Design of Bolts or Rivets Subject to Combined Shear and Tension

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THE 1963 AISC SPECIFICATION introduced, in Section 1.6.3, new provisions pertaining to the combined tension and shear stresses in bolts and rivets. The formulas given in this section permit the checking of a given design but leave the design itself to a rather cumbersome cut and try procedure. The purpose of this paper is to present a simple method which enables the designer or detailer to establish the required number of bolts or rivets in a bracket connection—the most common case in which rivets or bolts are under combined action of shear and tension (see Fig. 1). The specification provides different criteria for various rivet types, high tensile bolts of the friction or bearing type and for unfinished bolts; for this reason a different analysis is required for each of these elements. This paper will deal, however, only with the most commonly used types, that is, ASTM A325 type high tensile bolts and A141 rivets.

CASE I: A325 HIGH TENSILE BOLTS—FRICTION TYPE

For this case the specification prescribes that

$$F_v \leq 15{,}000\left(1 - f_t \frac{A_b}{T_b}\right) \qquad (1)$$

where F_v is the allowable shear stress in the bolt, f_t is the computed tensile stress in the bolt, A_b is the nominal body area of the bolt, and T_b is the proof load of the bolt.

Multiplying both sides of Equation (1) with A_b and noting that the product of f_t times A_b is the tensile force in the bolt, T, and further designating the product of $F_v \times A_b$ as V, the actual shear force in the bolt which is subject to tension and shear, Equation (1) can be rewritten in the form

$$V \leq V_{all}\left(1 - \frac{T}{T_b}\right) \qquad (2)$$

where $V_{all} = 15{,}000\ A_b =$ the allowable shear force in the bolt for a condition of pure shear without tension.

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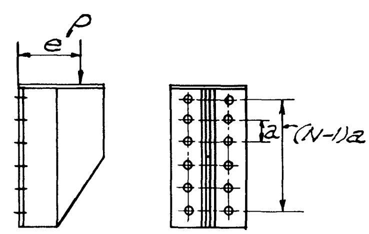

e = Eccentricity
a = Bolt or Rivet Spacing
m = e/a
N = Number of Bolts or Rivets per Row for Eccentricity e
n = Number of Bolts or Rivets per Row for Eccentricity e = 0

Figure 1

For ¾-in. dia. bolts:

$$\left.\begin{aligned} V_{all} &= 6.63 \text{ kips} \\ T_b &= 28.40 \text{ kips} \end{aligned}\right\} \qquad (3)$$

For ⅞-in. dia. bolts:

$$\left.\begin{aligned} V_{all} &= 9.02 \text{ kips} \\ T_b &= 36.05 \text{ kips} \end{aligned}\right\} \qquad (4)$$

The proof load T_b is taken from Table 2 of the *Specifications for Structural Joints Using ASTM A325 Bolts.*

There is no exact method known to establish the tensile force T in a bracket connection of the type shown in Fig. 1. The reason for this lack of an accurate theory lies in the fact that there is no definite basis for locating the neutral axis. It lies somewhere below the center line of the connection but common practice makes the safe and conservative assumption that the neutral axis is at the centroid of the fasteners (rivets or bolts). With this assumption the moment of inertia I and the section modulus of the fastener group shown in Fig. 1 can be written, respectively, as

$$I = \frac{N}{6}(N + 1)(N - 1)a^2$$

$$S = \frac{N(N + 1)a}{3}$$

where a = the spacing of the fastener and N = the number of fasteners in one row.

The moment, M, acting on the fastener group equals Pe, where P is the applied load and e its distance from the shearing plane of the connection. Hence the tensile force T in the extreme row (due to the moment) can be found from the relations

$$T = \frac{M}{S} \text{ or } T = \frac{3Pe}{N(N+1)a}$$

and with $m = e/a$,

$$T = \frac{3Pm}{N(N+1)} \tag{5}$$

It can further be stated that the actual shear force in the bolt is

$$V = \frac{P}{2N} \tag{6}$$

Substituting the values from Equations (5) and (6) into Equation (2),

$$\frac{P}{2N} \leq V_{all}\left(1 - \frac{3Pm}{N(1+N)T_b}\right) \tag{7}$$

If the number of bolts required in one row is defined as n for the condition that $e = 0$, then

$$P = 2nV_{all} \tag{8}$$

Introducing P from Equation (8) into Equation (7), after some simple transformation the relation between n and N becomes

$$N = \frac{n-1}{2} + \frac{1}{2}\sqrt{(n+1)^2 + 24mn\frac{V_{all}}{T_b}} \tag{9}$$

Introducing now the values from Equations (3) and (4) respectively into Equation (9), the specific expression for ¾-in. dia. bolts will be

$$N = \frac{n-1}{2} + \frac{1}{2}\sqrt{(n+1)^2 + 5.6mn} \tag{9a}$$

and for ⅞-in. dia. bolts

$$N = \frac{n-1}{2} + \frac{1}{2}\sqrt{(n+1)^2 + (6.0\ mn)} \tag{9b}$$

Equations (9a) and (9b) enable the designer to find the required number of bolts N for a given eccentricity represented by the ratio e/a as a function of the number n.

CASE II: A325 HIGH TENSILE BOLTS—BEARING TYPE

For this case the Specification prescribes that the tensile stress F_t in the bolt be less than $50{,}000 - 1.6f_v$, where f_v is the shear stress produced by the same force, and in no case shall the tensile stress exceed 40,000 psi:

$$F_t \leqq 50{,}000 - 1.6f_v < 40{,}000$$

Multiplying this expression with A_b, the cross-sectional bolt area, and observing that $F_t \times A_b = T$ (the allowable bolt tension) and $f_v \times A_b = V$ (the actual bolt shear), it is found that

$$V \leqq \frac{50{,}000}{1.60}A_b - \frac{T}{1.60} \tag{10}$$

Introducing for T and V the values from Equations (5) and (6) respectively, and replacing P with the value given in Equation (8), Equation (10) becomes

$$N^2 + N(1 - Kn) + Kn\left[\frac{6m}{1.60} + 1\right] = 0$$

from which

$$N = \frac{n}{2}K - \frac{1}{2} + \frac{1}{2}\sqrt{(1+nK)^2 + 15mnK} \tag{11}$$

where

$$K = \frac{1.6F_v}{50{,}000} \tag{12}$$

and F_v (in psi) is the allowable shear stress in the bolt (see Specification Sect. 1.5.2).

For high tensile bearing bolts with threads excluded from the shear plane, F_v = 22,000 and K = 0.704. Therefore,

$$N = 0.352n - \frac{1}{2} + \frac{1}{2}\sqrt{(1 + 0.704n)^2 + 10.56mn} \tag{13}$$

There are two limitations to this formula which are inherent in the wording of the Specification and which must be observed. The first limitation becomes apparent when Equation (13) is applied to the condition where $e = 0$, i.e., where $m = 0$. The answer must obviously be $N = n$, because by definition n equals the number of bolts for $e = 0$. The equation, however, gives a value for N which is less than n. The equation can, therefore, be used only up to an m value for which $N = n$. By equating N to n in Equation (11) the limiting m_L value is found to be

$$m_L = \frac{4(1-K)}{15K}(n+1) = 0.112(n+1) \tag{14}$$

Equation (13) is valid only for m values larger than those given by Equation (14). For m values below those N equals n. To put it in other words: up to the limiting eccentricity m_L the bolts which are required for direct shear will be sufficient to carry an additional tension. Only if the eccentricity exceeds the limiting value m_L will there be additional bolts required for bracket connections as compared to direct shear connections. The critical m_L value is represented in Table III at the intersection point between the straight horizontal and the curved lines.

The second limitation to Equation (13) is due to the Specification requirement that f_t shall not exceed the value of 40,000 psi. This leads to the following relationship:

$$f_t = \frac{T}{A_b} < 40{,}000 \qquad (15)$$

Substituting in this equation the value from Equations (5) and (8) and replacing V_{all} with A_bF_v,

$$N^2 + N = \frac{6nmF_v}{40{,}000} = 0 \qquad (16)$$

and

$$N = -\tfrac{1}{2} + \sqrt{\tfrac{1}{4} + \frac{6nmF_v}{40{,}000}} \qquad (17)$$

For A325 high tensile bolts-bearing type, F_v = 22,000 psi and

$$N = -\tfrac{1}{2} + \tfrac{1}{2}\sqrt{1 + 13.2nm} \qquad (18)$$

If Equation (18) requires a higher number of bolts than those obtained by Equation (13) the higher figure must be used. This will be the case if

$$m \geq 3.72n + 1.06 \qquad (19)$$

An inspection of Equation (19) indicates that for practical cases Equation (13) rather than Equation (18) governs. This means that the 40,000 psi limitation on the tensile stress generally does not affect the design of a bracket connection for ASTM A325 bearing bolts.

CASE III: RIVETS

For rivets the Specification requirements are similar to those for high tensile bearing bolts, and prescribe that:

$$F_t = 28{,}000 - 1.6f_v \leq 20{,}000$$

With an allowable shear stress, F_v, of 15,000 psi for rivets, Equation (12) leads to

$$K = \frac{1.6 \times 15{,}000}{28{,}000} = 0.86 \qquad (12a)$$

Substituting this value in Equation (11)

$$N = 0.43n - \tfrac{1}{2} + \tfrac{1}{2}\sqrt{(1 + 0.86n)^2 + 12.90mn} \qquad (13a)$$

Similarly the limiting m_L value according to Equation (14) is:

$$m_L = \frac{4}{15K}(1 - K)(n + 1) = 0.0435(n + 1) \qquad (14a)$$

Equation (17) can now be rewritten to

$$N = -\tfrac{1}{2} + \sqrt{\tfrac{1}{4} + \frac{6nm \times 15{,}000}{20{,}000}} \qquad (17a)$$

from which

$$N = -\tfrac{1}{2} + \tfrac{1}{2}\sqrt{1 + 18nm} \qquad (18a)$$

If Equation (18a) requires a higher number of rivets than those obtained from Equation (13a), the higher values must be used. This will be the case if

$$m \geq 2.05n + 0.68 \qquad (19a)$$

Equation (19a) defines the point of intersection of Equations (13a) and (18a). Note that Equation (18a) will usually govern only for n = 2 and m > 4.78 (see Table IV). For all other practical cases Equation (13a) governs, which means that the limitation of 20,000 psi for the tensile stress in a rivet is hardly ever a governing factor.

Summarizing, it may be stated that Equations (9a), (9b), (18) and (18a) permit the direct determination of the number of bolts or rivets in a bracket connection for a given load and eccentricity as a function of the number of bolts and rivets required for the same load without eccentricity.

To avoid the necessity of solving Equations (9a), (9b), (18) or (18a), Tables I through IV have been prepared, from which the required number of bolts or rivets can be obtained directly as illustrated in the following examples:

EXAMPLES

Procedure (See Fig. 1)

Given: Load = P

Allowable shear load for single bolt or rivet = V_{all}

Eccentricity = e

Rivet or bolt spacing* = a.

Find: Required number N of bolts or rivets.

Solution: (1) Establish $m = e/a$

(2) compute $n = P/2V_{all}$

(3) Read N from Tables I, II, III or IV.

Alternate Condition: If a bracket and the spacing and number N of the bolts or rivets is given and the carrying capacity of this bracket is to be determined, establish m and read from the given N the value of n from Tables I through IV, from which $P = 2nV_{all}$ (see Example 2).

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* *1. For irregular spacing use average a (see Example 2).*
2. For staggered spacing use distance between staggered rivets or bolts (see Example 3).
3. For 2 rows of rivets or bolts in each clip angle use ½ of P and double the number N obtained from tables (see Example 4).

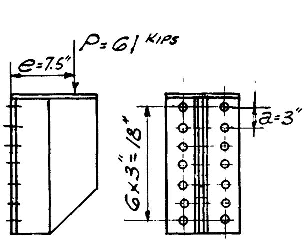


Figure 2

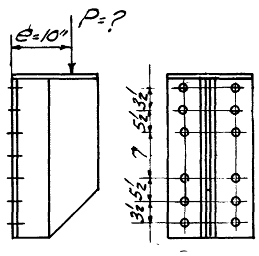


Figure 3

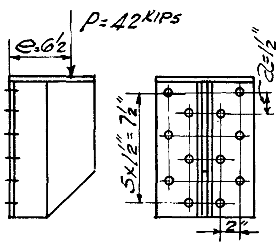


Figure 4

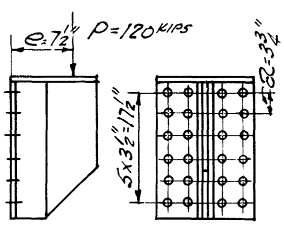


Figure 5

Example 1 (See Fig. 2)

Given: $P = 61$ kips, $a = 3$ in., and $e = 7\frac{1}{2}$ in.

Find: Number N of required $\frac{3}{4}$-in. dia. A325 high strength friction bolts.

Solution: (with Table I)

$$m = 7.5/3 = 2.5,$$
$$V_{all} = 6.63 \text{ kips}$$
$$n = 61/(2 \times 6.63) = 4.6$$
$$\text{From Table I: } N = 6.6$$

Use seven $\frac{3}{4}$-in. dia. high strength friction bolts in each clip angle.

Example 2 (See Fig. 3)

Given: Bracket shown in Fig. 3.

Find: Permissible load P for $\frac{7}{8}$-in. dia. A325 high strength friction bolts.

Solution: (with Table II)

$$a_{\text{average}} = \frac{2(3.5 + 5.5) + 7}{5} = 5 \text{ in.}$$
$$m = 10/5 = 2$$

From Table II, for $N = 6$ and $m = 2$, read $n = 4.2$

Permissible $P = 9.02 \times 2 \times 4.2 = 76$ kips.

Example 3 (See Fig. 4)

Given: $P = 42$ kips, $a = 1\frac{1}{2}$ in., and $e = 6\frac{1}{2}$ in.

Find: Number N of required $\frac{3}{4}$-in. high strength bearing volts.

Solution: (with Table III)

$$m = 6.5/1.5 = 4.35,$$
$$V_{all} = 9.72 \text{ kips}$$
$$n = 42/(2 \times 9.72) = 2.15$$
$$\text{From Table III: } N = 5.4$$

Use six $\frac{3}{4}$-in. dia. A325 high strength bearing bolts with threads excluded from shear plane in each clip angle.

Example 4 (See Fig. 5)

Given: $P = 120$ kips, $a = 3\frac{3}{4}$ in., and $e = 7\frac{1}{2}$ in.

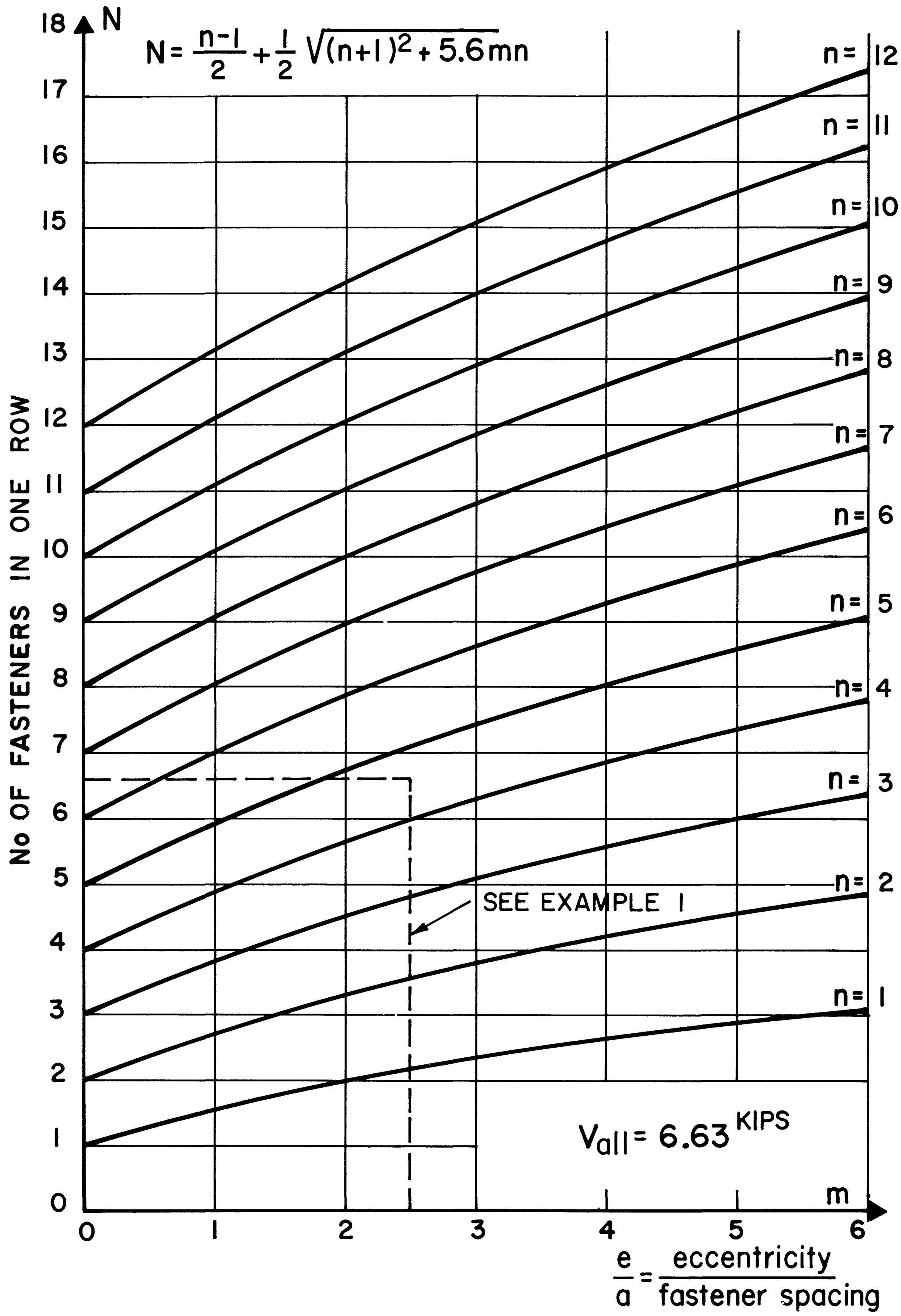
Find: Number N of required $^7/_8$-in. dia. rivets, type A141.

Solution: (with Table IV)

$$m = 7.50/3.75 = 2.0$$
$$V_{all} = 9.02 \text{ kips}$$
$$n = 120/(2 \times 2 \times 9.02) = 3.32$$
$$\text{From Table IV: } N = 5.9$$

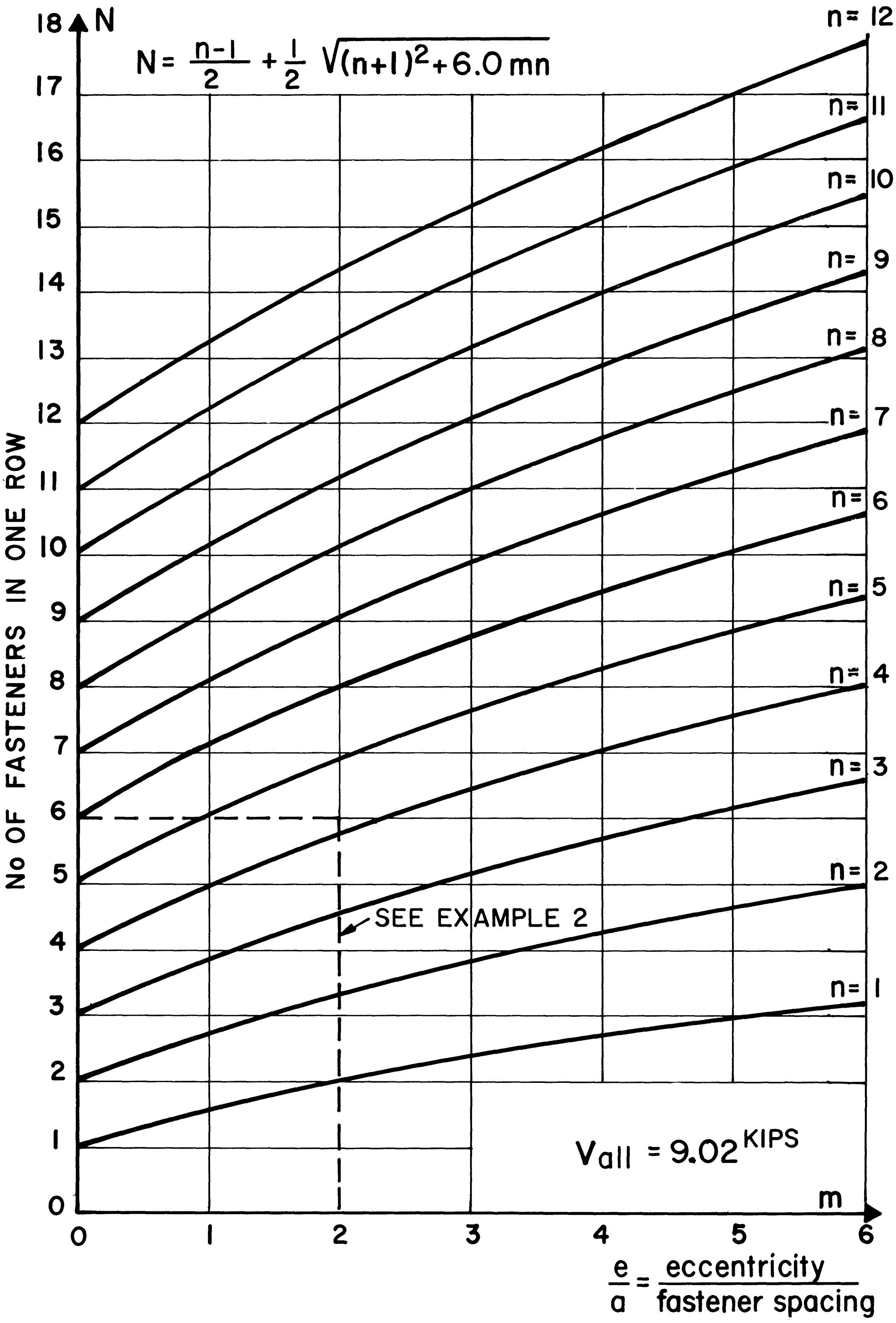
Use 2×6 = twelve $\frac{7}{8}$-in. dia. rivets, type A141, in each clip angle.

Table I.



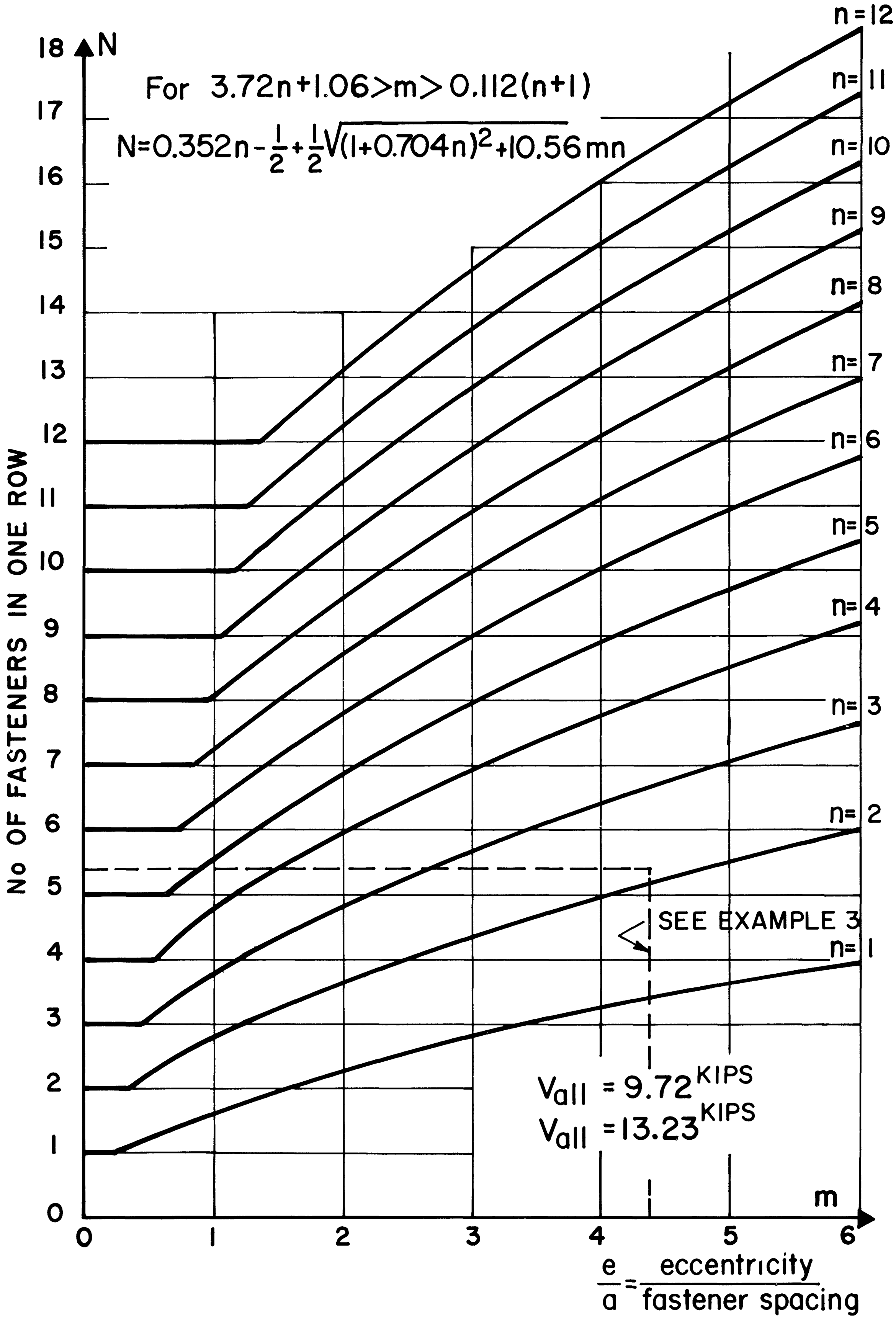
N
$N = \frac{n-1}{2} + \frac{1}{2}\sqrt{(n+1)^2 + 5.6mn}$
No OF FASTENERS IN ONE ROW
n = 12
n = 11
n = 10
n = 9
n = 8
n = 7
n = 6
n = 5
n = 4
n = 3
n = 2
n = 1
SEE EXAMPLE 1
$V_{all} = 6.63$ KIPS
m
$\frac{e}{a} = \frac{\text{eccentricity}}{\text{fastener spacing}}$

Table II.



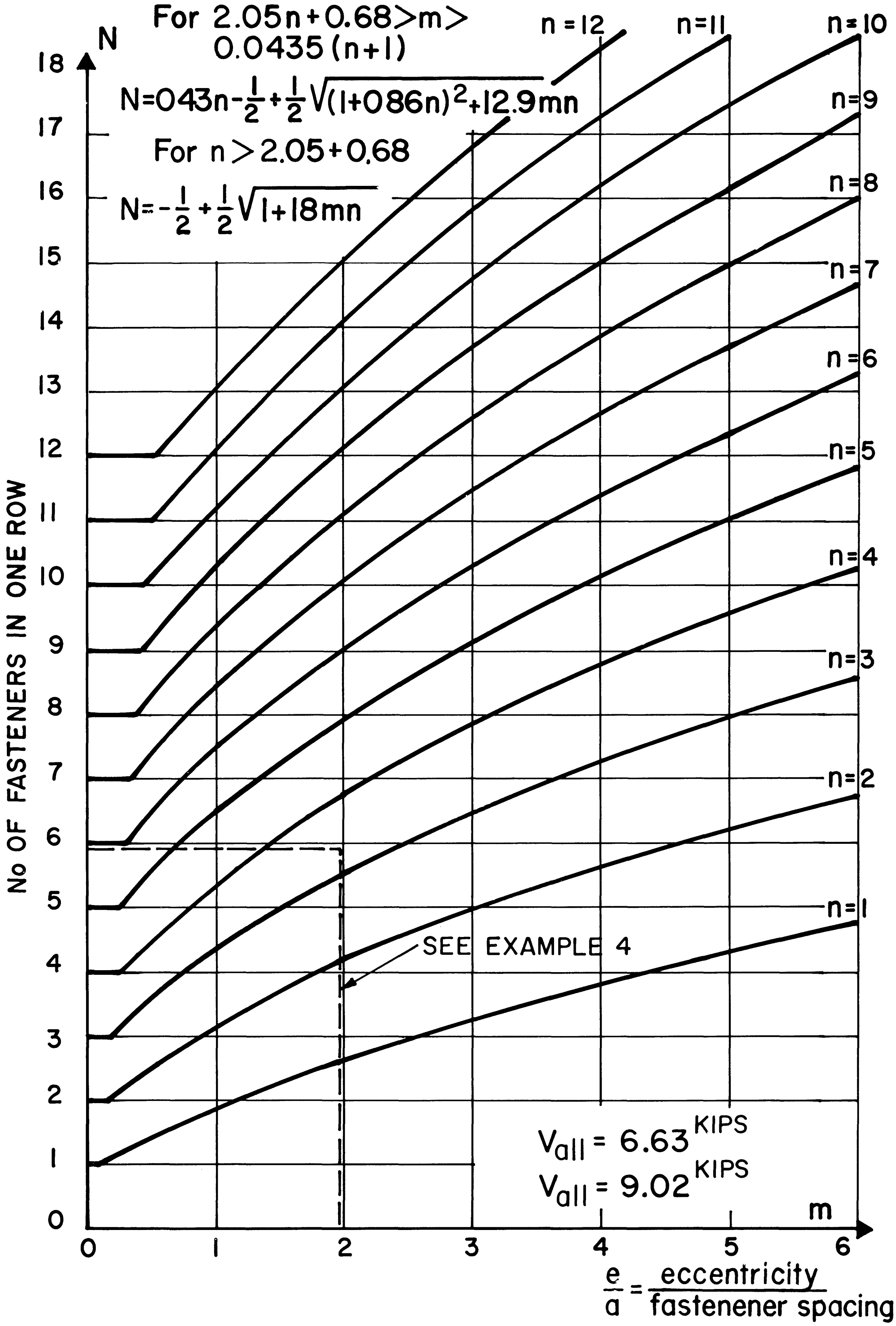
$N = \frac{n-1}{2} + \frac{1}{2}\sqrt{(n+1)^2 + 6.0\,mn}$
N
No OF FASTENERS IN ONE ROW
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
n= 12
n= 11
n= 10
n= 9
n= 8
n= 7
n= 6
n= 5
n= 4
n= 3
n= 2
n= 1
SEE EXAMPLE 2
$V_{all} = 9.02^{KIPS}$
m
0
1
2
3
4
5
6
$\frac{e}{a} = \frac{\text{eccentricity}}{\text{fastener spacing}}$

Table III.



For 3.72n+1.06>m>0.112(n+1)
N=0.352n-1/2+1/2√((1+0.704n)²+10.56mn)
N
No OF FASTENERS IN ONE ROW
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
n=12
n=11
n=10
n=9
n=8
n=7
n=6
n=5
n=4
n=3
n=2
n=1
SEE EXAMPLE 3
Vall = 9.72 KIPS
Vall = 13.23 KIPS
m
e/a = eccentricity/fastener spacing

Table IV.



For 2.05n+0.68>m>
0.0435 (n+1)
N=043n-1/2+1/2√((1+086n)²+12.9mn)
For n>2.05+0.68
N=-1/2+1/2√(1+18mn)
N
No OF FASTENERS IN ONE ROW
n=12
n=11
n=10
n=9
n=8
n=7
n=6
n=5
n=4
n=3
n=2
n=1
SEE EXAMPLE 4
Vall = 6.63 KIPS
Vall = 9.02 KIPS
m
e/a = eccentricity/fastenener spacing