A New Approach to Floor Vibration Analysis

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Floor vibrations analysis is one of few structural engineering topics that combine static and dynamic analyses, which makes it more interesting as well as more complicated. The subject is becoming increasingly familiar to structural designers as more and more lightweight floors exhibit varying degrees of vibration susceptibility. In the past, literature on this problem and ways to predict and prevent it were lacking. During the last 20 years, however, several methods have been developed to predict the susceptibility of floors to annoying transient and steady-state vibrations.

To date, methods developed have dealt independently with either the transient or the steady-state response of a floor system. The most popular methods of analysis have been: (1) Wiss and Parmelee rating factor R, (2) Modified Reiher-Meister frequency-amplitude scale and (3) Murray's acceptability criterion.³

Prior to discussing the validity of these three methods, it is believed that introducing a brief description of floor behavior under transient and steady-state conditions is warranted.

When a floor is impacted by a force (usually a falling weight), a transfer of energy occurs from the falling weight to the floor. This energy is caused by the velocity with which the weight strikes the floor. A simply supported floor beam, because of its boundary conditions, tends to oscillate upon the sudden gain of this energy at a frequency practically equal to its lowest natural frequency. Through oscillation, the beam (the floor) dissipates that energy in the form of heat (due to friction resulting from cyclical strain energy generated in the beam). The floor's rate of energy dissipation per cycle of oscillation is called damping. This damping is given for a particular floor as a percentage of the critical damping. Critical damping is the minimum amount of damping required to dissipate all the energy transferred upon impact and convert it to heat (or any other form of energy) in one half cycle, which means no oscillation. Floors behave differently under impact loads depending on the amount of damping. Figure 1 (a-c) illustrates the behavior of three systems subjected

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to a single impact load. These graphs represent systems with no damping, 10% and 100% damping, respectively. Depending on the method of construction, modern floors possess a damping ratio in the range of 2% to 10%. As energy is dissipated, oscillation amplitude decays gradually. The rate of decay is given by (see Fig. 1b)⁴

$$y_1 / y_1 = e^2 \pi^D$$
 (1)

where

 $y_1 =$ first cycle amplitude

 y_2 = second cycle amplitude

D = system's available damping



Fig. 1. Behavior under single impact of three systems with different damping characteristics

If under the application of a cyclic impact load the energy dissipated through damping is equal to the energy added from the load at each cycle, the system is then in a state of dynamic periodic equilibrium, or steady-state motion.

It has been demonstrated in numerous experimental research that human vibration perceptibility is a function of three variables: oscillation frequency, oscillation amplitude and damping ratio.⁵ The importance of oscillation frequency and amplitude has been demonstrated by Reiher and Meister in their work on steady-state vibration, from which they developed the famous Reiher-Meister scale. Damping importance has been demonstrated by Lenzen in Ref. 1: "Dr. Lenzen demonstrated repeatedly that if the amplitude decayed to a small percentage, say 20% of its initial amplitude in 5 cycles or less, human subjects felt only the initial impact, or no vibration. Others say the time to this decay (one half second) rather than the cycles to it are important." This statement demonstrates that human perceptibility of vibration is a function of the vibration amplitude rate of decay. Annoving vibrations are ones which do not decay rapidly.

CURRENT DESIGN METHODS³

Wiss and Parmelee Rating Factor

The R factor in this method was developed based on 40 laboratory tests with the R rating equation developed empirically by curve-fitting of the lab results (no theory involved),

$$R = 5.08 \left[\frac{FA_1}{D^{.217}} \right]^{.265}$$

with $F = 1.57 \sqrt{\frac{gEI}{WL^3}}, A_1 = (DLF) \frac{0.6L^3}{48EI}$ (2)

where

- F = Floor lowest natural frequency
- A_1 = Maximum amplitude under a single heel-drop impact
- D = Floor system's estimated available damping
- g =Gravity acceleration
- E = Young's modulus
- I = Floor beam's moment of inertia
- W = Total weight supported by the floor beam
- L = Floor beam's span

DLF = Dynamic load factor

if $R \le 2.5$ for a system, then the system is acceptable, and vice versa.

The main disadvantage of this formula is that the R rating is not sensitive to the damping ratio of a vibrating floor. This is in direct contrast to Lenzen's proven and accepted statement discussed in the previous section.

Modified Reiher-Meister Scale

Designers who have encountered floor vibration analysis are most familiar with the modified Reiher-Meister scale which relates a floor system's natural frequency and amplitude in steady-state motion to the floor's degree of human vibration perceptibility. Initially, damping was not considered by Reiher and Meister. Later, to compensate for damping effect on perceptibility, Lenzen modified the graph by increasing the ordinate scale by 10. Apparently, that seemed to correlate well with tests on floors having 5% or less damping. This increase was somewhat arbitrary and often caused unreliable results, particularly when used for floors subject to transient vibrations with damping ratios higher than 5%.

Murray's Acceptability Criterion

Murray recognized the vital importance of a floor's damping characteristic and developed an empirical formula establishing a minimum allowable damping ratio for a floor system. This ratio D is based on the floor's lowest natural frequency and on the initial amplitude due to a single heel drop impact of a person weighing 170-190 lbs. The equation establishes an absolute minimum allowable damping ratio of 2.5%.

$$D = 35 A_1 F + 2.5 \tag{3}$$

The results obtained from these three methods have a great deal of variation and are frequently in direct contradiction with field results of constructed floor systems.

A major cause of this variation and unreliability is the way the methods were developed, namely, statistical best fitting of accumulated data. The methods are also limited because none of them consider the effect of multiple heel drops applied at certain intervals to a floor system, which is a more accurate representation of a person walking across a floor. If the damping present in a system is not sufficient to overcome the vibration of the first heel drop before the second heel drop is applied, then the residual amplitude from the first excitation combines with that of the second to create a greater amplitude-and even greater amplitude from the third excitation. This phenomenon is referred to as resonance. In addition, all three methods treat floor beams, in estimating vibration amplitude, as undamped, single-degree-of-freedom oscillators. This approximation is adequate for systems with low damping. However, for systems with substantial damping, the inaccuracy could be significant.

DEVELOPMENT OF NEW ANALYSIS PROCEDURE

So as to study more closely the behavior of floors under impact loads, a computer program was developed to calculate the dynamic response of a *damped* single-degree-offreedom oscillator using Duhamel integral. The integral is calculated using numerical integration.

The program calculates the response of the simple damped system excited by any time-dependent, piecewiselinear external force. This made it possible to apply the exact heel-drop impact as measured by Omhart, instead of the approximate forcing function which has been used to date (see Fig. 2a).² It was also possible to study the response history of a floor system caused by consecutive heel-drops applied at equal time intervals. For example, Fig. 2b shows the impulse history of heel-drop loading applied at 0.6-second intervals. As discussed earlier, such loading is a more accurate representation of a floor excited by a person walking across it.

It was estimated that a person walking at a normal pace takes approximately 80-100 steps per minute. Therefore, t heel-drop period ranges between 0.6 and 0.75 second and f heel-drop frequency ranges between 1.33 and 1.67 cps. The program was designed to select automatically the frequency, within the above mentioned range, that produces the greatest vibration amplitude.

It has been suggested by some researchers that floor beams should be treated as multi-degree-of-freedom systems and the amplitude contribution of higher modes of vibration (other than the first mode) should be accounted for. It can be proven, if floor beams are analyzed as systems with infinite degrees of freedom (uniform mass), the contribution of even modes (i.e., 2, 4, 6...) cancel each other and the contribution of odd modes (i.e., 1, 3, 5...) are proportional to: $1/n^4$. Hence, the contribution of the 1st, 3rd and 5th modes are $1/1^4 = 1 = 100\%$, $\frac{1}{3}^4 = \frac{1}{81}$



Fig. 2. Comparison of exact and approximate heel-drop impact histories of available and new methods

= 1.2% and $\frac{1}{5^4} = \frac{1}{625} = 0.16\%$ respectively.⁴ This demonstrates the contribution of higher modes is negligible.

TEST RESULTS

A summary of test results made on 96 steel joist- and steel beam-concrete slab floor systems is reported in Ref. 3, Tables 1 and 2. In each of these tests, a single heel-drop impact was applied to the floor system and the motion recorded. The initial amplitude, frequency and damping were calculated from the motion records. Also, a rating was assigned to each system of either acceptable or unacceptable based on reported subjective evaluation by owners, occupants and researchers.

Based on the reported frequency and initial amplitude of each system, the stiffness and mass were calculated. Then, using mass, stiffness, available damping and heeldrop loading similar to that shown in Fig. 2b, all 96 tests were analyzed using the program described above. The analysis results are listed in Table 1a.

DISCUSSION

Examining the results listed in Table 1a shows that 45 of the 49 tests rated acceptable satisfy either one or both of the following conditions:

1)
$$\frac{A_2}{A_1} \le 1.15$$
 with $A_{\max} \le 0.05$ in.* (4)

2) Reiher-Meister scale rating for F and A_{max} falls within the slightly perceptible range $(A_{\text{max}} \cdot F \le 0.050)^{**}$

On the other hand, 43 of the 47 tests rated unacceptable do not satisfy either one of the two conditions mentioned above. This leads to the conclusion that there are three different vibration categories under which any floor system behavior can be classified:

1. A system with adequate frequency and damping to reduce the initial impact amplitude by 85% or more before the second heel drop is applied (0.6-0.75 sec.). Then, the amplitude of the first excitation A_1 will decay to 15% or less and the total second excitation ampli-

^{*} The calculated amplitude of all 96 tests converged to the maximum value after 2-5 heel-drops. The speed at which the amplitude converged was a function of the system's damping and frequency. Note the state of equal periodic deflection under equal periodic load is the steady-state condition described earlier in this article.

^{**} Note, all lines running parallel to the border lines in the Reiher-Meister scale are represented by $A \cdot F = \text{constant}$. For example, the line that separates slightly and distinctly perceptible ranges is given by $A \cdot F = 0.05$; and the line that separates distinctly and strongly perceptible ranges is given by $A \cdot F = 0.15$, etc.

	FREQUENCY (cps)	AMPLITUDE (in)	EVALUATION
O. DAMP	F f	A1 A2 Amax	A2/A1 Amax.F Rating
1a 2.0 1a 2.0 1a 2.0 1a 10.4 44 12.3 1a 10.4 44 12.3 55 4.1 1 1.4 2 10.9 34 8.0 55 10.2 4 8.0 5 10.8 7 10.9 34 8.0 55 10.7 78 7.1 90 10.9 10.9 3.0 10.9 3.0 10.9 3.0 11.0 7.3 12.1 10.5 3.0 0.0 12.2 10.5 3.0 0.0 12.3 11.0 7.3 12.0 7.4 9.0 12.2 10.1 13.4 9.0 12.3 11.1 13.4 9.0 12.2 10.1 13.4	4.0 1.38 4.7 1.56 10.6 1.52 10.6 1.52 10.7 1.50 10.0 1.43 10.0 1.50 10.0 1.50 10	0.007 0.011 0.014 0.015 0.105 0.040 0.040 0.044 0.044 0.005 0.005 0.005 0.004 0.005 0.005 0.005 0.005 0.016 0.016 0.016 0.016 0.011 0.011 0.011 0.011 0.012 0.012 0.012 0.012 0.013 0.013 0.014 0.014 0.012 0.013 0.013 0.015 0.019 0.019 0.019 0.017 0.011 0.011 0.011 0.017 0.012 0.013 0.013 0.013 0.014 0.012 0.013 0.013 0.012 0.013 0.013 0.013 0.013 0.012 0.014 0.012 0.014 0.012 0.013 0.013 0.015 0.017 0.017 0.013 0.016 0.006 0.007 0.007 0.	1.593 0.054 U 1.768 0.492 U 1.010 0.424 A 1.003 0.032 A 1.005 0.032 A 1.005 0.097 A 1.046 0.141 A 1.005 0.097 A 1.048 0.141 A 1.000 0.118 A 1.008 0.127 A 1.048 0.141 A 1.008 0.121 A 1.043 0.123 A 1.051 0.046 A 1.070 0.083 A 1.070 0.088 A 1.070 0.089 A 1.340 0.034 A 1.387 0.105 U 1.319 0.226 U 1.211 0.104 U 1.217 0.129 U 1.707 0.130 A 1.052 0.055 A 1.074 0.042 A 1.057 0.071 A 1.057 0.071 A 1.057 0.034 A 1.057 0.071 A 1.058 0.128 A 1.059 0.042 A 1.059 0.042 A 1.059 0.043 A 1.057 0.071 A 1.059 0.043 A 1.059 0.044 A 1.059 0.042 A 1.059 0.042 A 1.059 0.044 A 1.059 0.042 A 1.059 0.042 A 1.059 0.044 A 1.059 0.042 A 1.059 0.044 A 1.059 0.042 A 1.059 0.044 A 1.059 0.013 A 1.059 0.014 A 1.226 0.032 A 1.058 0.124 U 1.375 0.116 A 1.470 0.041 U 1.375 0.116 A 1.470 0.041 U 1.375 0.116 A 1.470 0.041 U 1.375 0.116 A 1.470 0.042 A 1.568 0.044 U 1.375 0.116 A 1.470 0.013 A 1.470 0.013 A 1.470 0.013 A 1.470 0.014 U 1.375 0.116 A 1.470 0.021 U 1.461 0.045 U 1.461 0.045 U 1.461 0.045 U 1.476 0.072 U 1.462 0.066 U 1.444 0.111 U 1.453 0.124 U 1.455

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 Table 1b.
 Comparison of Available and New Methods Results
to Field Ratings.

tude A_2 will be less than or equal to $1.15A_1$. Because the amplitude decays rapidly, vibration will not be felt and such a system will always be acceptable regardless of the value of $A_{max} \cdot F$ as long as A_{max} does not exceed 0.05 in. Note, this is in complete agreement with Lenzen's previously mentioned statement.

- 2. A system with insufficient frequency and damping to reduce the initial impact amplitude by 85% or more before the second heel drop is applied, but $A_{max} \cdot F \leq 0.050$. Then, the residual amplitude of the first excitation will be greater than 15% and the total second excitation amplitude will be greater than $1.15A_I$. In this system, vibration will be felt, but it will be within the slightly perceptible range. Thus, the floor will still be acceptable.
- 3. A system with insufficient frequency and damping to reduce the initial impact amplitude by 85% or more before the second heel drop is applied, with $A_{max} \cdot F > 0.050$. A_2 will be greater than $1.15A_1$. In this system, vibration will be felt and it will be within the distinctly or strongly perceptible ranges. Thus, the floor will be unacceptable.

Note here it has been mentioned in some literature that increasing a system's stiffness does not reduce vibration perceptibility. This statement is not accurate for the following reason:

Increasing a system's stiffness usually results in increasing its frequency. Now, assuming that m is equal to the total number of vibration cycles a system undergoes between two consecutive heel-drop impacts, then

$$m = \frac{\text{heel-drop impact period}}{\text{system's period}}$$
$$= \frac{\text{system's frequency}}{\text{heel-drop impact frequency}}$$
(5)

The rate of total amplitude decay between two impacts is given by:

$$\frac{Y_1}{Y_m} = \frac{Y_1}{Y_2} \times \frac{Y_2}{Y_3} \times \ldots \times \frac{Y_{m-1}}{Y_m}$$
(6)

But

$$\frac{Y_1}{Y_2} = \frac{Y_2}{Y_3} = \dots = \frac{Y_{m-1}}{Y_m}$$
(7)

Then, using Eqs. 6, 7 and 1

$$\left[\frac{Y_1}{Y_m}\right] = \left[\frac{Y_1}{Y_2}\right]^{m-1} = \left(e^{2\pi D}\right)^{m-1} \tag{8}$$

increasing the system's frequency increases m (Eq. 5), thus, the value $[e^2\pi^D]^{m-1}$ (Eq. 8), given a constant damping

ratio. This increases the total decay of the initial amplitude, which results in a smaller second amplitude, hence a smaller ratio A_2/A_1 . Increasing *m* also results in a smaller absolute maximum amplitude A_{max} , consequently, smaller product $A_{max} \cdot F$, which results in reduced vibration perceptibility.

Systems 3, 39 and 44 will be used as examples of Categories 1, 2 and 3, respectively. The calculated response history of these floor systems when subjected to an impulse history similar to that in Fig. 2b is shown in Fig. 3 (a-c). This response is discussed here.

System 3: $(F = 10.6 \text{ cps}; A_I = 0.04 \text{ in.}; \text{ damping} = 10.4\%)$

The damping ratio of 10.4% reduced the initial impact amplitude by 99% ($A_2/A_1 = 1.01$) within t = 1/f = 1/1.52 = 0.66 second. Also, $A_{max} = 0.04$ in. did not exceed 0.05 in. Therefore, the system satisfied Condition 1 and was rated acceptable, despite the relatively large value of $A_{max} \cdot F = 0.424$. This system was rated unacceptable by Wiss-Parmelee *R* factor, Modified Reiher-Meister scale and by Murray's acceptability criterion, though it was acceptable to the owner/users.

System 39: $(F = 8.0 \text{ cps}; A_1 = 0.0044 \text{ in.}; \text{ damping} = 4.1\%)$

The damping ratio of 4.1% reduced the initial impact amplitude by 72% $(A_2/A_1 = 1.28)$ within t = 1/1.60 = 0.63 second. The system's maximum impact amplitude was $A_{max} = 0.0060$ in. and $A_{max} \cdot F = 0.048$. The system satisfied Condition 2 and was rated acceptable because vibration will be felt, but it will be tolerable $(A_{max} \cdot F < 0.05)$. This system was rated unacceptable by Wiss-Parmelee R factor and acceptable by the Modified Reiher-Meister scale, Murray's criterion, as well as by the owner/users.

System 44: ($F = 6.7 \text{ cps}; A_I = 0.02 \text{ in.}; \text{ damping} = 2.0\%$)

The damping ratio of 2% reduced the initial impact amplitude by only 40% within t = 1/1.67 = 0.6 second. The system's maximum impact amplitude was $A_{max} = 0.0483$ in. and $A_{max} \cdot F = 0.32$. The system did not satisfy either condition and was rated unacceptable because vibration will be felt and will be intolerable ($A_{max} \cdot F > 0.05$). This system was rated unacceptable by Wiss-Parmelee R factor, Reiher-Meister scale and Murray's criterion, as well as by the owner/users.

Finally, ratings using the three methods discussed earlier, as well as ratings using the new method, are compared to the available subjective field ratings for all 96 floor systems. A (*) is assigned to each method when its rating does not agree with the field rating. The comparison is shown in Table 1b. Simple calculations show the Rrating factor disagrees with field results 41.7% of the time, Reiher-Meister 50% and Murray's criterion 13.5%. On the other hand, the new method disagrees with field results 8.3% of the time.



Fig. 3. Response history of Systems 3, 39 and 44 subjected to four consecutive heel-drops

CONCLUSIONS AND RECOMMENDATIONS

Based on lowest natural frequency and available damping, floor systems are classified under three different categories:

- 1. Systems that dissipate impact energy within 0.6–0.75 second. Regardless of motion magnitude, humans do not perceive it in such systems as annoying vibration because it decays rapidly.
- 2. Systems that do not dissipate impact energy quickly. But the magnitude and frequency of the motion are such that humans perceive it as slight and acceptable motion.
- 3. Systems that do not dissipate impact energy quickly. Also, the magnitude and frequency of the motion are such that humans perceive it as distinct and annoying motion.

Ninety six tests were investigated representing a wide range of floor systems. Span range from 23 to 95 ft, beam spacing range from 2 to 24 ft and slab thickness range from 2 to 7.5 in. Therefore, there are no limits on the applicability of these categories to floor beams. On the other hand, the computer program developed to calculate floor system response history is rather simple and can be installed on a small personal computer. In addition, the program calculates the response of a floor system excited by any timedependent force. Therefore, it is possible to study floor systems behavior under exciting forces other than the standard heel-drop impact. For example, the response of a floor system in a health club or a ballroom can be evaluated under the impact of a person jogging, doing aerobics or dancing. The exciting force frequency and magnitude, as well as the perceptibility thresholds, would be different in such cases.

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