Uniform Pin-based Crane Columns, Effective Lengths

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A parametric computer study is used to investigate the buckling of pin-based rectangular frames containing uniform crane columns. The study showed that the effective length factor of the lower segment of the heavily loaded column could be related to the nominal effective length factor obtained from the standard alignment chart. Easy-to-use formulae are presented and the K-factors for the other column segments are calculated from the heavily loaded segment.

An approximate frame, in which the beam is lowered to the level of the crane, was also investigated but gave results that were not consistent enough for design use.

The proposed procedure is illustrated with a numerical example.

INTRODUCTION

Overhead traveling cranes are a common feature of factory and warehouse buildings. In the case of heavy crane loads and tall buildings, stepped columns are used, but for buildings of small to modest proportions and lighter crane loads, the crane girders can be supported on brackets from uniform columns. In the plane of the frame, the column bases are often fully fixed on expensive moment-resisting foundations. However, the site conditions may dictate the use of pin-based columns with simpler foundations designed only for vertical reactions. Although pin-based frames require more steel, the overall costs may favor this option.

The 1986 LRFD version of the AISC "Specification for Structural Steel Buildings",[1] despite being written in limit-state format, still bases the design of all compression members on the slenderness ratio $K\ell/r$ in which K is the appropriate value of the effective length factor. The rules and alignment charts for calculating effective length factors, in this Specification and the SSRC "Guide to Design Criteria for Metal Compression Members",[2] relate to regular rectangular frames with symmetrical loads applied at the tops of the columns (see Fig. 1).

They do not apply to the general case of a frame supporting an overhead travelling crane [Fig. 2(a)], in which most of the load is applied at an intermediate point on the column, and the column loads vary as the crane moves back and forth across the work space.

The best source of the correct K-factors of the column segments (the change in the column load at the crane level requires a change in K) is a rational buckling analysis, but suitable computer programs are not as readily available as they should be. Also, the designer works in a very competitive environment and requires easy-to-use, reasonably accurate analytical techniques. If such a method were available that simply used modified existing information then the familiarity of the method could lead to its wider acceptance.

This paper presents a method, based on a parametric study of the problem defined in Fig. 2(a) and using a proven stability program,[3] whereby the conventional K-factor for the full-height uniform column K_c is modified to give the K-factor of the lower segment of the more heavily loaded column $K_{\ell,AB}$. The K-factors of the other column segments are then calculated relative to this solution.

There is a temptation to use an approximate frame Fig. 2(b), in which the beam is lowered to the level of the crane. This conforms geometrically to the standard model and in terms of members stiffnesses, even though the column loads can still be unsymmetrical. The approximate frame would appear to be reasonable provided the crane is located high up on the column, say $L_\ell/L_c \geq 0.8$. But how reliable is the approximate frame when the crane is near the mid-height of the column and if the design incorporates I_b less than I_c? The results of the investigation present an answer to this design question.

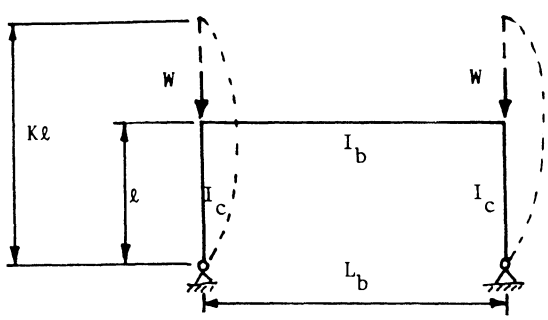

Fig. 1. The basic frame model for effective length factors from alignment charts

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THE PARAMETRIC STUDY

A pilot series of computer runs indicated the feasibility of relating K_c to $K_{\ell,AB}$ so an extensive parametric study was carried out[4] in which a number of values were used for the variables shown in Fig. 2(a). The investigation was made in two stages in order to evaluate the influence of the two principal factors affecting the stability of the heavily loaded column.

Figure 3 shows that the stability of any column, the heavily loaded column ABC in this case, depends on the interaction of two restraints, (1) the rotational restraint α_C (which is similar to the well known G-factors) and (2) the translational or sidesway restraint α_T which is generated by other less critical columns in the structural system. Here, the lesser loaded column, DEF in Fig. 2(a), generates α_T. When the crane is exactly in the middle of the building and the symmetrical frame is symmetrically loaded, then α_T equals zero.

Before analyzing the affects of α_C and α_T, a simplification was achieved by moving the crane loads onto the column center lines and eliminating the moment due to the eccentricity of the crane loads. This had negligible effect on the total buckling load of the whole frame and indicated that it was safe to eliminate the paramter e from the analysis. At the subsequent design stage, of course, the bending moments caused by Pe will be included in the combined stresses/forces equations, but these bending moments do not materially effect the values of effective length factors.

With this simplification it was possible to set up two series of analyses using the frames shown in Fig. 4.

The symmetrical system in Fig. 4(a) allows the effects of rotational restraints to be assessed and the unsymmetrical load case in Fig. 4(b) accounts for the effect of translational restraint.

THE RESULTS

From all the buckling analyses of the parametric study, the values of $K_{\ell,AB}$ were calculated and compared with K_c for the standard case of equal loads at the tops of the columns. It was found that $K_{\ell,AB}$ could be related to K_c in the following manner,

$$K_{\ell,AB} = K_c \times F_1 \times F_2 \qquad (1)$$

in which the factor F_1 measures the influence of the rotational restraint and the factor F_2 measures the further change in K due to the translational restraint. The presence of α_T improves the buckling capacity of AB thereby reducing its K-factor.

The following expressions were derived for F_1,

for $0.1 \leq G \leq 1.0$

$$F_1 = \frac{24.4}{(21.8 - \log G)\,(L_\ell/L_c - 1.0) + 24.4} \qquad (2a)$$

for $G > 1.0$

$$F_1 = \frac{7.6}{(6.8 - \log G)\,(L_\ell/L_c - 1.0) + 7.6} \qquad (2b)$$

in which G is the conventional $(I_c/L_c)/(I_b/L_b)$ and provided $(L_\ell/L_c) \geq 0.4$ and $n \leq 20$. These limits are generous and will cover all practical cases.

The expressions for F_2 were derived as,

for $(P_{\ell,\,max}/P_{\ell,\,min}) < 7$

$$F_2 = \frac{1}{(P_{\ell,\,max}/P_{\ell,\,min})^{0.15}} \qquad (3a)$$

for $(P_{\ell,\,max}/P_{\ell,\,min}) \geq 7$

$$F_2 = \frac{1}{1.27\,(P_{\ell,\,max}/P_{\ell,\,min})^{0.02}} \qquad (3b)$$

Equation (3a) will apply to most practical cases.

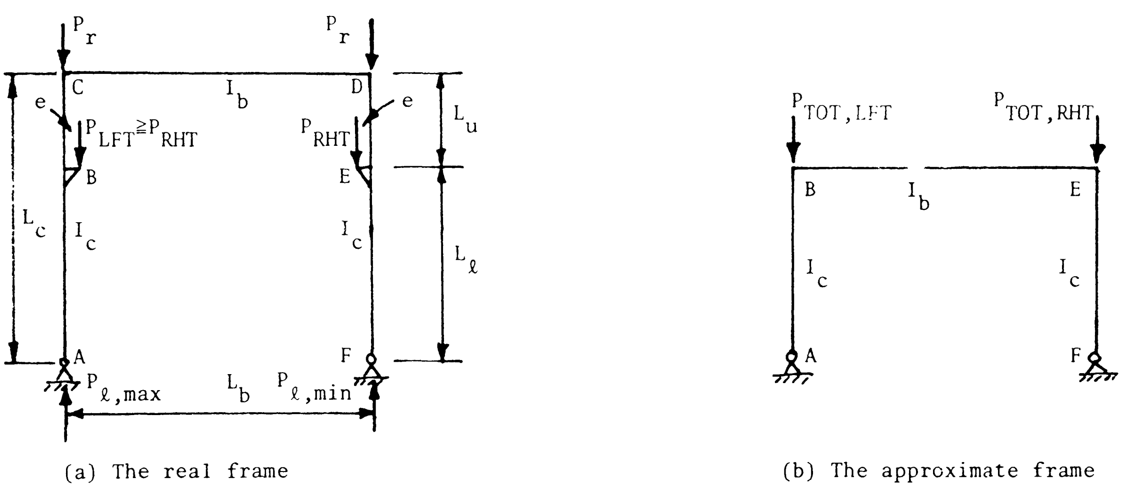

Fig. 2. The pin-based crane supporting frame

THE PROCEDURE

1. Calculate the conventional G-factor for the unloaded frame and determine K_c from the standard alignment chart.

   Calculate the values of L_ℓ / L_c and n and compare with the limits.
2. Depending on the value of G, evaluate F_1 from Eq. 2a or 2b.
3. Calculate, for the unsymmetrical load case, the ratio $P_{\ell,\,max}/P_{\ell,\,min}$ and evaluate F_2 from Eq. 3a or 3b
4. Calculate $K_{\ell,AB} = K_c \times F_1 \times F_2$
5. Calculate the K-factors for the other column segments from the relationship,

$$K_s = \frac{K_{\ell,AB} \times L_\ell}{L_s} \sqrt{\frac{P_{\ell,max}}{P_s}} \tag{4}$$

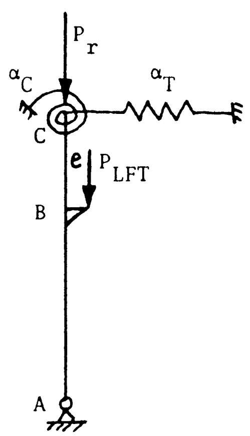

Fig. 3. The idealized crane column

NUMERICAL EXAMPLE

1. $G = \dfrac{2050 \times 10^6/12000}{2400 \times 10^6/20000} = 1.42$

$K_c = 2.45$

$n = \dfrac{(P_{LFT} + P_{RHT})}{2P_r} = \dfrac{700 + 200}{2 \times 234} = 1.92 < 20$

$\dfrac{L_\ell}{L_c} = \dfrac{10}{12} = 0.833 > 0.4$

2. $F_1 = \dfrac{7.6}{(6.8 - \log 1.92)\,(0.833 - 1.0) + 7.6} = 1.17$

3. $\dfrac{P_{\ell,max}}{P_{\ell,min}} = \dfrac{934}{434} = 2.15 < 7$

$F_2 = \dfrac{1}{2.15^{0.15}} = 0.89$

4. $K_{\ell,AB} = 2.45 \times 1.17 \times 0.89 = 2.55$

5. $K_{BC} = K_{DE} = \dfrac{2.55 \times 10000}{2000} \sqrt{\dfrac{934}{234}} = 25.5$

$K_{EF} = \dfrac{2.55 \times 10000}{10000} \sqrt{\dfrac{934}{434}} = 3.74$

COMMENT

The large K-factors for segments BC and DE are an automatic outcome of the fact that these upper segments are lightly loaded compared to the lower segments of the uniform columns. The fundamental formula

$$P_{cr} = \frac{\pi^2 E I_c}{(KL)^2} \tag{5}$$

ensures that as P_{cr} decreases then K increases.

These large K-factors will generate large effective lengths and slenderness ratios for the upper segments. It

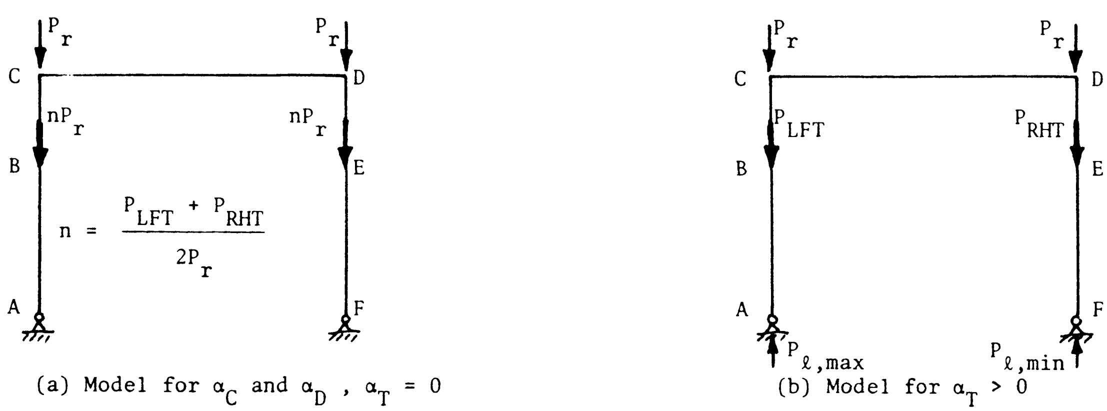

Fig. 4. Frame models used in the parametric study

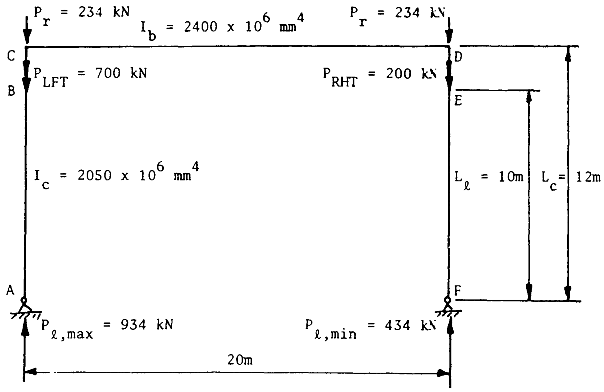

Fig. 5. The sample frame

is possible the slenderness ratios may exceed the limit of 200.

This DOES NOT mean that the design should be rejected. The result is perfectly consistent with the real buckling behavior of the whole frame and has nothing to do with the limit of 200 which was set arbitrarily many years ago when only isolated columns could be analyzed and designed.

The large slenderness ratios will, of course, generate small values of F_a (1978 ASD Specification) or ϕF_{cr} (1986 LRFD Specification), but the upper segments have very small values of f_a because they are the very lightly loaded portions of a uniform column.

The upper segments, by virtue of their small axial forces, are working in combination with the beam to stabilize the lower column segments. All members above the level of the crane are good restraining members and a characteristic of such members is their low values of axial force with corresponding large effective length factors.

The same reasoning explains why K_{EF} is larger than

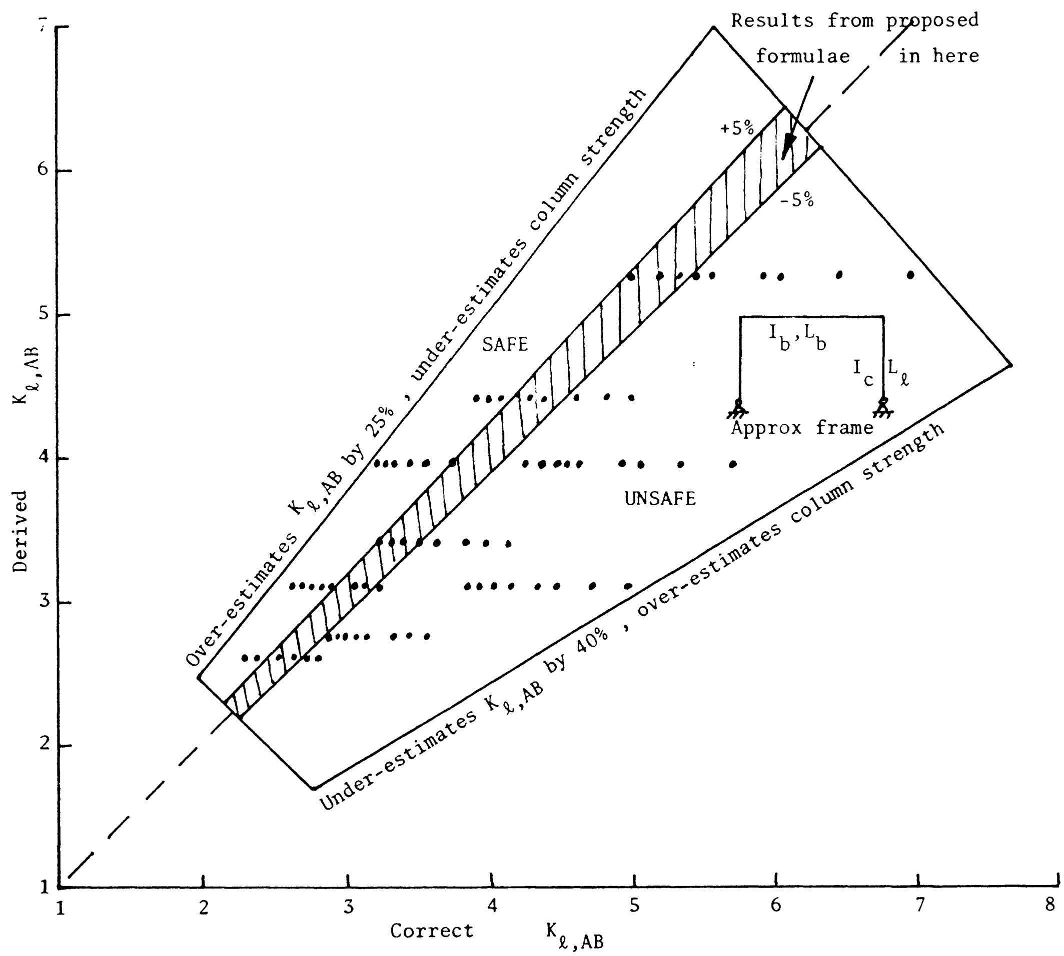

Fig. 6. Comparison of results from the proposed method and the approximate frame.

$K_{\ell,AB}$. Given that π^2, E, I_c and L_ℓ are the same for both AB and EF, then the smaller P_{cr} in EF requires, through Eq. 5, an increased K-factor. The full capacity of EF appears not to be utilized whereas, in fact, its reserve capacity in the unsymmetrical load case is the very reason for its ability to offer translational restraint to AB.

So here we find that large K-factors do not signify weakness but instead they represent strong members that are stabilizing the structure.

This is in complete contrast to the conventional interpretation where columns are designed as individual members whence large K-factors become synonymous with weakness.

In real frameworks, all members participate in the buckling process. The lighter loaded members (usually beams but sometimes other columns as seen here) support the heavily loaded columns. The ultimate restraining member is one with no axial force. It offers its full stiffness to stabilize other members and yet by definition from Eq. 5 its K-factor equals infinity.

If designers are to believe in the results from stability analyses, they must recognize that the over-simplified models, for isolated columns and sub-frames, do not represent the real world of frame buckling, and their thinking about effective length factors, effective lengths and slenderness will have to change.

THE APPROXIMATE FRAME

Figure 6 summarizes the comparison of the correct values of $L_{\ell,AB}$ with (a) the values from the derived formula and (b) values from the approximate frames. All values from the proposed procedure were found to lie within ±5% of the true values.

The results using the approximate frames and calculating $K_{\ell,AB}$ in the conventional way from the alignment chart led to a scatter in the range +25% to −40% of the correct values. Although some results fall within the ±5% zone, the scatter is too wide for design purposes. However, the study did show that provided the crane was high up on the column, L_ℓ/L_c greater than 0.8, the approximate frame over-estimated K_ℓ and so was conservative. For values of L_ℓ/L_c less than 0.8 the results rapidly leads to unsafe designs.

CONCLUSION

A parametric study was used to investigate the stability of uniform crane columns in pin-based frames. It was found that the K-factor of the lower segment of the heavily loaded column of the real frame could be related to the conventional K_c of the unloaded frame. Easy-to-use formulae were derived which give effective length values within 5% of their correct values. The K-factors of the other column segments could be calculated from the value for the heavily loaded segment.

A simplified frame with the beam at the level of the crane did not produce the same consistency of results and is not recommended for design use.

REFERENCES

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NOMENCLATURE

α_C	Rotational restraint at top of column
α_T	Translational/sway restraint at top of column
e	Eccentricity of crane load
E	Modulus of elasticity
F_1	Rotational restraint factor
F_2	Translational restraint factor
G	Stiffness ratio
I_b	Second moment of area of beam
I_c	Second moment of area of uniform crane column
K_c	Nominal effective length factor of full length column
$K_{\ell,AB}$	Effective length factor of the heavily loaded segment of column
K_s	Effective length factor of other column segments
L_c	Nominal length of column
L_ℓ	Length of lower segment of column
L_u	Length of upper segment of column
n	Ratio of symmetrical crane load to symmetrical roof load
P_{cr}	Buckling load in a member
$P_{\ell,max}$	Maximum reaction, at base of heavily loaded column
$P_{\ell,min}$	Minimum reaction, at base of lighter loaded column
P_{LFT}	Maximum crane load
P_{RHT}	Minimum crane load
P_r	Roof load
r	Radius of gyration