architectural and structural considerations for which no new concepts were presented. These are: functional requirements, size, shape, layout, and occupancy classifications which control loads. Also included are: roof and floor system design, rigid frame loads from floors, and tabulation of girder and column loads. New to these studies were the consideration of load factors of 1.70 for gravity loads and 1.30 for gravity plus wind loads. Reductions of load factor to this level were justified by the fact that many satisfactory structures designed by current allowable-stress design will have no greater actual factor of safety against ultimate load. Multiplication of working loads

formulated by conventional methods by these load factors gives the set of ultimate loads for which the plastic design of the frame is prepared.

PRELIMINARY ANALYSIS OF BRACED FRAMES

The design of braced frames is based on a preliminary analysis assuming that beam mechanisms form in all girders under the factored gravity loads. One new consideration in the conference was that the beam mechanisms formed entirely in the clear spans outside the column faces. This recognized that the bending moments referred to the column centerlines could be greater than the plastic moment of the girders as shown by the moment diagram of Fig. 1. This consideration has the advantage of both greater accuracy and economy in girder sizes without serious increases in column requirements. In combination with these girder moments, equilibrium of column moments can be achieved with half the unbalance assumed to act above and below the joint as shown in Fig. 2. This assumption is justified by a demonstrated small effect on column strength of inaccuracies of moment gradient in double curvature columns.

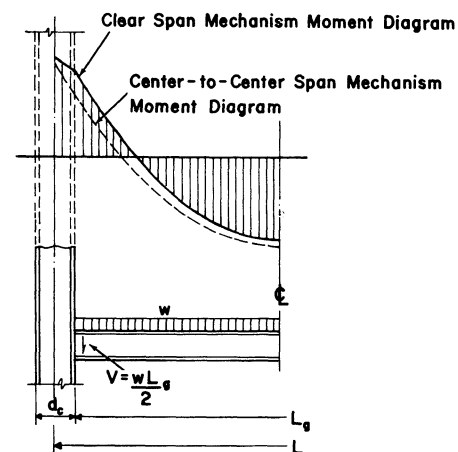


Fig. 1. Moment diagram for mechanism forming in clear span of girder

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PRELIMINARY DESIGN OF BRACED FRAMES

The forces from the preliminary analysis make it possible to select members for the gravity load case. Girders are selected on the basis of the required plastic moment, M_p . Checks must be made to assure control of local buckling and lateral buckling. New recommendations were presented for b/t and d/w ratios for bending members, along with recommendations for lateral bracing spacing and strength required. A range of recommendations was presented to cover steels having yield points up to 50 ksi.

Trial column sections are selected on the basis of the combination of thrust and moment from the preliminary analysis. Design aids presented in the conference included M_{pc} tables for selecting trial sections neglecting column instability effects. Moment-rotation curves make it possible to check slenderness in the plane of the frame, and tabulated values of a basic column formula enable rapid checks of out-of-plane slenderness and facilitate the design of axially-loaded interior columns.

Figure 3 schematically describes the forces in bracing members and adjoining frame members under a system of combined gravity and lateral loads, 1.3 times the working loads. The bracing system is assumed to behave as a pin-connected Pratt truss. Bracing forces from stories above are assumed to be carried down the frame by a couple comprised of axial force components in each pair of braced columns and by tension forces in each diagonal. Compression diagonals are assumed to be so slender that they will buckle at a negligible load and act as counters. Additional horizontal forces are assumed to be introduced in each floor through compression forces in the girders. This analysis made it possible to select diagonal bracing members and to check for necessary revisions in beam or column members resulting from axial forces induced by bracing. The lectures covered design to prevent sway due to both combined loading and frame buckling under vertical loading. Also covered were design based on limiting slenderness of bracing and working load deflection of the frame.

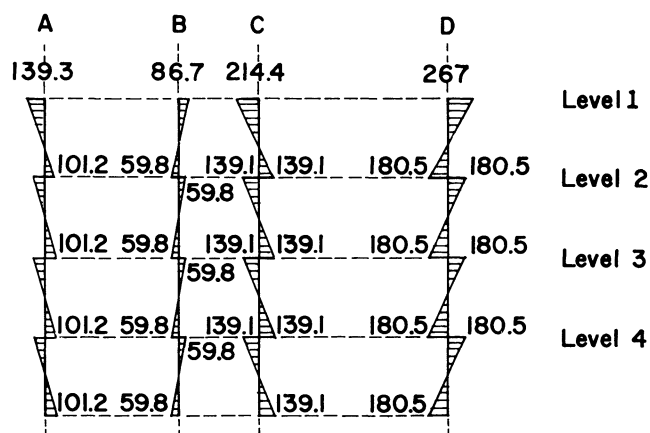


Fig. 2. Column moments due to gravity loads

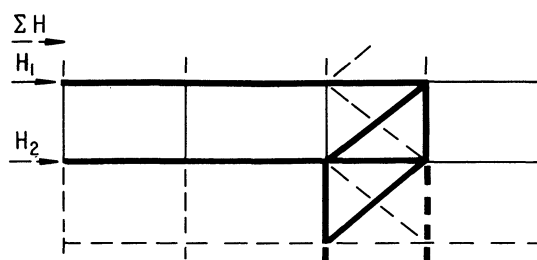
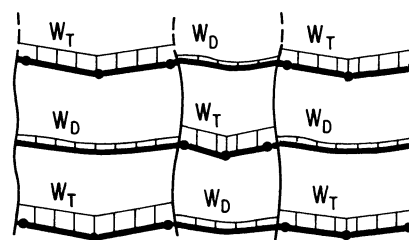


Fig. 3. Forces due to bracing action



W_T = Factored total load

W_D = Factored dead load only

Fig. 4. Effect of checkerboard loading on columns

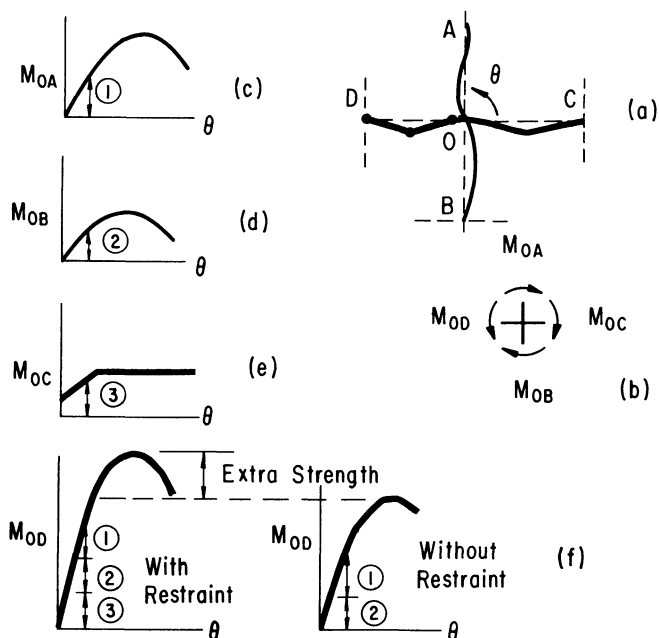


Fig. 5. Strength of restrained column

Further column problems studied involved the checkerboard loading concept where absence of live load from some bays can cause more severe bending in certain columns as indicated in Fig. 4. Economical solutions to this problem are facilitated by restrained column theory. This theory shows that the restraint provided by elastic beams without the live load increases the capacity of columns. New design aids based on this theory are column deflection curves and moment-rotation curves for columns prevented from sway. Interpretation of restrained column theory is illustrated by Fig. 5. Figure 5a shows columns **OA** and **OB** loaded by girder **OD** with full factored dead plus live load and girder **OC** which has only factored dead load and remains elastic. The plastic hinge moment at **O** in girder **OD** must be resisted by the moments **OA**, **OB**, and **OC** provided by the remaining girder and columns, as shown in Fig. 5b. Figures 5c and 5d show the moment-rotation curves of columns **OA** and **OB** if they were loaded separately. Figure 5e shows the moment-rotation curve of girder **OC** as a separate member. By adding together the moments for each given rotation of **OA**, **OB**, and **OC**, the rate of build-up of moment in member **OD** can be constructed as shown in Fig. 5f. A second sketch in Fig. 5f shows the comparison of the moment **OD** if the restraint provided by elastic beam **OC** were absent. The obvious extra strength provided by the restraint can be acknowledged as a part of the routine design procedure.

VERIFYING TESTS

Verification of restrained column theory was demonstrated by the subassembly test depicted in Fig. 6. A 10 ft long 6 WF 25 column of A441 steel was loaded axially

by means of a testing machine at the same time as bending moment was applied to its ends by means of hydraulic tension jack forces applied to stub beams at the top and bottom of the story. Two longer 12 B 16.5 beams of A36 steel in the bay at the opposite side of the column provided the restraint simulating elastic beams without live load. A moment versus joint rotation curve from the test compares well with the theoretical curve derived from the restrained column theory.

A three-story, two-bay braced frame using 12 B 16.5 girders and 6 WF 20 and 6 WF 25 columns was tested by applying combined horizontal and vertical loads with hydraulic jacks. The frame had an overall height and span of 30 ft each. Figure 7 shows a load-deflection curve of this test compared with a theoretical prediction. Good agreement is obvious. The photograph in Fig. 7 shows the loading frame used to support the specimen laterally so a single plane frame could be tested alone. Also shown is the system of gravity load simulator devices which allow the application of truly vertical loads even though the frame sways laterally in its plane.

Other tests were performed to demonstrate the basic material and component properties. These were tensile tests, residual stress measurements, beam test, composite beam test, and stub column test.

BRACED FRAMES—DESIGN EXAMPLES

Three braced frames shown in Fig. 8 were designed as examples and compared with allowable stress designs. The frames were a three-story two-bay, a ten-story three-bay, and a twenty-four story three-bay frame. Figure 9 shows the members selected for the ten-story frame and Fig. 10 shows the comparison of the steel

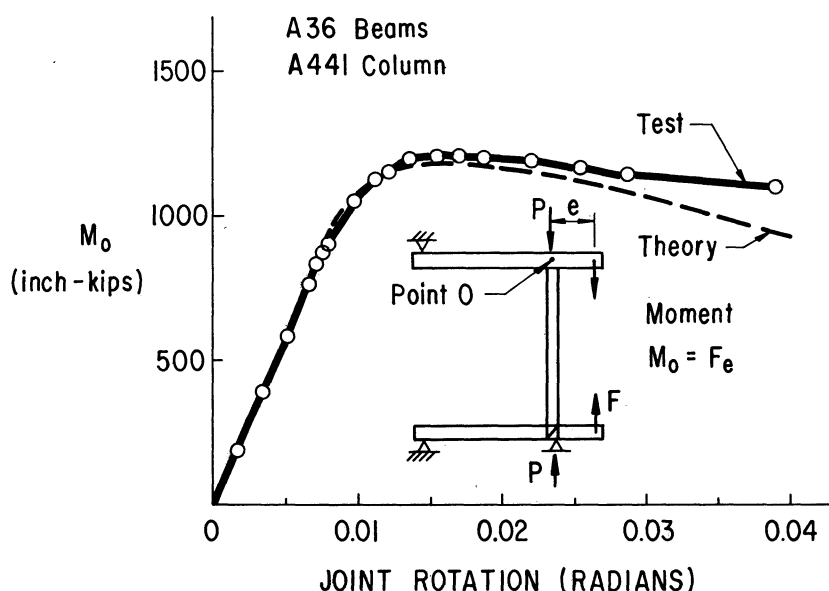
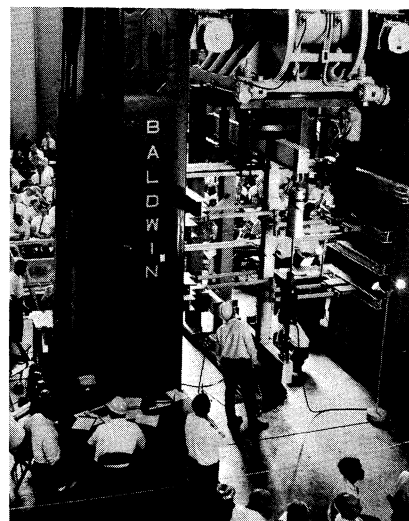


Fig. 6. Results of subassembly test



weights required for the plastic design and an allowable stress design of the same frame. Savings of steel of 8 percent, 8 percent, and 6.5 percent were indicated for the three frames designed.

PRELIMINARY ANALYSIS OF UNBRACED FRAMES

In the design of unbraced frames for gravity load, the preliminary architectural and structural considerations up to the tabulation of loads and selection of members

would be similar to those described for braced frames. In evaluating the design for resistance to combined horizontal and vertical loads, different preliminary analysis procedures are required. From considerations of equilibrium in a given story, the required resistance of girders and columns can be calculated.

In the conference, a method for determining the sum of column end moments in a story was presented. Figure 11 shows a free body diagram of the several columns in a

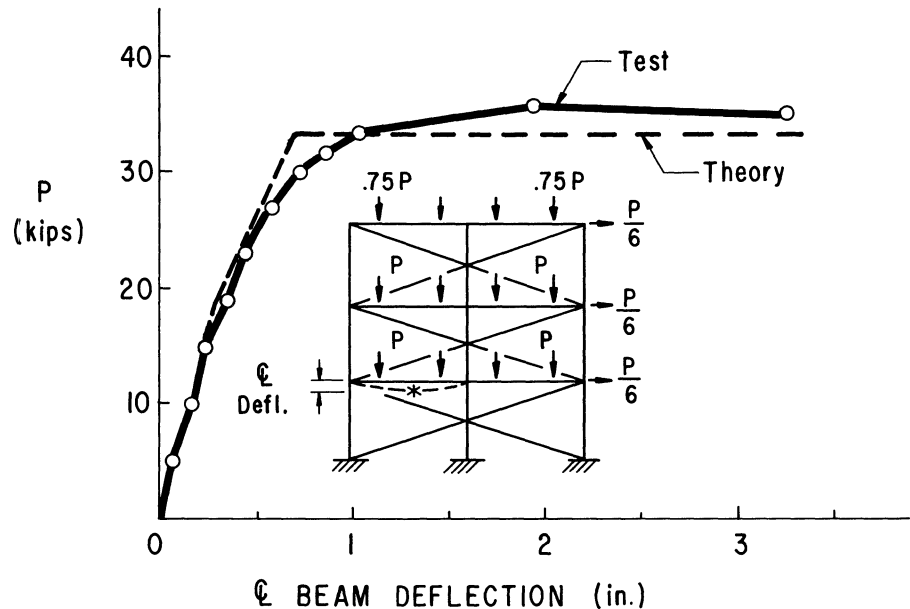
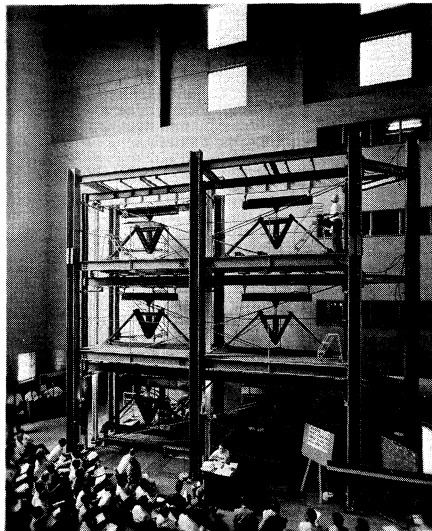
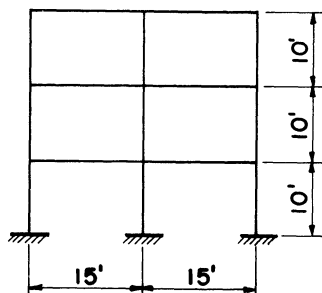
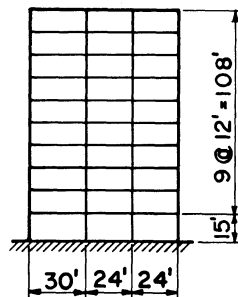


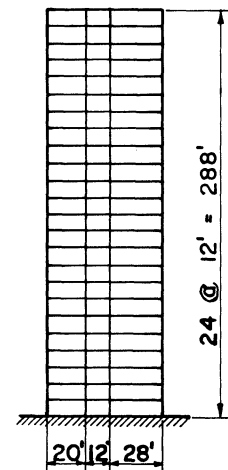
Fig. 7. Results of braced frame test



FRAME A



FRAME B ---10 STORIES



FRAME C ---24 STORIES

- LECTURES AND DEMONSTRATIONS

Fig. 8. Types of braced frames in Design Examples

story subjected to a resultant horizontal shear ΣH from all the stories above and a sum of column loads ΣP from all the stories above. The story has a sway Δ and a height h . The horizontal shear and the vertical loads in the swayed position together cause an overturning moment which must be resisted by the sum of the column end moments ΣM_c . Without knowing the individual end moments, their required sum can be determined from the following equation:

$$\Sigma M_c = -(\Sigma H)h - (\Sigma P)\Delta \quad (1)$$

Figure 12 shows a free body diagram of the girders on one level which receive column moments from the bottoms of the columns above and from the tops of the columns below. For an estimate, it is assumed that half the total moments are at the top and bottom of each set of columns. Then the sum of the clockwise end moments on all girders in a level (for wind from left) are:

$$\Sigma M_g = -\frac{1}{2}[(\Sigma M_c)_{n-1} + (\Sigma M_c)_n] \quad (2)$$

where $n-1$ refers to the story above and n to the story below the girders. The sway value Δ which affects ΣM_c in both equations is unknown at the time of preliminary analysis but can be purposely overestimated to select adequate members and then revised if later deflection checks show this to be necessary.

Once the sum of girder end moments required is known, the selecting of girders can begin. This is aided by solutions for the sway resistance of a loaded girder. Figure 13 shows a moment diagram of a girder with both uni-

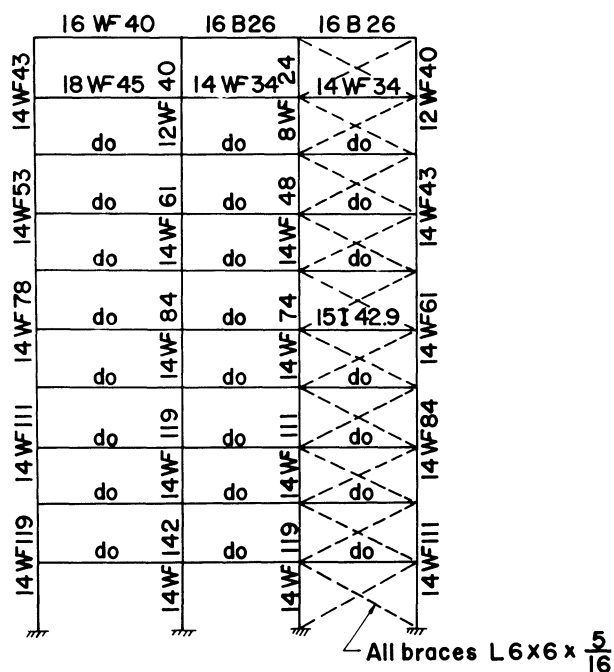


Fig. 9. Member sizes of frame B designed by plastic method

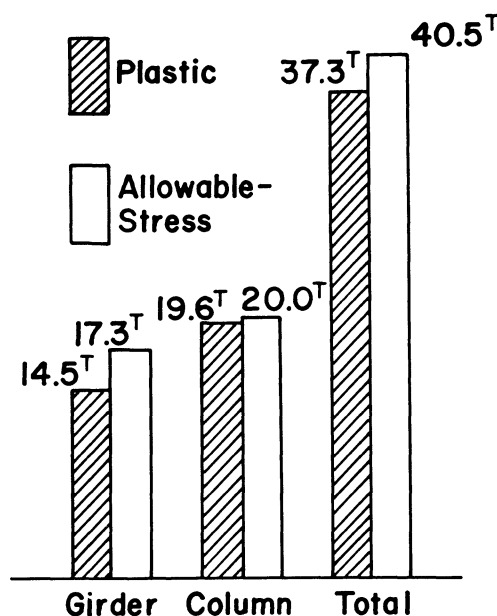


Fig. 10. Weight comparison for frame B

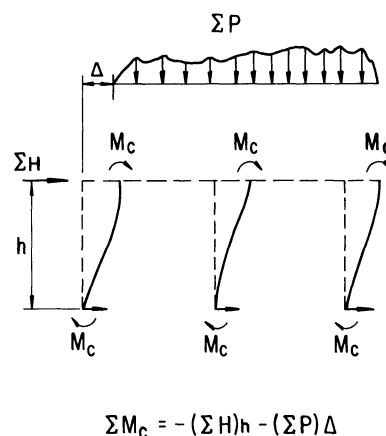


Fig. 11. Horizontal shear equilibrium in a story of an unbraced frame

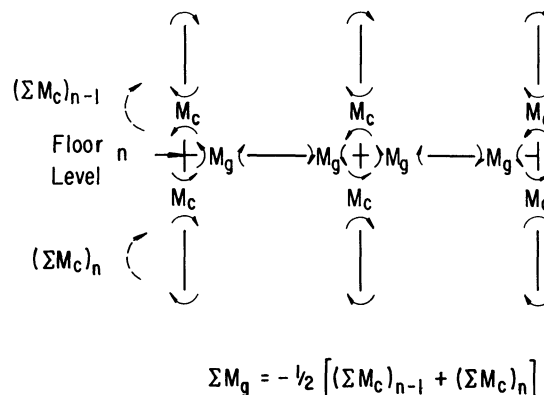


Fig. 12. Sum of girder moments in a story of an unbraced frame

form loads and sway moments. The limit of capacity is reached when a plastic hinge forms at the lee column face and another at some point between the center and the windward column face. To carry the anti-symmetrical wind moments along with the symmetrical gravity moments requires a larger M_p than is required for the gravity loads alone. Equilibrium solutions based on the moment diagram of Fig. 13 permit the determination of required M_p , moments at both column centerlines, and moments at both column faces for a given factored load and sum of clockwise girder moments. A chart for the determination of these functions is given in the lecture notes.

PRELIMINARY DESIGN OF UNBRACED FRAMES

Using the preliminary analysis and a girder selection chart, preliminary girder sizes and girder end moments may be determined. It is then necessary to determine the individual column end moments which have so far been grouped together as a sum. A moment balancing method was presented in the conference for this purpose. This is simply an orderly process for calculating and keeping track of moment equilibrium at each joint.

Having column end moments, it is then possible to select preliminary column sections using the same basic M_{pc} tables and moment-rotation curves as were used for columns in braced frames. A further check is needed to determine whether actual effects of sway deflection are no greater than assumed in determining column and girder moments.

CHECKING PROCEDURE FOR SWAY

After loads, girder sections, and column sections are determined from the preliminary design, the column restraint provided by girders can be determined. The conference provided equations for restraint functions based on the stiffness, length, and plastic moment of girders. The resistance to sway of a single story can be analyzed by considering the horizontal force versus sway characteristics of subassemblages consisting of a column and the girders framing to it. Figure 14 is typical of design aids prepared for the design of columns using the subassemblage concept. For a given size column with a given axial load, it gives the horizontal force versus sway for a number of different strengths of restraining members. The curve ABC shows the behavior of a particular subassemblage which has a restraining moment function 120 times the end rotation until a plastic hinge forms in the restraining beam (point A). Then the restraining strength is cut in half until another plastic hinge is formed (point B). The remaining part of the curve is the behavior of an unrestrained column. The resistance to superimposed horizontal force must decrease with increasing sway because more of the capacity is required to resist the overturning moment caused by

the vertical load P . The actual process of using the curves is to use overlays of transparent paper for tracing lines for the particular subassemblage studied. The slope and extent of each line is determined from the earlier calculations of restraint functions. This process is followed for each column in a story. Then the sum of the column resistances for a given amount of sway can be added to give the resistance of the whole story for the same sway. Figure 15 shows the force versus sway graphs for four columns and then the curve for the four columns added together giving the total story resistance which is seen to be about 194 kips. At working load, near 114 kips, the sway is seen to be about 0.002 times the story height.

This procedure gives a solution to the strength of a single story considering the effects of inelasticity and sway. Being able to solve the problem will make it possible to formulate practical design procedures.

OTHER CONSIDERATIONS

Other considerations in the basic lectures of the conference were frame buckling and the application of high-strength steels to plastic design. Frame buckling is a problem which can occur when a symmetrical structure is loaded by symmetrical gravity loads only. Sway buckling can occur sometimes at lower loads than would cause failure if the structure remained in a vertical position. Except in the higher stories, the design for combined wind plus gravity load will provide the necessary resistance to frame buckling under gravity load alone. Proportioning the upper stories to resist frame buckling was also discussed.

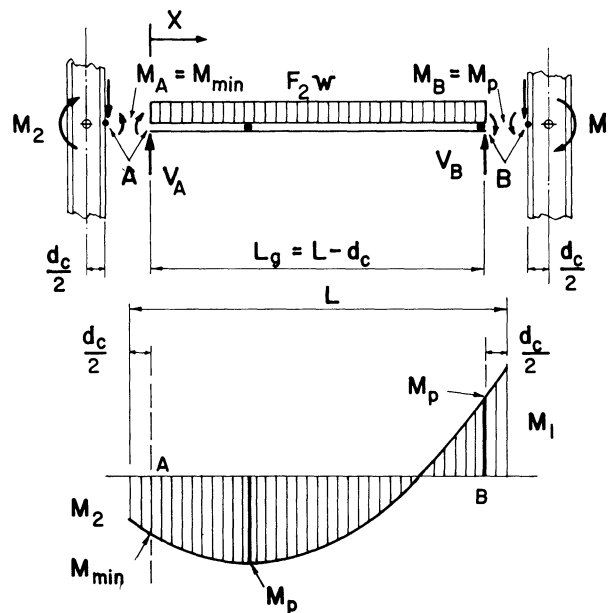


Fig. 13. Sway moments on a transversely loaded girder

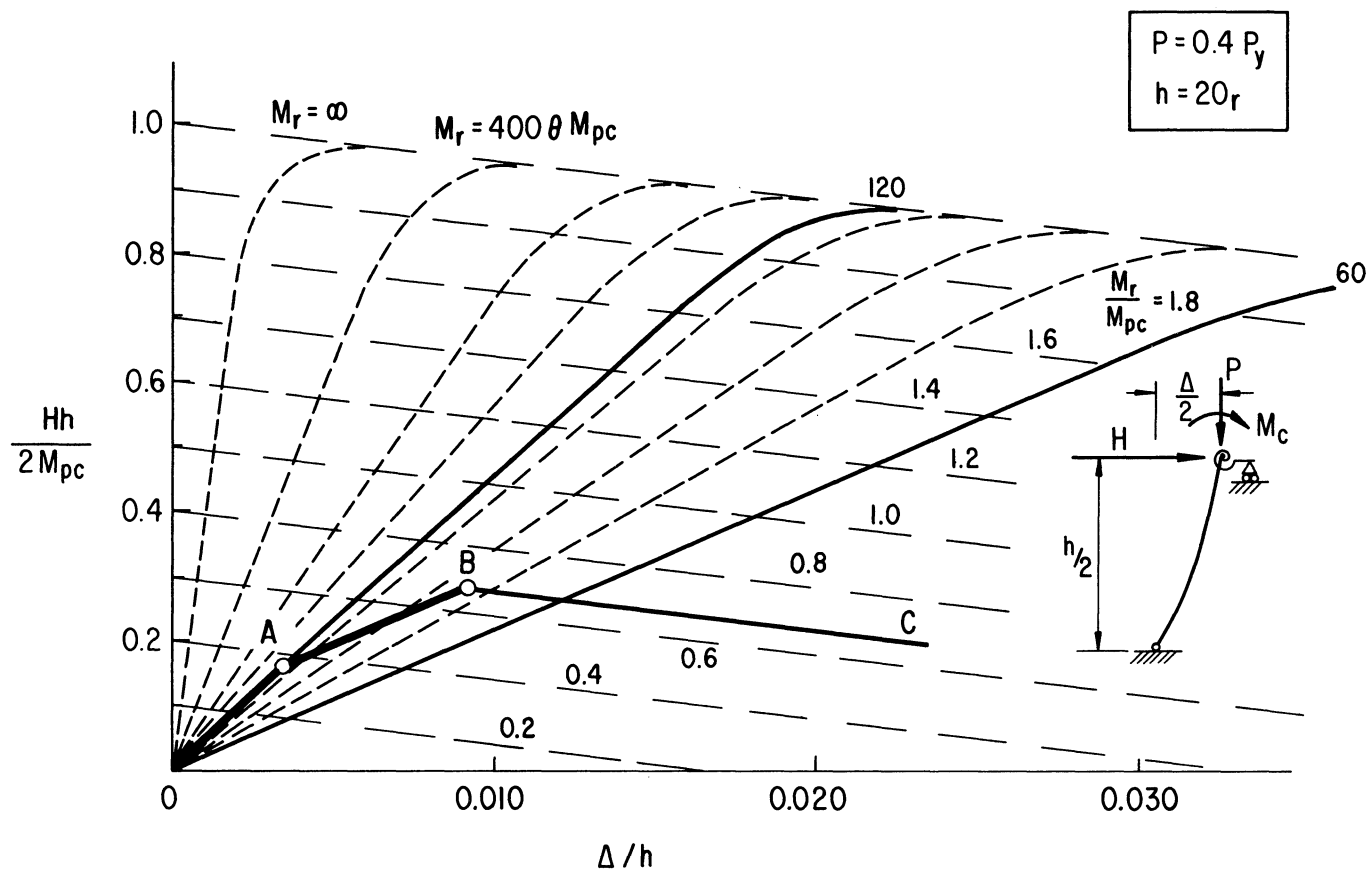


Fig. 14. Curves for the design of columns using the subassembly concept

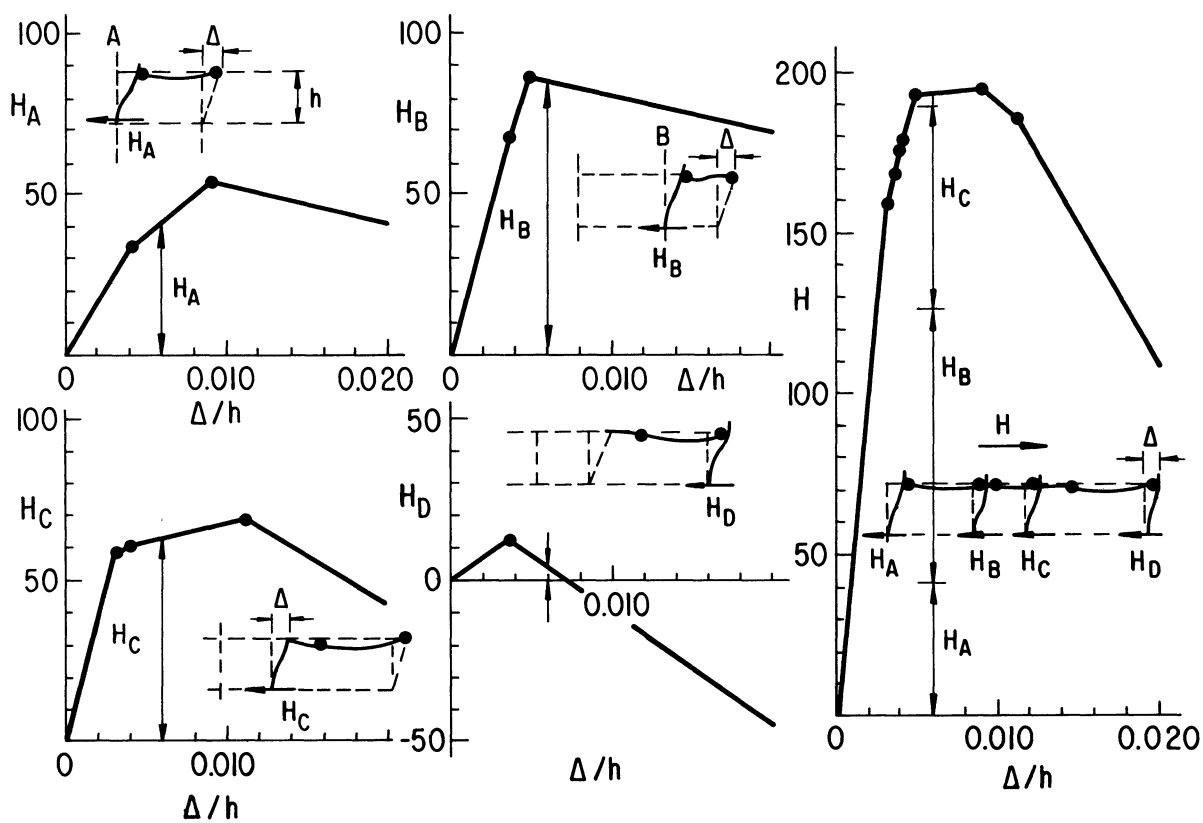


Fig. 15. Story shear resistance obtained from sum of subassembly shears

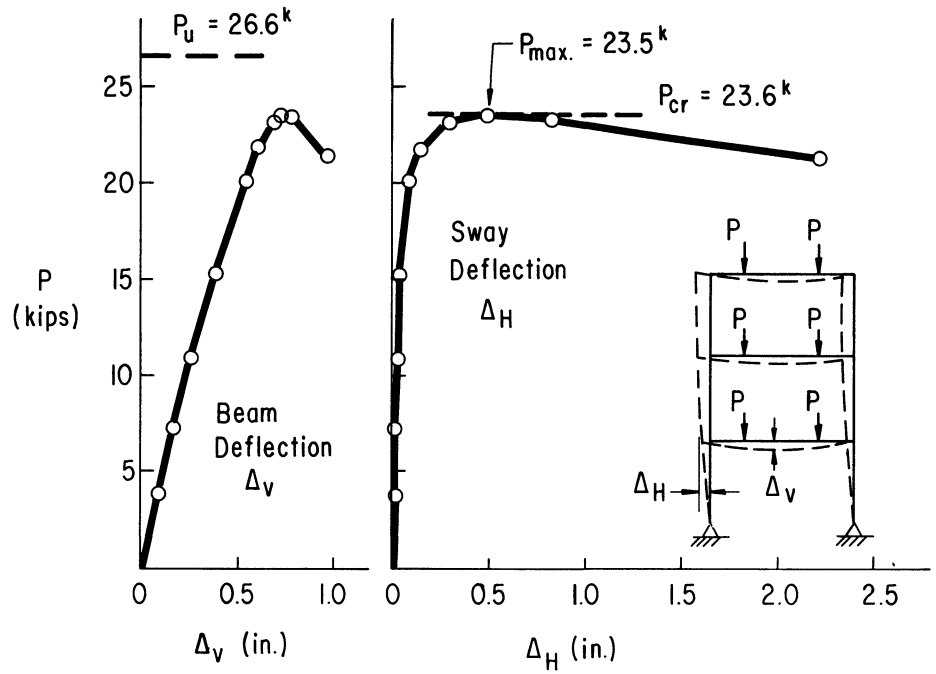
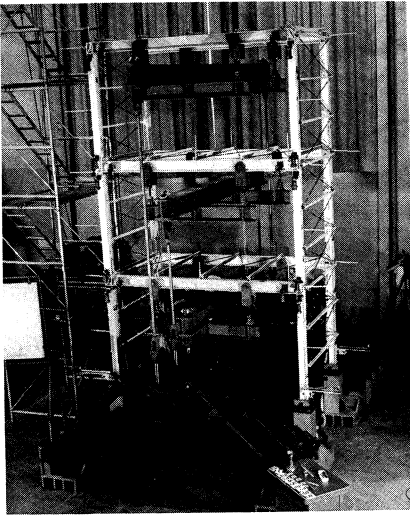


Fig. 16. Results of frame buckling test

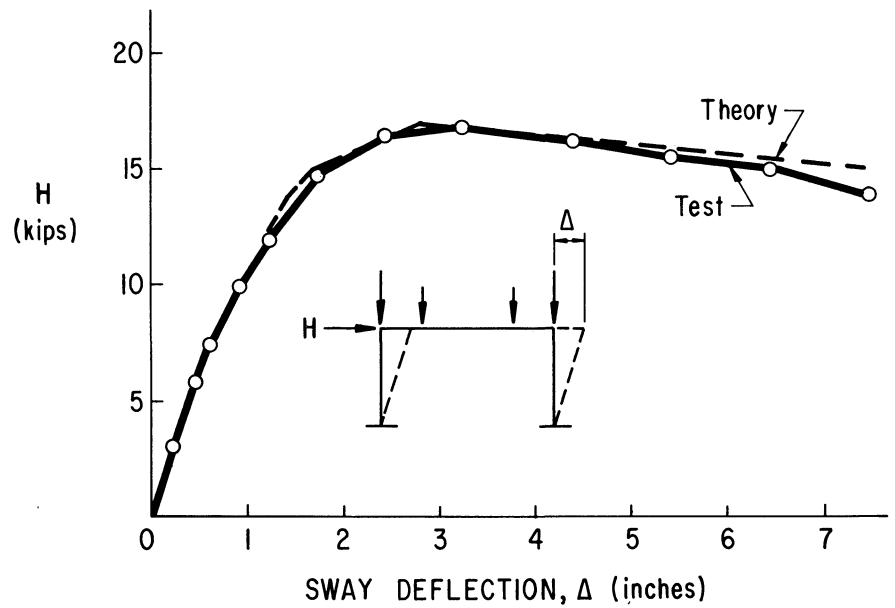
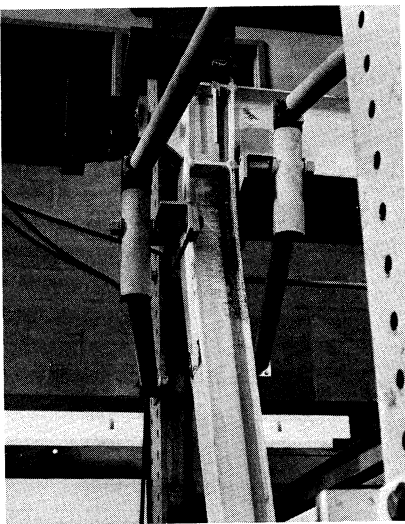


Fig. 17. Results of high-strength steel frame test

Studies of the behavior of individual components in every case included members up to 50 ksi yield point (A441 and A242 steels). The proper proportions were determined to assure adequate performance of plastically designed structures using these materials.

Comparative plastic and allowable stress designs of unbraced frames with the same dimensions given in Fig. 8 were prepared. Savings in steel by plastic design were indicated as 12.3 percent for frame **A**, 13.4 percent for frame **B** and 6.8 percent for frame **C**.

To the basic lectures of the conference were added guest lectures on: structural research at other laboratories, plastic design in other countries, composite construction, earthquake-resistant design, minimum weight design, and practical design problems.

TESTS OF UNBRACED FRAMES

Figure 16 shows the results of a frame buckling test. The vertical load versus beam deflection is shown in Fig. 16a. The maximum load was enough below the plastic theory load to cause a mechanism that the engineer would experience some concern. It is desirable that the plastic theory load be reached. Figure 16b shows the sway deflection caused by the vertical loads as compared with a theoretical prediction of the frame buckling load. Because the theory is accurate, the designer can recognize the possibility of frame buckling and allow for it in design. A photograph of the frame tested is given in Fig. 16c. The frame consisted of two identical bents having a 10 ft span and a total height of 17 ft.

A test of an unbraced portal frame having A441 columns and A36 girder under combined vertical and horizontal loading was performed to demonstrate the plastic behavior of high-strength steel. This frame had a span of 15 ft and a height of 9 ft. Figure 17a shows that the horizontal load versus sway behavior closely approximates the theoretical prediction shown as a dashed line. The photograph of the frame shows the large inelastic deformation of the A441 column which was possible without any unexpected consequences. Except for the higher loads, the investigators could not observe behavior which would appear any different from a frame made entirely of A36 steel. Because of the high concentrated loads at the column tops, this frame could also be looked on as a single story of a taller one-bay multi-story frame.

Figure 18 shows the behavior of the final demonstration test of a two-bay, three-story frame subjected to combined vertical and horizontal loading. The specimen had 6 W 20 columns and had 12 B 16.5 beams on the two floor levels plus 10 B 11.5 beams at the roof level. Its overall height and width were both 30 ft. In the graph of horizontal load versus sway deflection, the solid curve of test results falls slightly above the theoretical curve which includes the effect of the sway displacement of vertical loads. A second theoretical curve which neglects the effect of sway displacement of the vertical loads falls considerably above the experimental curve showing the inadvisability of using first order theory for the design of multi-story frames. A photograph of the test setup and specimen accompanies the test curve.

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CONCLUSIONS

Conclusions reached as a result of the studies and tests discussed in the conference were:

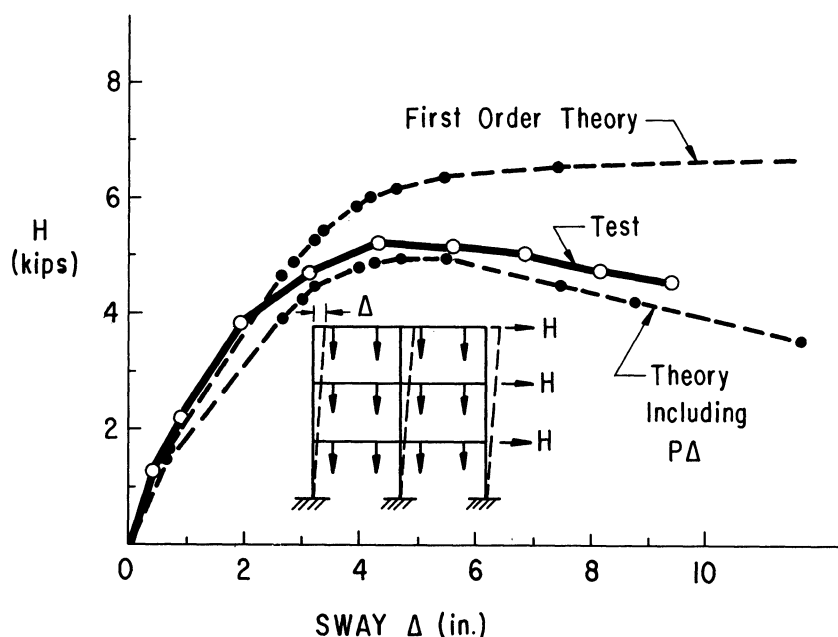
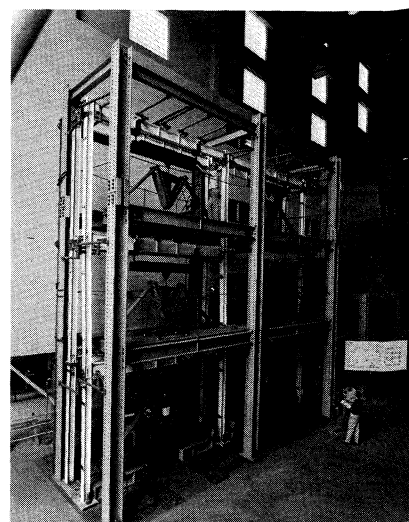


Fig. 18. Results of unbraced frame test



1. The method presented for the design of braced multi-story frames is successful. A savings of steel and design time is possible.
2. Plastic hinges will develop in high-strength steels such as A441 steel. Proper proportions of members will assure adequate rotation capacity for the development of plastic mechanisms in structures.
3. Plastic design of unbraced multi-story frames is feasible. Completion of current research is expected to result in a successful method for the design of typical frames. Less savings of steel may be expected than for a braced frame, and sway deflection can govern the design rather than strength considerations alone.

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