On Beam-Column Moment Amplification Factor

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ABSTRACT

In designing beam-columns in a steel frame, the LRFD Specification uses two moment amplification factors (B_1 and B_2) to account for the second-order $P-\delta$ (member instability) and $P-\Delta$ (frame instability) effects. For beam-columns with joint translation restricted and subjected to end moments only, the B_1 factor usually underestimates the $P-\delta$ effect. However, in some uncommon practical cases, under high axial compression ratio ($P/P_e > 0.7$) with double curvature bending, it is conservative to estimate the $P-\delta$ effect. This is due mainly to the fact that, for simplicity, the LRFD C_m expression excludes the influence of the axial load ratio P/P_e. In this paper, an improved C_m expression, written in terms of both the axial load ratio P/P_e and the end moment ratio M_a/M_b, is proposed. The proposed moment amplification factor has been verified by an extensive comparison with the exact analytical solution. It is found that the $P-\delta$ moment predicted by the proposed formula is more accurate than that predicted by the B_1 factor in the present LRFD Specification.

INTRODUCTION

There are several computer-based methods available for an elastic second-order analysis.[4,6-9,11] For design purposes, a simple procedure is desirable. One simplified procedure is the AISC LRFD method[1] in which the $P-\delta$ effect is approximated by the B_1 factor while the $P-\Delta$ effect is approximated by the B_2 factor. For members subjected to axial compression combined with uniaxial bending, the following LRFD bilinear interaction equation is used to design these beam-columns.

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\frac{M_u}{\phi_b M_n} \le 1.0 \text{ for } \frac{P_u}{\phi_c P_n} \ge 0.2 \tag{1}$$

$$\frac{P_u}{2\phi_c P_n} + \frac{M_u}{\phi_b M_n} \le 1.0 \text{ for } \frac{P_u}{\phi_c P_n} < 0.2 \tag{2}$$

where

P_u = required compressive strength

P_n = nominal compressive strength determined from the LRFD column-strength curve

M_n = nominal flexural strength determined from the LRFD beam-strength curve

ϕ_c = resistance factor for compression = 0.85

ϕ_b = resistance factor for flexure = 0.9

M_u = required flexural strength, i.e., design moment of beam-columns.

For structural members in a frame, M_u is the maximum second-order moment in the beam-column.[12] It may be determined directly from an elastic second-order analysis. In structures designed on the basis of elastic first-order analysis, the LRFD Specification recommends the following procedure for the determination of the second-order moment M_u in lieu of a second-order analysis.

$$M_u = B_1 M_{nt} + B_2 M_{lt} \tag{3}$$

where

M_{nt} = maximum moment in the member assuming there is no lateral translation of the frame, calculated by using a first-order elastic analysis.

M_{lt} = maximum moment in the member as a result of lateral translation of the frame only, calculated by using a first-order elastic analysis.

B_1 = $P-\delta$ moment amplification factor, given by

$$B_1 = \frac{C_m}{1 - \dfrac{P_u}{P_e}} \ge 1.0 \tag{4}$$

where $P_e = \pi^2 EI/(KL)^2$, in which K is the non-sway effective length factor in the plane of bending.

B_2 = $P-\Delta$ moment amplification factor, given by

$$B_2 = \frac{1}{1 - \Sigma P_u\left(\dfrac{\Delta_{oh}}{\Sigma HL}\right)} \tag{5a}$$

or

$$B_2 = \frac{1}{1 - \dfrac{\Sigma P_u}{\Sigma P_e}} \tag{5b}$$

where

Δ_{oh} = translational deflection of the story under consider-

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ation, calculated by using a first-order elastic analysis

ΣP_u = required axial load strength of all columns in a story

ΣH = sum of all story horizontal forces producing Δ_{oh}

L = story height

P_e = $\pi^2 EI/(KL)^2$, in which K is the sway effective length factor in the plane of bending.

Use of the B_1 and B_2 factors is a convenient way to approximate the $P-\delta$ and $P-\Delta$ secondary effects. However, for beam-columns subjected to end moments only, the B_1 factor usually underestimates the $P-\delta$ effect, while in some uncommon practical cases, under high axial compression ratio with double curvature bending, it may overestimate the $P-\delta$ effect.

In this paper we will first discuss the exact amplification factor due to $P-\delta$ effect. Then we will highlight the approximation and simplification associated with the amplification factor B_1 used in the present LRFD Specification. Finally, we will propose an improved C_m factor. For simplicity, we will consider moments about only one axis and will denote P_u and M_u as P and M_{max}, respectively, and the resistance factor will not be considered.

EXACT AMPLIFICATION FACTOR FOR BEAM-COLUMNS SUBJECTED TO END MOMENTS ONLY

For the beam-column subjected to end moments M_a, M_b and axial force P as shown in Fig. 1, the closed-form elastic moment amplification factor A_f can be derived from the governing differential equation for a beam-column.[5,10] The elastic maximum moment M_{max} can be written as

$$M_{max} = A_f |M_b| \tag{6}$$

where, for $0 \leq \bar{x} \leq L$,

$$A_f = \sqrt{\frac{(M_a/M_b)^2 + 2(M_a/M_b)\cos\gamma L + 1}{\sin^2\gamma L}} \tag{7a}$$

and for $\bar{x} < 0$ or $\bar{x} > L$,

$$A_f = 1.0 \tag{7b}$$

in which $\bar{x}$ is the location of the maximum moment and it can be determined by

$$\tan(\gamma\bar{x}) = -\frac{(M_a/M_b)\cos\gamma L + 1}{(M_a/M_b)\sin\gamma L} \tag{8}$$

and γ is given by

$$\gamma = \frac{\pi}{L}\sqrt{P/P_e} \tag{9}$$

M_a and M_b in Eqs. 7 and 8 are end moments such that $|M_a| \leq |M_b|$. M_a/M_b is positive when the member is bent in reverse curvature, and negative when bent in single curvature.

The exact moment amplification factor, Eq. 7, is plotted in Figs. 2 through 4, and some of the values are also listed in Tables 1 and 2 for more detailed comparisons. For convenience in the comparisons, A_f is expressed in the familiar form as:

$$A_f = \frac{C_m^{\,exact}}{1 - \dfrac{P}{P_e}} \tag{10}$$

where, for $0 \leq \bar{x} \leq L$,

$$C_m^{\,exact} = \sqrt{\frac{(M_a/M_b)^2 + 2(M_a/M_b)\cos\gamma L + 1}{\sin^2\gamma L}}\left(1 - \frac{P}{P_e}\right) \tag{11a}$$

and for $\bar{x} < 0$ or $\bar{x} > L$,

$$C_m^{\,exact} = 1 - \frac{P}{P_e} \tag{11b}$$

THE B_1 FACTOR FOR BEAM-COLUMNS SUBJECTED TO END MOMENTS ONLY

The exact amplification factor (Eq. 7), denoted by B_1 in the LRFD Specification,[1] can also be written as

$$B_1 = C_m \sec(\gamma L/2) \tag{12}$$

where C_m is referred to as the equivalent moment factor and is given by

For $0 \leq \bar{x} \leq L$

$$C_m = \sqrt{\frac{(M_a/M_b)^2 + 2(M_a/M_b)\cos\gamma L + 1}{2(1 - \cos\gamma L)}} \tag{13a}$$

For $\bar{x} < 0$ or $\bar{x} > L$

$$C_m = \frac{1}{\sec\dfrac{\gamma L}{2}} = \cos\frac{\gamma L}{2} \tag{13b}$$

To simplify Eqs. 12 and 13, two approximations were made in the LRFD Specification. First, Eq. 13 is replaced by the following expression proposed by Austin:[3]

$$C_m = 0.6 - 0.4\,(M_a/M_b) \tag{14}$$

Second, sec $\gamma L/2$ is replaced by $1/(1 - P/P_e)$. The B_1 factor has therefore the form

$$B_1 = \frac{C_m}{1 - \dfrac{P}{P_e}} \tag{15}$$

For some combinations of P/P_e and M_a/M_b, Eq. 15 results in $B_1 \leq 1.0$. But, by definition, B_1 should not be less than unity, i.e., the second-order moment cannot be less than the first-order moment. To this end, LRFD imposed the condition $B_1 \geq 1.0$. The LRFD Specification therefore recommends B_1 factor as

$$B_1 = \frac{C_m}{1 - \frac{P}{P_e}} \geq 1.0 \tag{16}$$

The restriction on C_m, $C_m \geq 0.4$, imposed in ASD Specification[2] has been removed in the LRFD.

However, these two simplifications result in the following two major shortcomings:

1. The B_1 factor is mostly unconservative as compared to the exact amplification factor. For example, for the beam-column under equal moments ($M_a/M_b = -1.0$), Eq. 16 reduces to

$$B_1 = \frac{1}{1 - \frac{P}{P_e}} \geq 1.0 \tag{17}$$

   This equation is compared with the exact amplification factor for the case of equal end moments in Fig. 3 and in Table 2. It is seen that the larger the P/P_e value, the more unconservative it becomes. For instance, when $P/P_e = 0.5$, the error is about −11%; while when $P/P_e = 0.7$, the error becomes −15%. For a small axial force, such as $P/P_e \leq 0.2$, the error is less than 5%. For $P/P_e > 0.2$, the accuracy of the B_1 factor is decreased. Also, in Fig. 2, the B_1 factor always gives an unconservative result for the single curvature case ($M_a/M_b < 0$) for all values of P/P_e considered.
2. The B_1 factor is conservative for a few uncommon practical cases, i.e., under high axial compression ratio with double curvature bending. For example, in Fig. 2, the B_1 factor is conservative for the case of double curvature for P/P_e greater than 0.7. The conservativeness increases as the value of P/P_e increases.

THE PROPOSED MOMENT AMPLIFICATION FACTOR A_f^*

Against the background of this information, an improved moment amplification factor is proposed for estimating the elastic second-order $P-\delta$ effect of beam-columns subjected to end moments without joint translation:

$$A_f^* = \frac{C_m^*}{1 - \frac{P}{P_e}} \geq 1.0 \tag{18}$$

where

$$C_m^* = 1+0.25(P/P_e)-0.6(P/P_e)^{1/3}(M_a/M_b+1) \tag{19}$$

Equation 18 is compared with the exact solution of Eq. 7 in Figs. 3 and 4 and in Tables 1 and 2. Good agreements are generally observed. This proposed formula has the following advantages:

1. The moment amplification factors predicted by Eq. 18 (Figs. 3 and 4, and Tables 1 and 2) are more accurate than those predicted by the B_1 factor in the present AISC LRFD Specification.
2. When $P/P_e = 0.0$, Eqs. 18 and 19 automatically reduces to $C_m^* = 1.0$ and $A_f^* = 1.0$, respectively.
3. When the beam-column is in single curvature bending ($M_a/M_b = -1.0$), Eq. 18 becomes

$$A_f^* = \frac{1 + 0.25(P/P_e)}{1 - \frac{P}{P_e}} \tag{20}$$

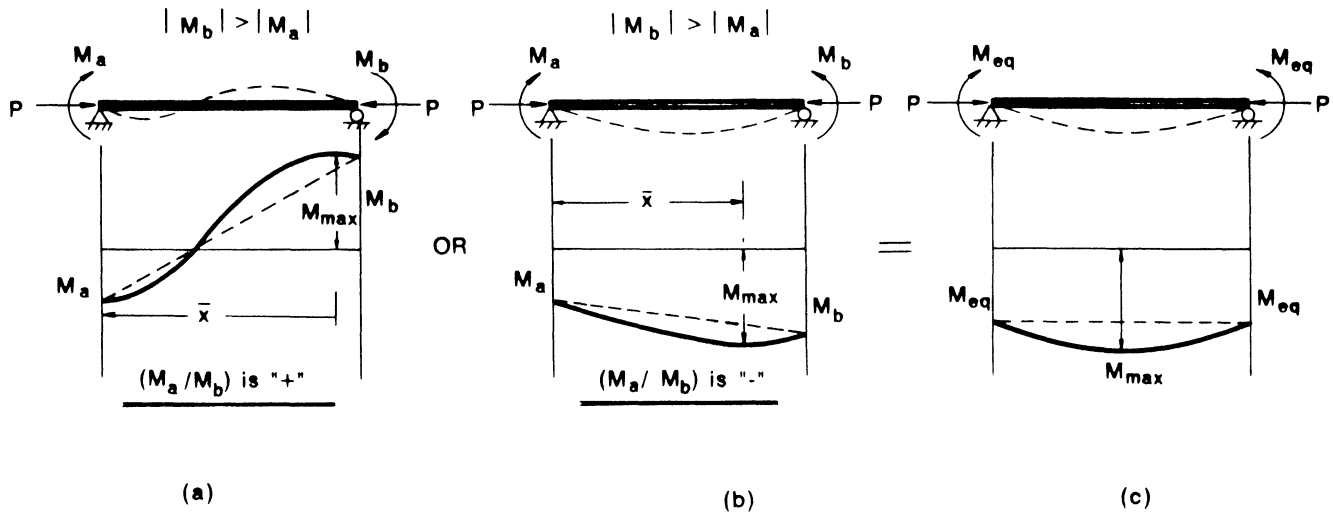

Fig. 1. Maximum moment in a beam-column subjected to end moments and axial force.

This equation can also be derived analytically from Eq. 7a. For the equal end moment case, $M_a/M_b = -1.0$, Eq. 7a reduces to:

$$A_f = \sec \frac{\gamma L}{2} \tag{21}$$

Using the power series expansion for sec ($\gamma L/2$), Eq. 21 can be expressed as

$$A_f = 1+\frac{1}{2}\left(\frac{\gamma L}{2}\right)^2+\frac{5}{24}\left(\frac{\gamma L}{2}\right)^4+\frac{61}{720}\left(\frac{\gamma L}{2}\right)^6+\ldots \tag{22}$$

Substituting $\gamma L/2 = \pi/2\sqrt{P/P_e}$ into Eq. 22, we obtain

$$A_f = 1+\frac{1}{2}\left(\frac{\pi}{2}\sqrt{\frac{P}{P_e}}\right)^2 + \frac{5}{24}\left(\frac{\pi}{2}\sqrt{\frac{P}{P_e}}\right)^4 + \frac{61}{720}\left(\frac{\pi}{2}\sqrt{\frac{P}{P_e}}\right)^6 +\ldots \tag{23}$$

or

$$A_f = 1+1.2337P/P_e+1.2683(P/P_e)^2+1.2727(P/P_e)^3+\ldots \tag{24}$$

or, as a close approximation

$$A_f \approx 1+1.25(P/P_e)[1+(P/P_e)+(P/P_e)^2+\ldots] \tag{25}$$

Thus, Eq. 21 can be conveniently expressed as:

$$A_f = \frac{1 + 0.25P/P_e}{1 - P/P_e} \tag{26}$$

This is the same as the approximate Eq. 20 for the moment amplification factor for a beam-column subjected to equal end moments only.

EFFECT OF THE B_1 FACTOR ON INTERACTION EQUATIONS

Figure 5 shows a comparison of the LRFD interaction equations with the exact solutions and the proposed amplification factor for a beam-column subjected to axial compression combined with end moments only. In Fig. 5, the solid lines are based on the exact second-order moments, and the proposed A_f^* factor, and the dashed lines are based on the LRFD B_1 factor. It is seen that the B_1 factor gives a somewhat unconservative result, while the proposed A_f^* factor gives an almost identical result with that of the exact second-order moments.

Since beam-column interaction equation involves both axial force and bending moments, the difference in moment amplification factor will not cause a significant effect in actual design.[12] It is found that, in most cases, the unconservativeness of the interaction equation due to the approximate nature of the B_1 factor is less than 5%. For example, for a braced member subjected to axial compression combined with equal end moments ($M_a/M_b = -1$), with $KL/r = 120$, $E = 29{,}000$ ksi, $F_y = 36$ ksi, $P/P_n = 0.43$, and $M/M_n = 0.41$, we find the following differences:

Using the LRFD B_1 factor, Eq. 16, we obtain $B_1 = 1.568$, and the interaction equation leads to the value:

$$0.43 + \frac{(8)(1.568)(0.41)}{9} = 1.001 \tag{27}$$

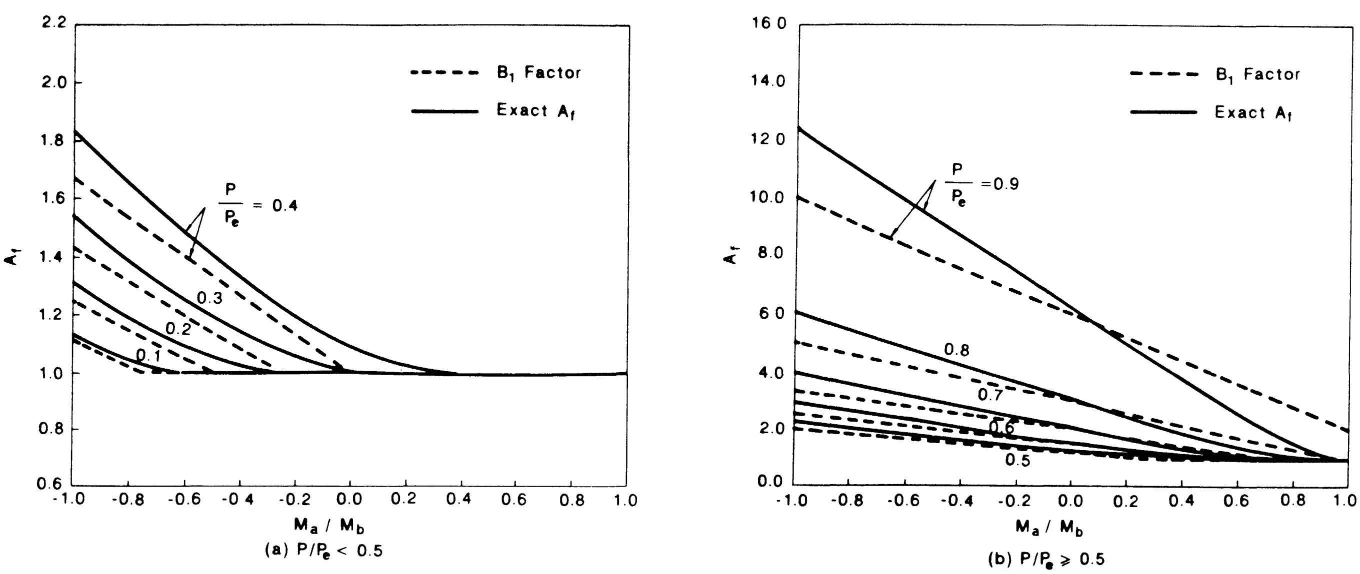

Fig. 2. Moment amplification factor for beam-column subjected to end moments and axial force (the B_1 factor and the exact A_f).

Table 1. Comparison of Amplification Factors (for End Moment Case)						
P/P_e	M_a/M_b	Exact A_f	LRFD B_1	B_1/A_f	Proposed A_f^*	A_f^*/A_f
0.1	−1.0	1.137	1.111	0.977	1.139	1.001
	−0.6	1.002	1.000	0.998	1.015	1.013
	−0.2	1.000	1.000	1.000	1.000	1.000
0.2	−1.0	1.310	1.250	0.954	1.312	1.002
	−0.6	1.093	1.050	0.961	1.137	1.040
	−0.2	1.001	1.000	0.999	1.000	0.999
0.3	−1.0	1.533	1.429	0.932	1.536	1.002
	−0.6	1.255	1.200	0.956	1.306	1.041
	−0.2	1.061	1.000	0.943	1.077	1.015
0.4	−1.0	1.832	1.667	0.910	1.833	1.001
	−0.2	1.198	1.133	0.946	1.244	1.038
	0.4	1.000	1.000	1.000	1.000	1.000
0.5	−1.0	2.252	2.000	0.888	2.250	0.999
	−0.2	1.423	1.360	0.956	1.488	1.046
	0.2	1.222	1.040	0.927	1.107	0.986
	0.6	1.000	1.000	1.000	1.000	1.000
0.6	−1.0	2.884	2.500	0.867	2.875	0.997
	−0.2	1.782	1.700	0.954	1.863	1.045
	0.2	1.319	1.300	0.985	1.357	1.029
	0.8	1.000	1.000	1.000	1.000	1.000
0.7	−1.0	3.941	3.333	0.846	3.917	0.994
	−0.2	2.400	2.267	0.944	2.496	1.040
	0.2	1.694	1.733	1.023	1.786	1.054
	1.0	1.000	1.000	1.000	1.000	1.000
0.8	−1.0	6.058	5.000	0.825	6.000	0.990
	−0.2	3.657	3.400	0.930	3.772	1.031
	0.6	1.458	1.800	1.235	1.544	1.059
	1.0	1.000	1.000	1.000	1.000	1.000
0.9	−1.0	12.419	10.000	0.805	12.250	0.986
	−0.6	9.937	8.400	0.845	9.933	1.000
	−0.2	7.462	6.800	0.911	7.616	1.021
	1.0	1.000	2.000	2.000	1.000	1.000

Table 2. Comparison of Amplification Factors (M_a/M_b = Single Curvature					
P/P_e	Exact A_f	LRFD B_1	B_1/A_f	Proposed A_f^*	A_f^*/A_f
0.00	1.000	1.000	1.000	1.000	1.000
0.10	1.137	1.111	0.977	1.139	1.002
0.20	1.310	1.250	0.954	1.312	1.002
0.30	1.533	1.429	0.932	1.536	1.002
0.40	1.832	1.667	0.910	1.833	1.001
0.50	2.252	2.000	0.888	2.250	0.999
0.60	2.884	2.500	0.867	2.875	0.997
0.70	3.941	3.333	0.846	3.917	0.994
0.80	6.058	5.000	0.825	6.000	0.990
0.90	12.419	10.000	0.805	12.250	0.986
0.95	25.149	20.000	0.795	24.750	0.984
0.99	127.006	100.000	0.787	124.750	0.982

Using the exact amplification factor, Eq. 7, we obtain A_f = 1.708, and the interaction equation leads to:

$$0.43 + \frac{(8)(1.708)(0.41)}{9} = 1.052 \qquad (28)$$

Using the proposed amplification factor, Eq. 18, we obtain A_f^* = 1.710, and the interaction equation leads to:

$$0.43 + \frac{(8)(1.710)(0.41)}{9} = 1.053 \qquad (29)$$

The difference between the LRFD and the exact or the proposed amplification factor is about 9%, while the difference between the values of interaction equation is 5%.

CONCLUSIONS

1. In a braced frame, for beam-columns subjected to end moments only, the B_1 factor usually underestimates the $P-\delta$ effect, and only in some uncommon cases, i.e., under high axial compression ratio (P/P_e > 0.7) with double curvature bending, it becomes conservative in estimating the $P-\delta$ effect.
2. Although the proposed C_m^* factor includes the effects of both axial force and end moments, it is still simple and easy to use. The proposed moment amplification factor A_f^* has been verified by comparison with the exact solutions. The $P-\delta$ moment predicted by the proposed Eq. 18 is more accurate than that predicted by the B_1 factor in the present LRFD Specification.
3. The difference in moment amplification factor will not affect significantly the actual beam-column design. In most cases, unconservativeness in the interaction equa-

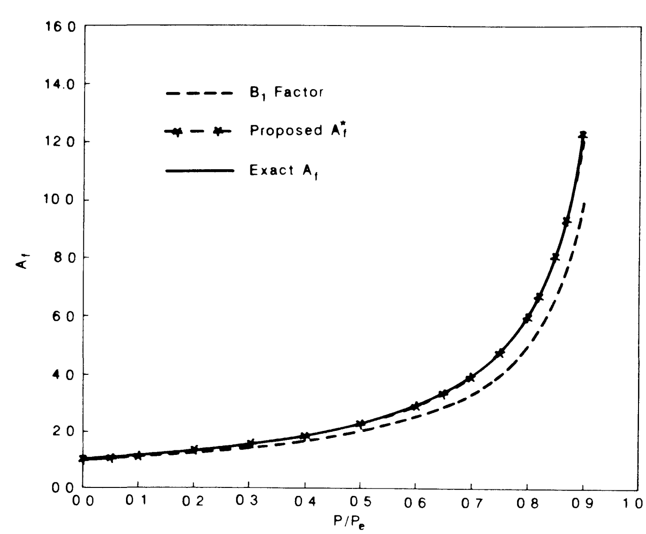

Fig. 3. Moment amplification factor for beam-column subjected to equal end moments (M_a/M_b = −1.0) and axial force.

tion due to the difference in B_1 factor is less than 5%, although the difference in moment amplification may be more than 10%.

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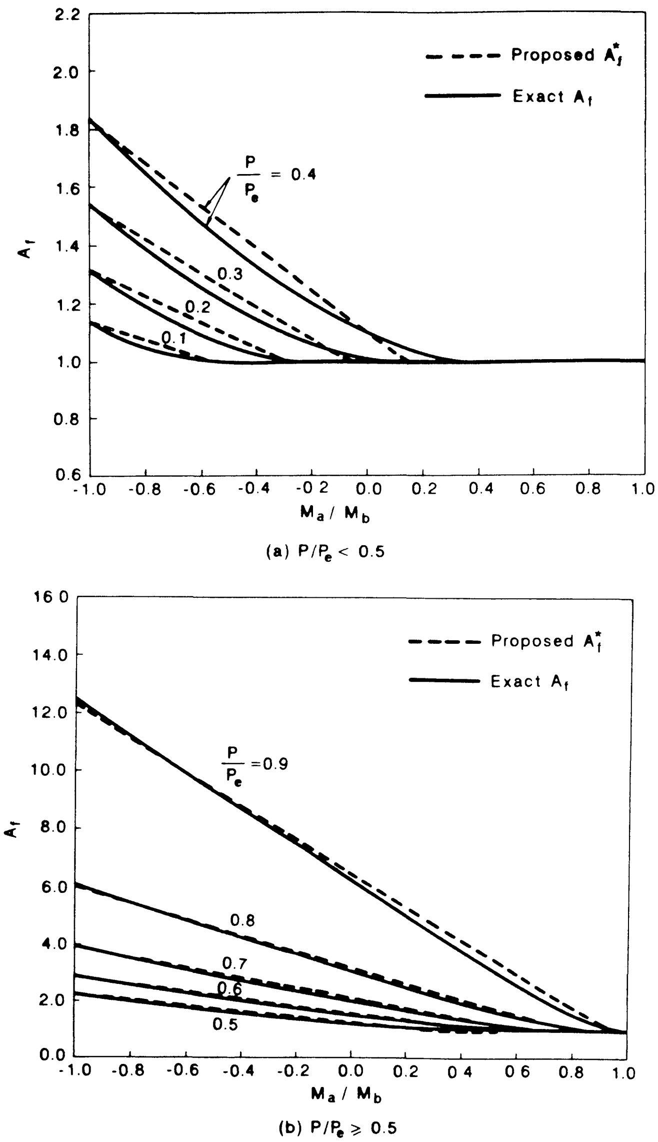

Fig. 4. Moment amplification factor for beam-column subjected to end moments and axial force (the proposed A_f^* and the exact A_f).

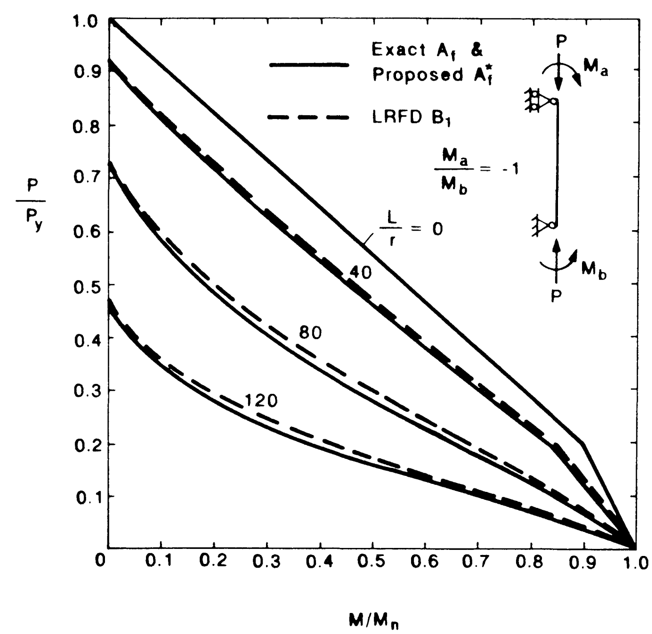

Fig. 5. Comparison of the LRFD interaction equations.