Preliminary Minimum Weight Design of Moment Frames for Lateral Loading

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This paper introduces a new concept for the design of regular rectangular moment frames under lateral loading. The concept is based on the assumption that the entire framework is composed of imaginary rectangular modules, with each module consisting of four uniform prismatic members rigidly connected at their ends. This concept entails the selection of the properties of the constituent elements of the modules in such a way as to reduce the total material consumption to a minimum, and at the same time maintain the least possible drift for the given loading and geometry. In addition to controlling the sideway deflections of the structure, the method presented not only reduces the otherwise complicated task of structural analysis to direct member size selection, but also it achieves substantial cost reductions by reducing the total weight of the structure to a minimum. The use of this concept results in an extremely simple yet powerful method of design for the class of structure treated in this study. Several numerical examples are presented to illustrate the practical applications of the concept. The validity and the consistancy of the proposed method in both the elastic and plastic ranges of behavior are established by the standard techniques of structural analysis.

INTRODUCTION

Conventional definitions regard moment frames as structural forms composed of rigidly connected members such as beams and columns. The classical methods of design associated with this definition entail the selection of the individual members based on the complete structural analysis of the entire system. This approach is inclined more towards analysis than selection and the analyst has little or no control over the final results. Consequently, the solutions are seldom the most economical with respect to the total weight of the structure. Regardless of whether the design criteria is the working stress or the limit state, the traditional method often results in tedious analysis and heavier members.

Amongst others, minimum weight design of the class of tall, multi-story frames subjected to large lateral forces has been investigated, in a general form, by Heyman,[1,5] Foulkes,[2,3] Livesley,[4] Stone,[6] Kalker,[7] Tanaka,[8] and Prager.[9] In contrast to previous works on the numerical methods of minimum weight design, the purpose of the present study is to establish an extremely simple, yet explicit, form of solution for the minimum weight design of the aforementioned class of structures.

Since the magnitudes and the distributions of the gravity floor loads are almost identical in all stories of high-rise buildings and the effective dissipation of fixed-end moments is confined to elements of close neighborhood, the minimum-weight treatment of the structure becomes less significant in the absence of lateral forces. In contrast to gravity loads, the effect of the lateral forces is accumulative in the members of lower stories and therefore a sensitive factor in the minimum-weight design. The minimum-weight treatment becomes even more significant when the system is designed to carry lateral forces only. This class of structure has many practical applications in earthquake hazard reduction projects for existing buildings and industrial facilities. Although the present study is concerned primarily with this particular class of structure and loading, its application may be extended, with some modification, to similar forms in the same category.

To develop the concept, it is first assumed the entire framework, including grade beams, is composed of imaginary rectangular subframes which fit into the actual bays of the structure. A pictorial presentation of the concept is shown in Fig. 1.

To avoid recourse to complicated analytical procedures, the design criteria are next studied with respect to a generalized subframe, hereafter referred to as the "basic module." The design criteria for each basic module is the selection of safe elements for the four basic members in such a way as to minimize its total weight. Naturally, if the

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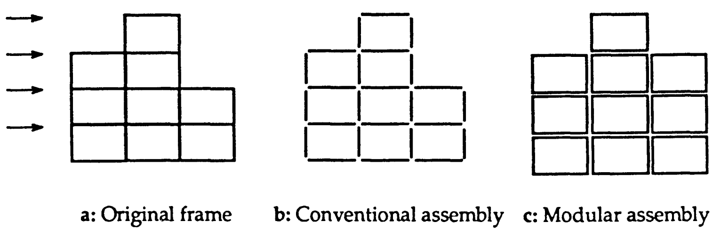


Fig. 1. Laterally loaded moment frame and member assemblies

weight of each constituent module is minimum, then the sum of the weights of all modules would also be a minimum. The rules governing the combination of the modules both in the vertical and horizontal directions, stated later in the study, provide for the complete design of the structure. The rules of module combination are established for both elastic and plastic modes of application. The rules of the elastic mode are concerned with selection of appropriate stiffnesses as well as moments of resistance and are thought to be best suited for prefabricated steel beams and columns as well as reinforced concrete frames, where section resistances and moments of inertia can be worked out readily for the desired values. The laws governing combination of modules in the plastic range are based on conditions of failure mechanisms and static equilibrium, where the process of member selection is only concerned with strength of the section and is suited ideally for steel frames, where it is difficult to select the desired section resistance and moment of inertia at the same time.

DEVELOPMENT OF THE "BASIC MODULE"

Minimum weight treatment

The basic module is defined as a symmetric rectangular subframe composed of four rigidly connected prismatic members simply supported at the lower nodes and carrying a concentrated lateral force, as shown in Fig. 2a,

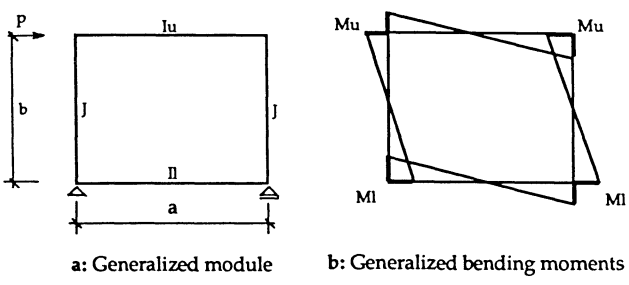


Fig. 2. Laterally loaded basic module

where a and b are defined as the length and height of the module respectively. I is the moment of inertia of the vertical members and I_u and I_l are the moments of inertia of the upper and lower horizontal beams respectively. Figure 2b shows the corresponding bending moment distribution where M_u and M_l describe the maximum bending moments at the upper and lower nodes, respectively. The problem at this stage is to select I_u and I_l in such a way as to minimize the total weight of the module.

Now supposing that $M_u > M_l$, then the weight G of the module for an acceptable solution may be written as:

$$G=C[(2b+a)\cdot Mu + a\cdot Ml] \tag{1}$$

where C is an arbitrary constant of proportionality. Using simple statics it can be shown that the sum of the column end moments should equal the story moment $\overline{M}=Pb$, i.e.,

$$M_u + M_l = Pb/2 \tag{2}$$

Substituting for M_l from Eq. 2 in Eq. 1 and rearranging it, gives:

$$G = C\,[Pab/2 + 2bM_u] \tag{3}$$

Now since it has been assumed $M_u > M_l$, then for a nonzero moment solution, Eq. 3 may be studied within the following ranges of variation.

$$Pb/2 > M_u \geq Pb/4 \tag{4}$$

$$Pb/4 \geq M_l > O \tag{5}$$

Comparing the effects of the extreme ranges of the inequality (4) in the weight Eq. 3 it follows the total weight of the module is a minimum when $M_u = M_l = Pb/4$ and $I_u = I_l = I$.

This finding leads to the first fundamental statement of the problem:

"The minimum weight design of the basic module is one involving a subframe of uniform moment of resistance."

The practical importance of this statement becomes more significant when it is seen these modules may be combined to create minimum weight structures of multimodule formation. Because of double symmetry, the problem becomes quasi-static and therefore independent of the relative values of the moments of intertia I and J. Consequently, since the weight function is also independent of the stiffnesses of the members of the module, the minimum weight statement holds true for all stages of loading up to and including plastic collapse.

Deformation analysis

To study the deformations of the basic module, consider the deflected shape of the subframe of Fig. 3 with arbitrarily selected moment hinges along the vertical members. It has already been shown that node moments for the minimum weight design are given by $M = \overline{M}/4 = Pb/4$.

Furthermore, it may be shown easily by standard techniques of structural analysis that the total sway of frame is given by:

$$\delta = [\,(b_1^3+b_2^3)/JE+a\cdot(b_1^2+b_2^2)/2IE]\cdot P/6 \tag{6}$$

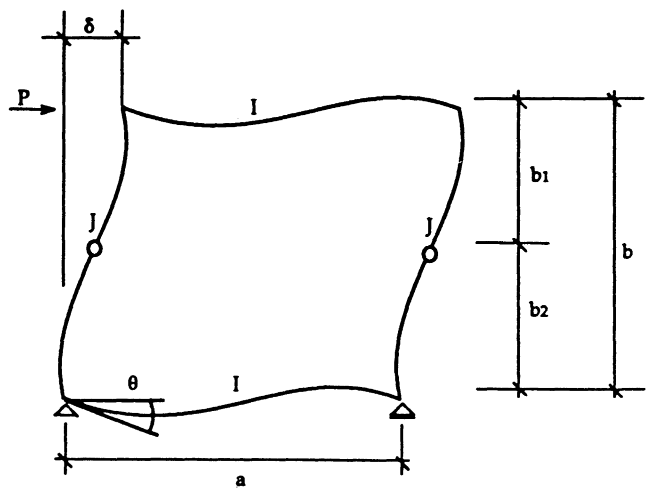


Fig. 3. Generalized deformations of basic module

The condition for minimum deflection with respect to location of the point of contraflexure is given by:

$$\partial\delta/\partial b_1 = 0 \qquad (7)$$

Substituting for b_2 from $b_2 + b_1 = b$ in Eq. 6 and performing the differentiation (7) gives $b_1 = b/2$ or $b_1 = b_2$. Because of double symmetry, moment reversal occurs mid-distance of all four members, indicating the proportioning of the members of the module based on minimum weight criteria also results in minimum drift for the given geometry and loading. This obvious result is the second fundamental statement of the concept, i.e:

"The minimum drift design of the module is one involving a subframe with symmetrical moments of intertia for its horizontal and vertical members"

With $b_1 = b_2 = b/2$ Eq. 6 is simplified to read:

$$\delta=(a/I+b/J)\cdot Pb^2/24E \qquad (8)$$

with corresponding node rotation:

$$\Theta = Pba/24EI \qquad (9)$$

Equations 8 and 9 may be expressed more conveniently in terms of the bay moment $\overline{M} = Pb$ as:

$$\delta = (a/I + b/J)\,\overline{M}b/24E \text{ and } \Theta = \overline{M}a/24EI \qquad (10)$$

It is significant to note the rotation Θ of the basic module, which is constant for all four nodes, is independent of the rigidities of the vertical members; but the module drift δ is directly controlled by the magnitudes of the stiffnesses of both vertical and horizontal elements. These deformation characteristics are used in the preceding section to explain the principle of multiples for combining two or more modules in any direction.

MODULE COMBINATION

Having studied the characteristics of the basic module for minimum weight and deflection, the problem of the design of a generalized regular frame, such as shown in Fig. 1, may now be looked upon as one involving the combination of as many basic modules as there are bays or subframes. If any two modules designated i and j are to be combined to form a new structure while retaining their fundamental characteristics then these modules should merge in such a way as to satisfy the conditions of modular compatability and equilibrium in the elastic range of application. To further simplify the rules govening the merging of modules, two distinct classes of module combination are described:

Vertical Combination

For the vertical combination of two or more modules of equal length, the only requirement to be satisfied is that angles of rotation of all modules should be equal, or, in more practical terms, the sign and magnitude of the ratio $\overline{M}/I$ should remain constant for all such subframes. To demonstrate the application of the concept for vertical combination, a simple problem is worked out in this example:

Example 1

Design the four-story frame of Fig. 4 for minimum weight and story drift.

Solution:

Figure 4b shows the accumulative story shears and the corresponding bay moments of each story. The moment of inertia of the uppermost module is taken as I. The member end moments which are obtained by dividing the respective bay moments by four, as in Fig. 4c. By proportioning the $\overline{M}/I$ values for the four levels, the corresponding moments of intertia of the fictitious modules are also presented as multiples of I in the same diagram.

Figure 4d represents the final design by simply adding the values of the merging members. The symbolic lowermost horizontal member describes the strength and stiffness required for the footing and the grade beams. The story sways can easily be worked out simply by substituting the corresponding module values from Fig. 4c into Eq. 10. The total story sway of each level is the sum of the story deflections of the levels below that story.

Horizontal Combination

To combine two or more basic modules of equal height in a horizontal sense it becomes necessary to achieve drift and rotation compatability for all adjoining modules in the elastic range of loading. Now consider the structural actions of N independent basic modules of equal height in their most general form. It is then seen that, for these

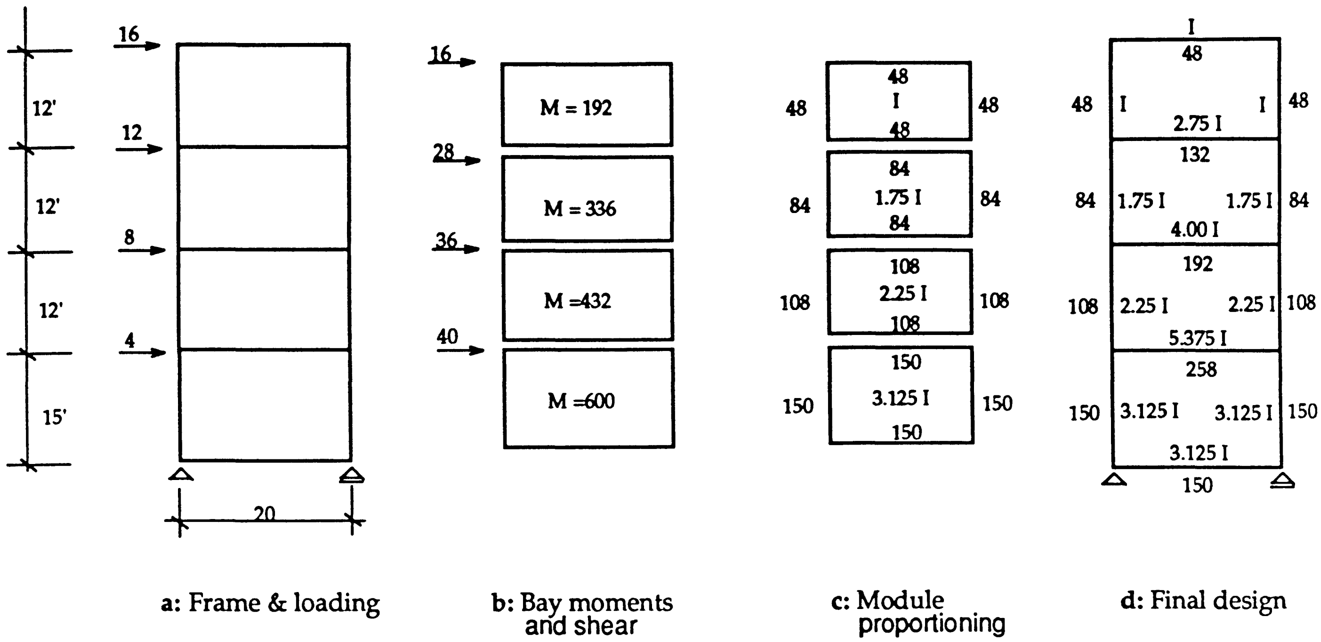


Fig. 4. Example 1 and solution

modules to be compatible with each other, the following relationships should hold;

$$\Theta_i \times \Theta_1 \text{ i.e.; } M_i/K_i = M_1/K_1 \tag{11}$$

$$\delta_i \times \delta_1 \text{ i.e.; } M_i/\bar{K} = M_1/\bar{K}_1 \tag{12}$$

$$\bar{M} = \sum_{i \times 1}^{n} \quad \bar{M}_i = Pb \tag{13}$$

where the member stiffnesses K and $\bar{K}$ are defined as a/I and b/J respectively. Combining relationship (11) with equilibrium Eq. 13 gives:

$$\bar{M}_i = (K_i/\Sigma K_i) \cdot \bar{M} \tag{14}$$

and comparing relationships (11) and (12) gives:

$$K_i/\bar{K}_i = K_1/\bar{K}_1 \tag{15}$$

Equations 14 and 15, as they stand, represent an infinite number of solutions, including the degenerate case of $\bar{M}_I = M$ and $\bar{M}_i = O$. This indicates an additional condition should exist to allow for non-zero solutions. The third condition may be sought in terms of practical restraints, such as use of a section of uniform strength for the horizontal elements or columns. Example 2 is presented to illustrate the application of the concept to horizontal combination of the basic modules.

Example 2

Design the four-bay frame of Fig. 6 for minimum weight, first with uniform beams, then with uniform columns.

Solution 1: Uniform horizontal members.

$M = 36 \times 15 = 540$ kip-ft

$I_i = I$ for $i = 1,2,3,4$, and $\Sigma a_1 = 54$ ft.

Then $J_i = (a_1/a_i) \cdot I$

$\bar{M}_i = (a_i/\Sigma a_i)M = 10 \cdot a_i$

This simple arithmetic is summarized in Fig. 7a. The module solutions of Fig. 7a are then merged to arrive at the final results of Fig. 7b.

Example 3.

Example 2 with uniform vertical members. The condition for interior columns of equal resistance requires that,

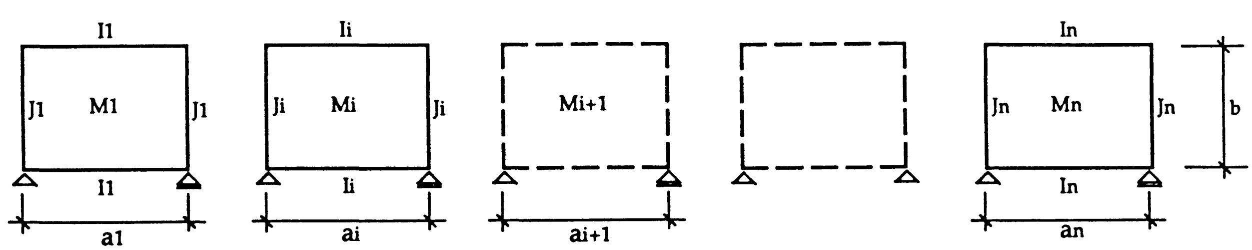


Fig. 5. Module generation for horizontal combination

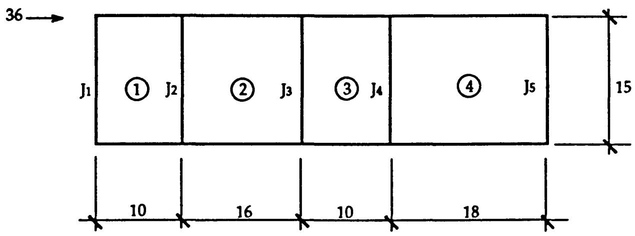


Fig. 6. Example 2, horizontal combination of modules

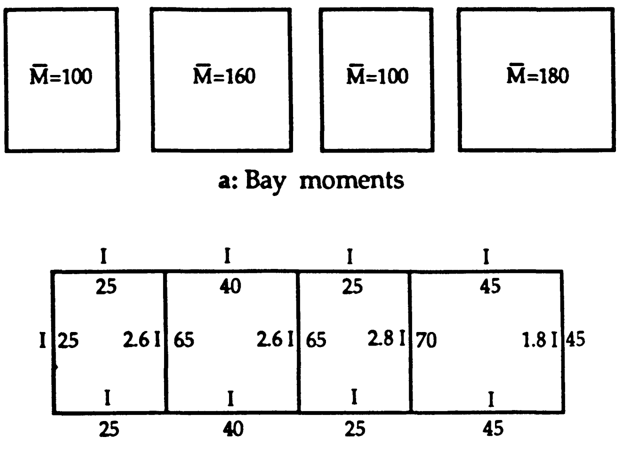


Fig. 7. Example 2 solution

$$\overline{M} = M/n = 540/4 = 135 \text{ kip-ft}$$

$$I_i = (a_i/a_1)\,I_1$$

$$\text{Let } J = I_1 = I$$

Figures 8a and 8b describe the modular and final solutions of this case, respectively.

In passing, it is interesting to note the total consumption of material for the first solution is 7,950 units, (moment volume), whereas for the second solution it is 7,695. Obviously, the final choice would depend on many other factors, including drift control, stability criteria, availability of materials and ease of construction.

DESIGN OF MULTI-BAY MULTI-STORY FRAMES

The principle of multiples expounded to combine basic modules in vertical and horizontal direction may be utilized readily to design generalized multi-bay, multi-level frames with constant bay lengths and story heights at each level. The application of the concept to this class of structure is best illustrated by this numerical example:

Example 4

Design the four-story frame of Fig. 9 for minimum weight. Allow columns of equal strength for the interior bays.

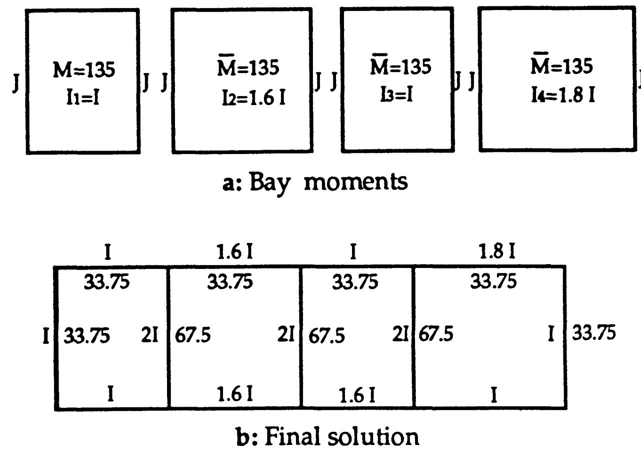


Fig. 8. Example 3 solution

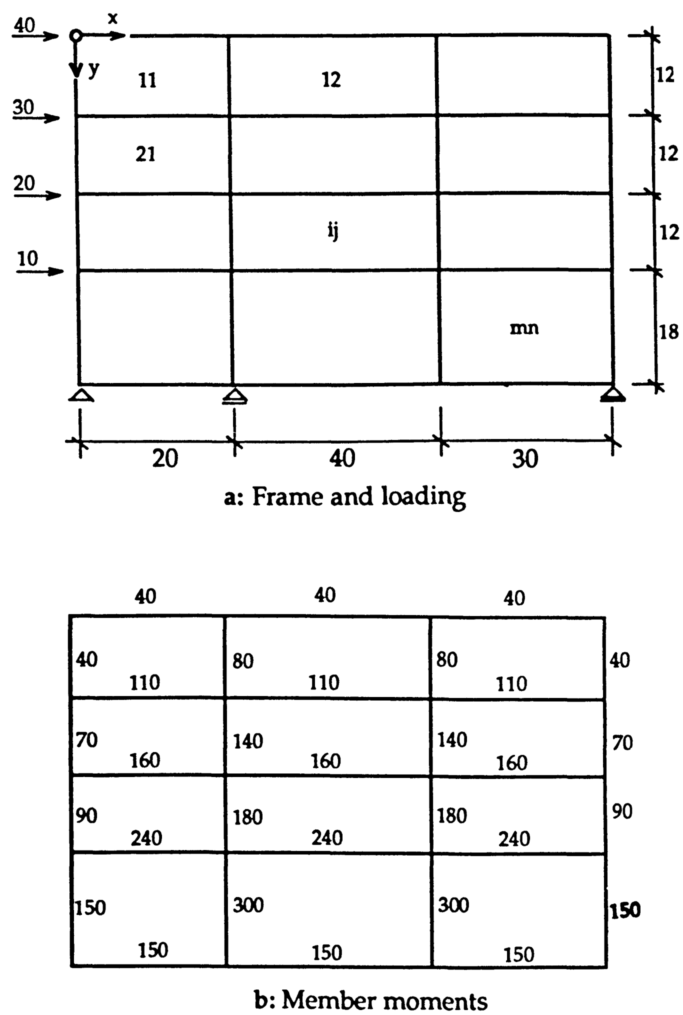


Fig. 9. Example 4 solution

Solution:
Consider the reference system xy with the origin at the top left hand corner of the frame with increasing $y=1,..i,..n$ downwards and increasing $x=1,...j,...m$ towards right, therefore $P_1 = 40$, $b_1 = 12$, $a_1 = 20$, $m = 3$ and $n = 4$. Story shear $= V_i = P_y$ and story moment $M_i = V_i \cdot b_i$.

So the interior columns may be of equal strength, $M_{ij} = M_i/m$. It follows immediately that the column-end moment of the exterior vertical members is given by $\overline{M}_{ij}/4 = V_i \cdot b_i/4m$ and those of the interior columns by $\overline{M}_{ij}/2 = V_i \cdot b_i/2m$. By the same token, it follows that the beam-end moments would be,

$$(\overline{M}_{ij}+\overline{M}_{i+1,j})/4=(V_i \cdot b_i+V_{i+1} \cdot b_{i+1})/4m$$

and

$$\overline{M}_{ij}/4=V_i \cdot b_i/4\text{m}$$

for upper-most level beams and lower-most level beams (roof & grade) respectively. The results of these simple formulas are shown in Fig. 9b. Next, let column inertia J for the upper-most modules be $J_1 = J$ then $J_i = J \cdot (M_i/M_1)$, which in turn results in J_i and $2J_i$ for the moments of inertia of the exterior and interior columns of each level, respectively. Next, if $I_{11} = I$ then $I_{1j} = (a_j/a_1)I$, for the roof beams and $I_{mj} = (a_j/a_1)$. (M_m/M_1) for the grade beams. To obtain the appropriate moments of inertia of the beams of intermediate levels, the corresponding I values of the upper and lower modules of a common beam are added together to give $(a_j/a_1\ (M_i+M_{i+1})/M_i$. The results of the inertia analysis are shown in Fig. 10.

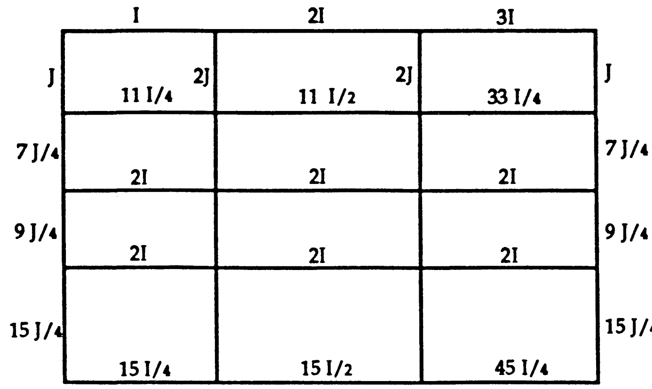


Fig. 10. Inertia analysis, Example 4

PLASTIC DESIGN

The significance of the applications of the concept as described in the preceding section is not only that it provides a simple means of obtaining minimum weight solutions in the elastic range, but also it results in structural frames of uniform strength. This implies all members sustain equal levels of stress at all stages of loading up to and including first-yield through full-plasticity. Evidently, if all member ends can become fully plastic simultaneously, then the entire structure would fail through a state of over collapse. The possibility of plastic collapse through formation of more than a sufficient number of hinges by itself constitutes an ideal condition for a true limit-state mini-

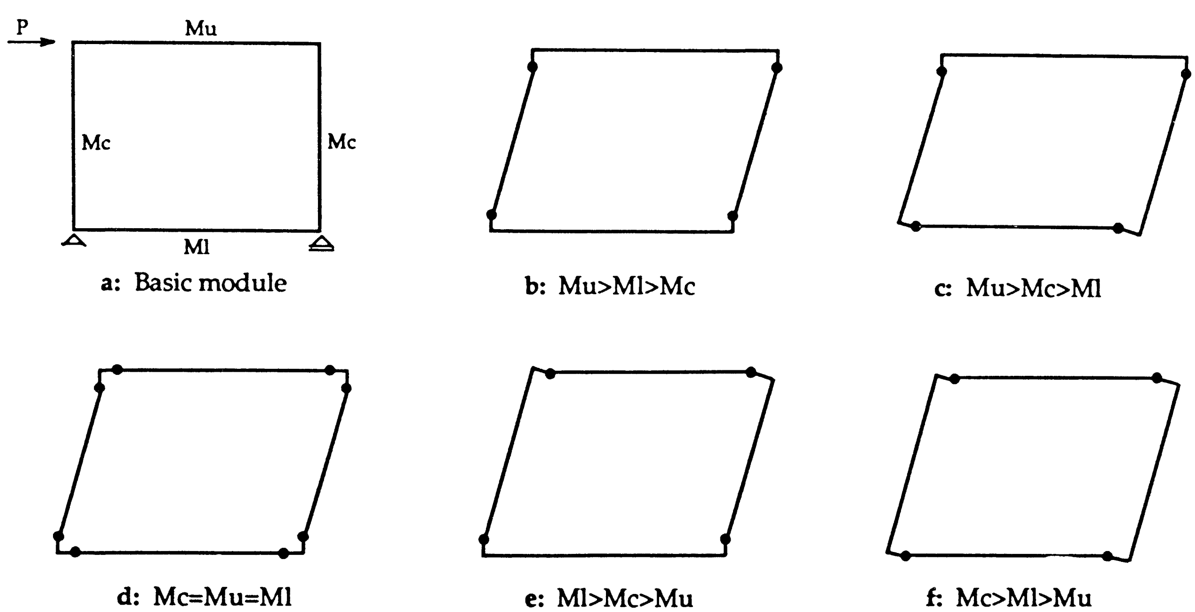


Fig. 11. Plastic collapse modes of basic module

mum weight treatment. However, to study the structural behavior of the module under plastic conditions, consider the collapse of a subframe with plastic moment capacities M_c, M_u & M_l for the column, the upper beam and the lower beam respectively as depicted in Fig. 11a.

With the vertical members having identical moments of resistance, it is possible to postulate four independent collapse mechanisms, as in Figs. 11b, 11c, 11e and 11f, depending on the relative magnitudes of the member strengths M_c, M_u and M_l. According to Foulkes[2] theorem for a frame composed of N prismatic members with different full plastic moments, the minimum weight design will be the one in which the frame collapses by the formation of at least N simultaneous mechanisms. Thus, the solution $M_c = M_l = M_u = M = Pb/4$ represents a condition of minimum weight design with four active and four inactive plastic hinges (as in Fig. 11d), which is a further conformation of the validity of the application of the concept in both elastic & plastic ranges of loading.

To combine basic modules of a regular framework that are collapsing in the same direction, the only rule to observe is that of equilibrium of the common nodes. This can be achieved readily by simply adding the collapse moments of the merging members. The following example demonstrates the application of the concept in the plastic range.

Example 5

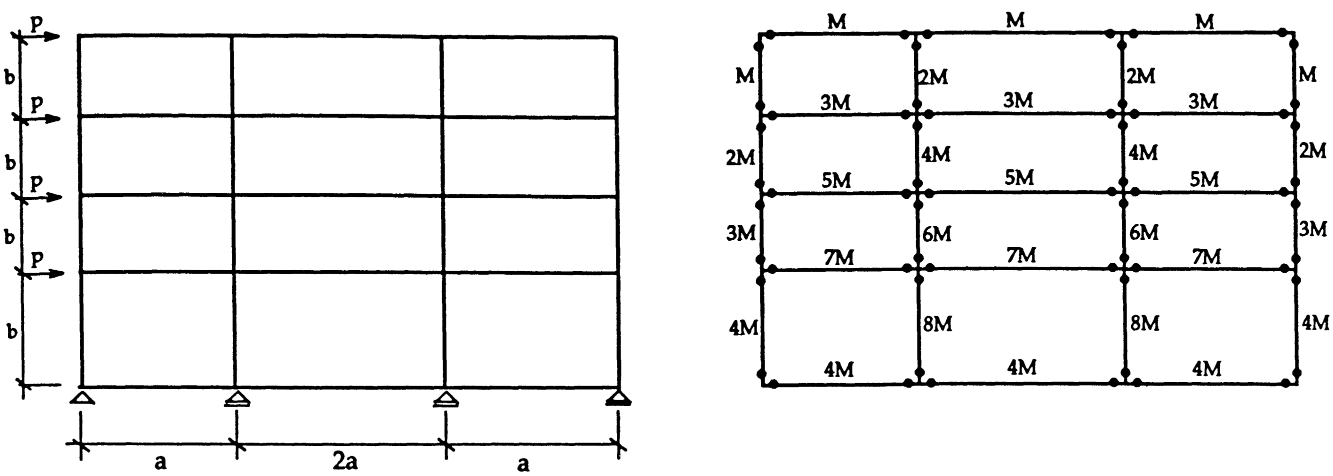
Obtain the collapse load of the symmetric frame of Fig. 12 for minimum weight and equal bay moments.

Solution:

For equal bay moments, each module of the upper story will carry a lateral shear of $P/3$ and the corresponding member end moments will be $M=Pb/12$. Then, the roof beam will be subjected to uniform plastic end moments M and the exterior and interior plastic column end moments will be M and $2M$ respectively. The second story from top is subjected to a shear force of $2P$. Therefore, the plastic end moments of the members of the modules of this story would be twice those of the corresponding upper level. The merging beams of the fourth level will carry a moment of $3M$, i.e., a summation of the components M and $2M$ from its upper and lower modules respectively. The moments of the remaining members are written down by comparison and inspection as shown in Fig. 12b.

CONCLUSION

A direct practical method of design has been developed for regular moment frames subjected to lateral loading. The method is applicable to both elastic and plastic ranges of stress and results in a minimum weight solution for all stages of loading. It has been shown this method of material selection also results in minimum drift for the given loading and geometry. Several numerical examples have been described to illustrate the simplicity and usefulness of the proposed concept. The theorem of the concept, as it stands, does not take into account the effects of axial deformations or loss of stiffness due to axial forces and therefore is recommended for preliminary member sizing before satisfying other pertinent design criteria. Using this method brings to mind the applications of the well established portal method of analysis for laterally loaded moment frames, implying that designs obtained by the portal method cannot be very far from minimum weight solutions.



a: Frame and loading

b: Plastic solution

Fig. 12. Example 5

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