Design Aid for Required Moment of Inertia of Simple Beams

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INTRODUCTION

The AISC *Manual of Steel Construction* and other publications include tables which permit calculation of beam deflections for simple loading cases after a preliminary section has been selected. To design for a specific deflection criterion, these tables may be used in iterative fashion for a series of trial sections, or the formulas may be inverted to solve for a required moment of inertia when the limiting deflection is known. In either case, use of the formulas is cumbersome and prone to error, especially in converting to consistent units.

Presented here is a table of formulas which permit direct solution for required moment of inertia for several simple loading cases, for the two most common deflection criteria, $L/240$ and $L/360$. The author has found these formulas much simpler and quicker to use, because the constants in the formulas already incorporate the conversion factors for commonly used units.

USING THE TABLE

Table 1 lists formulas for required moment of inertia, expressed in terms of W and L, where

I = required moment of inertia, in.^{4}
W = load on span, kips
L = length of span, ft

The formulas are based on steel's elastic modulus of 29000 ksi, but can easily be used for other materials as well, by multiplying the formulas by $(29000 \div E)$ where
E = elastic modulus of beam material.

DESIGN EXAMPLES

Example 1

An A36 steel beam with a simple span of 40 ft supports a uniform dead load of 0.35 kips per foot, plus a concentrated live load of 1 kip at midspan. Maximum allowable total load deflection is $L/240$, and maximum allowable live load deflection is $L/360$. Assuming full lateral support, design the beam.

$$M = 0.35\,\frac{(40)^2}{8} + \frac{(1)40}{4} = 70 + 10 = 80 \text{ ft-kip}$$

$$S_R = \frac{80 \times 12}{24} = 40 \text{ in.}^3$$

$$I_R\,(\text{TL}) = [0.0155\,(0.35 \times 40) + .0248\,(1)]40^2 = 387 \text{ in.}^4 \text{ (controls)}$$

$$I_R\,(\text{LL}) = 0.0372\,(1)40^2 = 59.5 \text{ in.}^4$$

Scanning AISC's Allowable Stress Design Selection Table and Moment of Inertia Selection Table (which begin on pages 2-6 and 2-18, respectively, of the eighth edition of the *Manual*), it will be seen that the most economical section which satisfied both stress and deflection criteria is W18×35.

Example 2

An A36 steel beam with a simple span of 30 ft supports a uniform dead load of 1 kip per foot and a uniform live load of 1 kip per foot, plus a partition load of 0.2 kips per foot. To ensure proper operation of the partition, maximum deflection under live load plus partition weight is limited to 0.375 inches. Assuming full lateral support, design the beam.

$$M = \frac{2.2(30)^2}{8} = 248 \text{ ft-kip} \quad S_R = \frac{248 \times 12}{24} = 124 \text{ in.}^3$$

$$0.375 \text{ in.} = L/960$$

$$I_R = (960/360)\,0.0233\,(1.2 \times 30)\,30^2 = 2013 \text{ in.}^4$$

Scanning AISC tables, most economical section is W24×76.

SUMMARY

This design aid is intended solely for use by professional engineers competent to evaluate its accuracy and limitations. The author disclaims any and all responsibility for errors due to its use or misuse.

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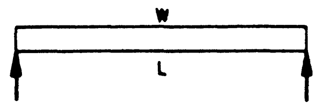
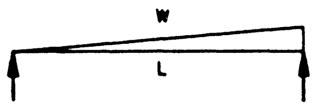
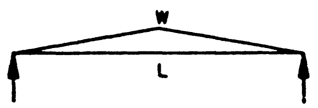
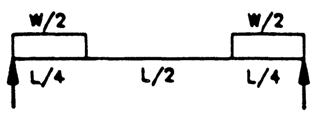
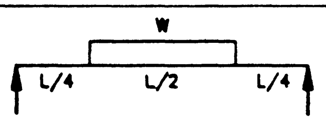
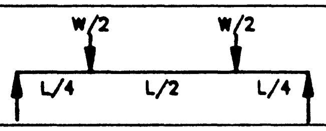
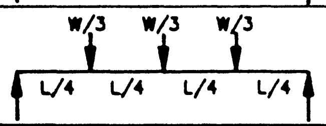
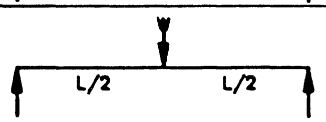
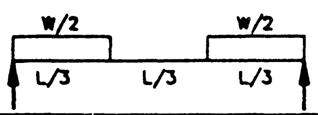
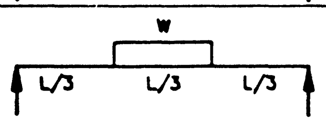
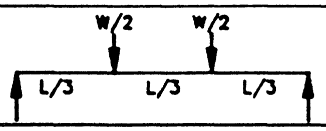
TABLE 1

LOADING	Δ	MINIMUM I STEEL		M
		$\Delta = L/240$	$\Delta = L/360$	
	$\frac{5}{384}\,\frac{WL^3}{EI}$	$.0155\ WL^2$	$.0233\ WL^2$	$\frac{WL}{8}$
	$.01304\,\frac{WL^3}{EI}$	$.0155\ WL^2$	$.0233\ WL^2$	$.1283\ WL$
	$\frac{1}{60}\,\frac{WL^3}{EI}$	$.0199\ WL^2$	$.0298\ WL^2$	$\frac{WL}{6}$
	$\frac{23}{3072}\,\frac{WL^3}{EI}$	$.0089\ WL^2$	$.0134\ WL^2$	$\frac{WL}{16}$
	$\frac{19}{1024}\,\frac{WL^3}{EI}$	$.0221\ WL^2$	$.0332\ WL^2$	$\frac{3WL}{16}$
	$\frac{11}{768}\,\frac{WL^3}{EI}$	$.0171\ WL^2$	$.0256\ WL^2$	$\frac{WL}{8}$
	$\frac{19}{1152}\,\frac{WL^3}{EI}$	$.0197\ WL^2$	$.0295\ WL^2$	$\frac{WL}{6}$
	$\frac{1}{48}\,\frac{WL^3}{EI}$	$.0248\ WL^2$	$.0372\ WL^2$	$\frac{WL}{4}$
	$\frac{25}{2592}\,\frac{WL^3}{EI}$	$.0115\ WL^2$	$.0172\ WL^2$	$\frac{WL}{12}$
	$\frac{205}{10368}\,\frac{WL^3}{EI}$	$.0236\ WL^2$	$.0353\ WL^2$	$\frac{5WL}{24}$
	$\frac{23}{1296}\,\frac{WL^3}{EI}$	$.0211\ WL^2$	$.0317\ WL^2$	$\frac{WL}{6}$

Suggestion: Add a column for deflection for a given I, e.g., for uniform load, $\Delta = 0.00075\ W\ L^3/I$.