Column Stability in Type 2 Construction

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ANALYSIS FOR the stability of columns has become of increasing importance under the impetus of the AISC Specification. The discussion of "effective length" contained in the AISC Commentary[1] makes reference to columns in continuous frames classified as Type 1 construction in the AISC Specification. Described herein is a method whereby the Commentary discussion is extended to include structures commonly designed as simply framed (Type 2 construction), but which rely upon the connection for frame action to resist lateral forces.

The alignment chart provided by the Column Research Council[2] offers a rapid and convenient method for evaluating the K factor for columns with rigid beam connections. Where structural connections are partially rigid, modification to this procedure must be made. Figure 1 illustrates the relationship between the deflected shape of the column and its effective length. Any additional rotation of the top of the column because of the incomplete (partial) rigidity of the connection increases the effective length. This is indicated in Fig. 1a by the higher position of the intersection of the extension to the elastic curve and the projection of the undeformed column axis.

A similar effect would result from a reduction in the stiffness provided by a girder with fully rigid connections. The analogy between a girder with partially rigid connections and a more flexible girder with rigid connections forms the basis for obtaining a modified girder stiffness,[3] given as

$$k_r = \frac{3(I/L)}{4(L'/L) - (L/L')} \qquad (1)$$

where

k_r = Relative stiffness of beam or girder with partially rigid connections
L' = $L + 3EIZ$
L = Span of beam or girder

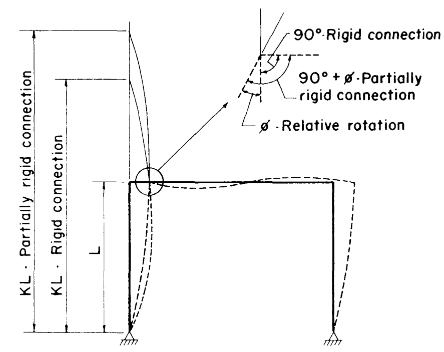

Figure 1

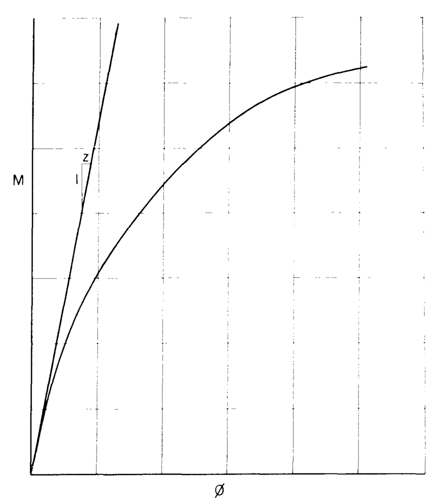

Figure 2

E = Modulus of elasticity
I = Moment of inertia of beam or girder
Z = Moment-rotation characteristic of connection

Equation (1) must be substituted for I_g/L_g in the expression for G_A and G_B when using the alignment chart of the Column Research Council. Each parameter used in deriving Equation (1) depends upon the geometric and physical characteristics of the girder with the exception of Z, which defines the relationship between moment and rotation for the connections. The factor Z for a connection is analogous to the flexibility factor,

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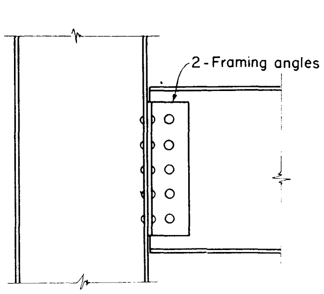
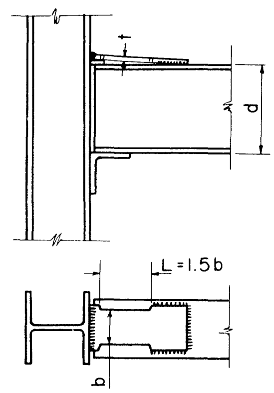
Table I. Type I Connection



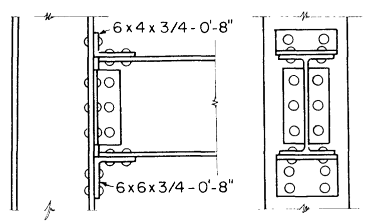
No. of Rows of Fasteners	$Z \times 10^5$, rad/kip-in
3	3.1
4	1.3
5	0.35
6	0.20
7	0.11
8	0.075
9	0.052
10	0.035

Table IV. Type IV Connection



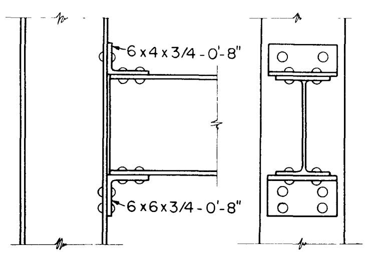
d, in.	$Z \times 10^5$, rad/kip-in.
8	0.21
10	0.13
12	0.093
14	0.068
16	0.052
18	0.041
21	0.030
24	0.023
27	0.018
30	0.015
33	0.012
36	0.010

Table II. Type II Connection



Depth of Beam, in.	Type II Connection, $Z \times 10^5$, rad/kip-in.
8	0.046
10	0.036
12	0.028
14	0.024
16	0.020
18	0.015
21	0.013
24	0.011
27	0.0087
30	0.0076
33	0.0065
36	0.0054

Table III. Type III Connection



Depth of Beam, in.	Type III Connection $Z \times 10^5$, rad/kip-in.
8	0.046
10	0.036
12	0.028
14	0.023
16	0.018
18	0.014
21	0.012
24	0.010
27	0.0078
30	0.0066
33	0.0055
36	0.0046

$L/4EI$, for a prismatic beam. The value of Z can be determined from the moment-rotation curve of the connection, as indicated in Fig. 2. Although the moment-rotation characteristic of a partially rigid connection is not linear, it may be approximated as the reciprocal of the initial tangent to the moment-rotation curve for use in the technique proposed in this paper. The goal here is not to predict the residual moment in a connection, but to measure the capacity of the connection to provide rotational resistance to the joint for consideration of lateral frame stability.

Lothers[4] presents formulas to determine Z for various common connection types. His analysis agrees favorably with Rathbun's tests.[5] By using the Z value obtained from Lothers' analysis, or by any analytical or empirical determination, the alignment chart can be utilized to determine effective length for columns which are members of partially rigid frames. Consider the Type I connection shown in Table I. Lothers defines Z for this connection in his text. As a practical matter the values can be approximated for design purposes by the use of Table I. This table is applicable for angle thicknesses and fastener sizes of usual proportions.

Likewise for a connection consisting of top and bottom clip angles (Type II, Table II), or with a web framing connection (Type III, Table III) Lothers provides the Z equations. Tables II and III give some typical Z values for these connections using 6 x 4 x $\frac{3}{4}$ in. x 0 ft-8 in. top angles and 6 x 6 x $\frac{3}{4}$ in. x 0 ft-8 in. bottom angles.

Another type of commonly employed "soft" wind connection is Type IV in Table IV. This type of connection obtains its required rotational capacity by leaving a portion of the top plate unwelded and free to deform under applied wind moments. The action is shown idealized in Fig. 3. By assuming that the neutral axis of the connection occurs at mid-depth of the beam and that the connection material behaves elastically the Z function can be derived as follows:

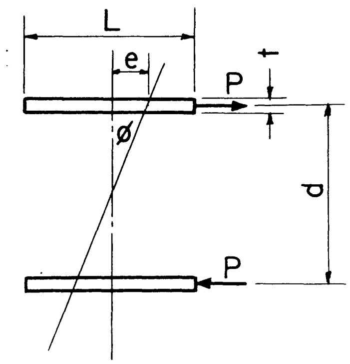


Figure 3

$$Z = \phi/M \qquad \phi = \frac{e}{d/2} = 2e/d \qquad e = PL/AE$$

Since $P = M/d$, $A = b \times t$, and $L = 1.5b$, then $e = 1.5M/(d \times t \times E)$ and

$$Z = \frac{2 \times 1.5 \times M}{d \times t \times E \times d \times M} = \frac{3}{t \times d^2 \times E}$$

If $E = 29 \times 10^3$ ksi, then

$$Z \cong \frac{10^{-4}}{t \times d^2} \frac{\text{rad}}{\text{kip-in.}}$$

For various beam depths, d, and a $\frac{3}{4}$-in. plate thickness, the Z values in Table IV may be used:

After determining the proper Z value for a given connection, Equation (1) can be solved for the apparent stiffness of the beam or girder. The apparent stiffness values for all beams or girders framing into a joint are utilized to determine the G_A and G_B factors (described in the CRC Guide[2] and also the AISC Commentary,[1] Section 1.8) in computing the effective column length factor. Thus after obtaining the apparent stiffness values of the beams, the procedure for determining K of the column is exactly the same as for members of fully rigid frames.

To facilitate the determination of k_r, Equation (1) has been plotted in Fig. 4. In this chart

$$C_p = \frac{I(\text{in.}^4)}{L(\text{ft})} \times Z \times 10^5 \quad \text{is the abscissa}$$

and

$$C_e = \frac{k_r}{I/L} \quad \text{is the ordinate}$$

C_p is the flexibility index of beam and connections and C_e, the efficiency coefficient for the assembly, is the ratio of effective beam stiffness (of beam with partially rigid connections) to idealized stiffness (of the same beam with fully rigid connections).

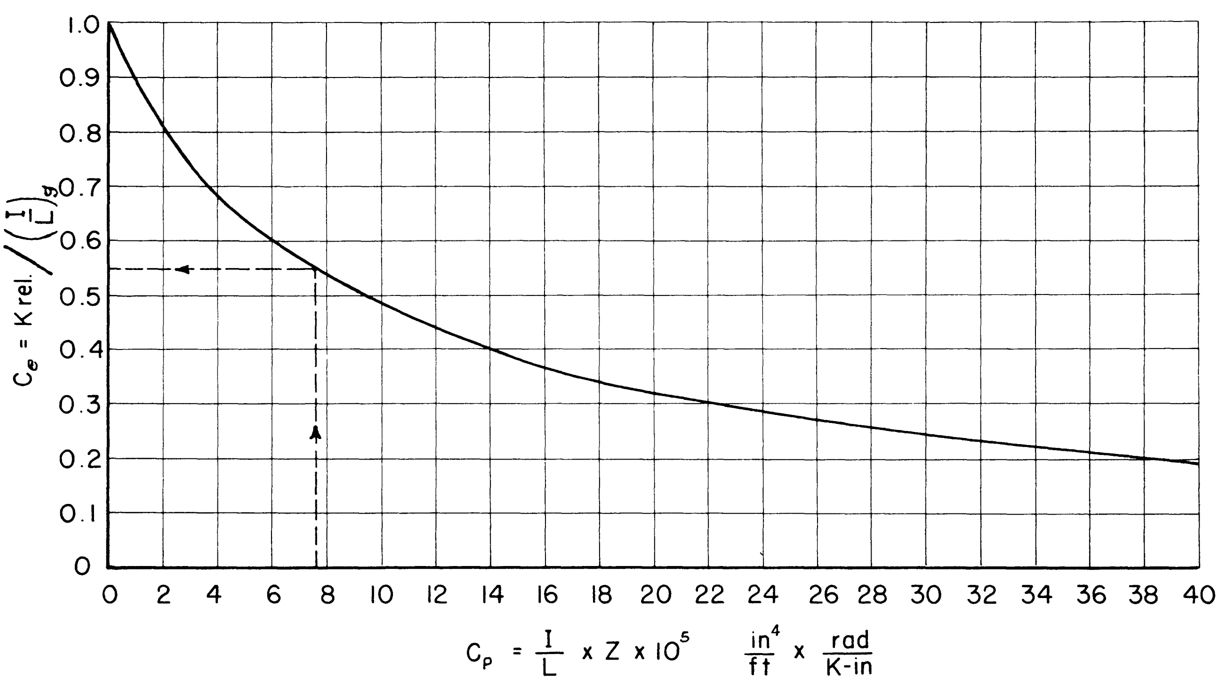


Figure 4

EXAMPLES

The following examples are provided to demonstrate the application of the method proposed here. It is presupposed that the connections utilized in each example would be capable, in an actual situation, of developing the required moment capacity to resist imposed lateral forces.

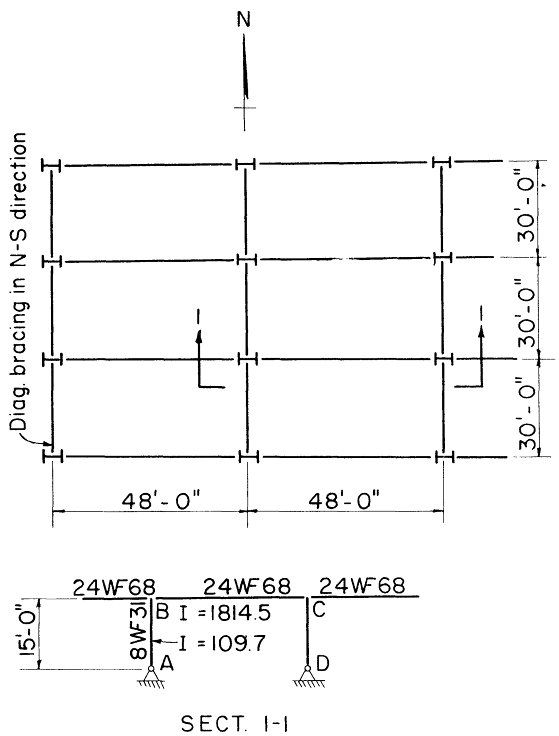


Figure 5

Example 1

Determine the effective length factor for column **AB** in Fig. 5.

Given: (1) No lateral bracing in E-W direction. (2) Many bays in E-W direction, i.e., framed connections provide sufficient moment capacity to resist moments induced by lateral forces. (3) All columns receive restraint against sidesway in N-S direction.

Solution:

$$Z = 0.20 \times 10^{-5} \frac{\text{rad}}{\text{kip-in.}} \quad \text{(see Table I)}$$

$$C_p = \left(\frac{I}{L}\right) Z \times 10^5 = \frac{1814.5}{48} \times 0.20 = 7.58$$

$$C_e = 0.55 \text{ (see Fig. 4)}$$

From AISC Commentary, (Manual, p. 5–118)

$$G_A = \frac{\Sigma I_c/L_c}{\Sigma C_e(I_g/L_g)} = \frac{109.7/15}{2[0.55 \times (1814.5/48)]} = 1.75$$

$$G_B = 10$$

$$\therefore K_x = \mathbf{2.0}$$

$$\frac{K_x l_x}{r_x} = \frac{2.0 \times 15 \times 12}{3.47} = 104 \quad \text{(governs)}$$

$$\frac{K_y l_y}{r_y} = \frac{15 \times 12}{2.01} = 90$$

Assume column load = 90 kips

F_a = 12.47 ksi (AISC Specification, Manual p. 5-68)
P_a = 12.47 × 9.12 = 113 kips > 90 kips (**OK**)

Example 2

Determine the effective length factor for column **BD** in Fig. 6.

Given: (1) All girder-to-column connections are Type III (see Table III). (2) No lateral bracing is provided in the plane shown. Assume bracing is available in a direction perpendicular to the plane shown. (3) Construction is Type II, i.e., girders are proportioned for simple beam moment, and joints are designed for moment due to wind only.

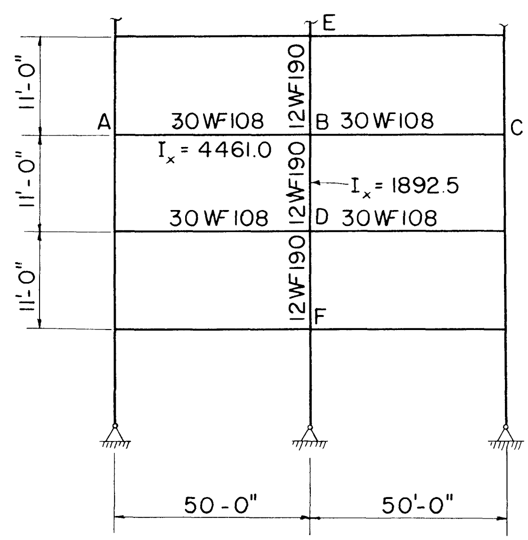


Figure 6

Solution:

$$Z = 0.0066 \times 10^{-5} \frac{\text{rad}}{\text{kip-in.}} \quad \text{(see Table II)}$$

$$C_p = \left(\frac{I}{L}\right) Z \times 10^5 = \left(\frac{4{,}461}{50}\right) 0.0066 = 0.59$$

$$C_e = 0.91 \text{ (see Fig. 3)}$$

From AISC Commentary (Manual p. 5–118):

$$G_A = G_B = \frac{\Sigma I_c/L_c}{\Sigma C_e(I_g/L_g)}$$

$$G_A = \frac{2(1{,}892.5/11)}{2[0.91(4461/50)]} = 2.12$$

$$\therefore K_x = \mathbf{1.63}$$

$$\frac{K_x l_x}{r_x} = \frac{1.63 \times 11 \times 12}{5.82} = 37$$

$$\frac{K_y l_y}{r_y} = \frac{11 \times 12}{3.25} = 41 \text{ (governs)}$$

F_a = 19.11 ksi (AISC Specification, Manual p. 5–68)
$P_a = 19.11 \times 55.86 = 1{,}065 \text{ kips} > 990 \text{ kips}$ (**OK**)

It is interesting to note that had fully rigid connections been used in each of the examples, instead of those specified, the corresponding K factors would be 1.85 and 1.58 respectively.

CONCLUSIONS

Presented here is a method for determining the effective length of columns in unbraced frames when partially rigid beam to column connections are employed.

It is evident that in an actual design problem, the effective length of a column is related to column stiffness, beam stiffness and connection stiffness. In a particular situation the designer must make the structural and economical decision between decreased column and beam material with a fully rigid connection or increased column and beam material together with a partially rigid connection. The connection selected must take into consideration not only design loads but also fabrication preferences.

REFERENCES

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