Connection Flexibility and Beam Design in Non-sway Frames

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ABSTRACT

Attention is drawn to the fact that virtually all commonly used types of steelwork beam-to-column connections function as semi-rigid. Representation of this behavior is best achieved by means of the connection's moment-rotation (M–ϕ) curve. For the types of connection normally used in non-sway frames, where "simple framing" is customarily employed, this means true behavior differs from that assumed in design. Beam-end moments are produced, the magnitude of which depends upon the M–ϕ characteristics of the particular connection types used. The implications of this for the design of beams in non-sway frames is discussed.

Connection behavior is reviewed and the results of a series of tests on a variety of types, conducted under identical arrangements, used to illustrate the range of M–ϕ behavior expected. Special tests incorporating fabrication induced imperfections have been included so as to assess the likely variability of M—ϕ data. A series of ultimate strength analyses on laterally supported beams provided with various forms of end connection are reported. The results provide quantitative indications of the ability of end restraint to reduce deflections and to increase load carrying capacity. Some results from a preliminary study of laterally unsupported beams are also presented. These suggest proper allowance for end restraint, which in this case should include both in-plane and out-of-plane components, will also lead to worthwhile economies.

The use of bracing to provide lateral stiffness in multistory steel frames permits the designer to specify "simple" beam-to-column connections, whose primary purpose is the transfer of gravity beam loads by means of shear. In such cases, it is customary to design the beams as simply supported and to consider only nominal moments in the columns arising from out-of-balance beam reactions acting at notional eccentricities to the column centerline. Such an approach has the attraction of extreme simplicity: beams and columns may be proportioned separately, while any connection type capable of transmitting the beam reactions may be selected. This has permitted connection design to be left to the fabricator, thereby enabling him to select the most suitable practical arrangements.

This use of simple framing does, however, lead to certain contradictions and inconsistencies between the structural behavior assumed for the purposes of design and the performance of the structure as built. This is true even when only the steel skeleton is considered, i.e., additional structural interactions with floors, cladding, partitions etc., are neglected. The principal reason for this inconsistency is that all commonly used types of steelwork beam-to-column connections possess some degree of rotational stiffness.[17] This results in some interaction between beams and columns, the exact nature dependent upon the balance of stiffness between the members and the connections.[21] This, in turn, will be a function of load level as the member stiffnesses are affected by plasticity and instability effects, while the joint stiffnesses also vary with a number of different factors.[17]

The tendency worldwide towards the introduction of limit-states-based steelwork codes has created a need for more realistic assessments of the true performance of structures and components. In the case of the latter, research has certainly brought better understanding, much of which is reflected in the new generation of structural steelwork codes. Rules for plate girder design,[9] the multiple column curve concept,[3] new procedures for designing laterally unrestrained beams[18] and more accurate interaction equations for beam-columns[15] are some examples of this.

Parallel advances for assemblies of members or structural systems have, understandably, not progressed as rapidly. Much of the reason for this lies in the need to understand first the behavior of key components, e.g., joints, bracings etc. Now that the attention of researchers has turned more firmly towards these topics, it should be possible to make similar improvements in design approaches for systems.

An important initial step in this direction is the recognition by the AISC LRFD Specification[1] of partially restrained (PR) construction. This acknowledges the ability of connections which are less than fully rigid to transmit

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limited moments and to provide some measure of end restraint. As yet, detailed provisions are not included in the specification. Designers who wish to avail themselves of this opportunity[14] must employ a combination of working from research data, resorting to first principles or making substantial simplifying assumptions. However, it seems likely that, with technical committees of SSRC and ECCS currently working towards the preparation of North American and European design documents, this situation will soon change.

This paper reports on some of the findings of the research conducted in Sheffield during the past two years as it affects the title problem. Much of the Sheffield work has been directed at the rather more difficult column aspects of the subject, and a pair of papers[5,19] describes recent findings. Attention here is focussed on the implications of the fundamental studies for designers wishing to use the concept of PR construction when designing beams more economically than with the present simple approach. Both strength and deflection aspects of laterally restrained members are treated and some preliminary findings for laterally unrestrained beams are also presented.

CONNECTION BEHAVIOR

When seeking to use the PR approach, the single most important aspect of connection behavior is the moment-rotation (M–ϕ) characteristic. This is the relationship between the moment transmitted by the joint and the rotation of the beam relative to the column, i.e., the joint rotation. Figure 1 illustrates typical M–ϕ curves for very flexible, intermediate and very stiff connections. Each is nonlinear with connection rotational stiffness, as indicated by the slope of the curve, which decreases as rotations increase. Despite some attempts at numerical modelling,

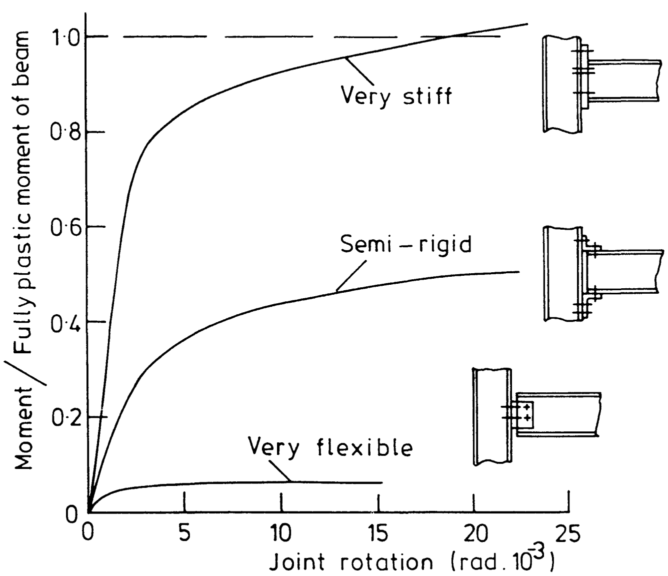

Fig. 1. Typical moment-rotation (M–ϕ) curves

the only really satisfactory way to obtain M–ϕ curves is physical testing at full scale. Of course, this situation may change as finite element approaches improve—some have been encouraging[12,20]—but problems of slip, lack of fit, bolt preload, etc., are not easily modelled theoretically.

Utilization of the existing M–ϕ data—amounting to some 500 individual test histories—requires systematic collection and review of the material.[16] A particularly useful initiative in this area is the work of Goverdhan,[10] which provides digitized versions of many of the curves, as well as an appraisal of suitable curve-fitting expressions. Such expressions must, however, be used with caution and extrapolation beyond the range covered by the test data used as the original basis is not recommended. A safer approach is the use of functions operating between certain limits which may at least be approximated analytically.[25] Further work on evaluation and classification of the existing M–ϕ data is still required before authoritative curves can be provided for designers. This is a prime task of the SSRC/ECCS working groups. One of the most promising approaches would seem to be through a computerized data bank, a pilot version[17] of which is now being converted into an operational tool.

Appraisal of existing M–ϕ data is made difficult by its diversity. It incorporates different test set-ups, beam and column sizes, test methods, techniques to measure connection rotation, bolt tightening procedures, etc. Moreover, test reports frequently omit important details. (Preparation of an SSRC Technical Memorandum on the conduct and reporting of connection M–ϕ tests should alleviate the problem in the future.) The influence of the various joint parameters is hard to assess and the construction of an M–ϕ curve for a particular arrangement—beam, column, angles/plates, bolts, etc.—from existing data requires skill and judgement. Even then it is likely to be approximate. While this may well be all needed for design (assuming system performance is not particularly sensitive to limited variations in connection behavior) it is not suitable for more fundamental work. Accordingly, the Sheffield study generated its own M–ϕ data from in-house tests[6,7] for subsequent use in the analysis and evaluation of the subassemblage[5] and frame tests. The results of these connection tests are of interest in their own right, since they represent comparative data on different joint types tested under identical conditions. This contrasts with the general body of data for which the emphasis has been on a particular joint type with variations in the joint parameters, e.g., end-plate thickness, number of bolts, etc.

Twenty two tests have been conducted using five different types of connection, all of them between a 254×102UB22 beam and a 152×152UC23 column. A cruciform[17] arrangement using pairs of beams was employed. The member cross sections are the same as those used for the later subassemblage and frame tests—hence their small size—and just permit the use of realistic joint arrangements. For the stiffer arrangements, however, the thickness and thus the flexibility of the column flanges, even when heavily stiffened as in the case of the extended end plate tests, meant performance was inferior to that expected from a more balanced member/connection arrangement. The tests were arranged in two series:

1. A main series to produce the basic M–ϕ curves for the connections to be employed subsequently.[6]
2. A subsidiary series to investigate the effect of constructional inaccuracies on M–ϕ behavior.[7]

The end product of the first series is the set of M–ϕ curves shown in Fig. 2. These are results of averaging and smoothing the several individual results produced for each type—the cruciform test arrangement does, of course, provide a pair of results for every test. A full appraisal of the results has been presented elsewhere[6] and the following selected findings are adjudged of greatest interest in the present context:

1. Web angles were capable of generating only a small fraction of the beam's moment capacity—6% at a joint rotation of 0.01 rad. Only after the beam flange came into direct bearing with the column face (at unacceptably large rotations) was significant rotational stiffness developed. The initial, steeper part of the M–ϕ curve in Fig. 2 persisted for such small rotations that it is unlikely to be of real assistance in enhancing beam strength—although for columns that attain maximum load at very small end rotations even this seemingly unimpressive M–ϕ behavior can have substantial benefits.[5,19] Because of possible beam rotation due to bolt slip, bolt tightness affects M–ϕ behavior more than in any of the other connection types tested, an observation previously made in the work of Richard[20] on single web angle connections.
2. Both flange-angle and combined seat and web-angle connections exhibited an approximately bilinear M–ϕ response, with the initial stiffness maintained up to a moment of about 15% of the beam capacity.
3. Results for flush end-plate connections were affected significantly by column flange flexibility, with the absence of this component accounting for the enhanced performance of the connections to the column web. Moments of approximately 30 and 60% of beam M_p were achieved at joint rotations of 0.01 rad. These values are clearly significant in the context of design approaches which treat such joints as shear only connections. Omission of the flange welds on a full depth end plate had negligible effect on M–ϕ behavior.

If connection stiffness is to be utilized when assessing frame performance, it is important that minimum valves be present, even if the connections have been poorly produced. Tests in which deliberate imperfections have been introduced have been conducted[7] and results compared with the equivalent perfect series.[6] Full details are available elsewhere.[7] The most significant findings are:

1. Slip in angle connections due to the use of oversize holes has little effect on moment capacity, but does increase connection flexibility. If such connections are relied on to transfer rotational restraint, one possibility

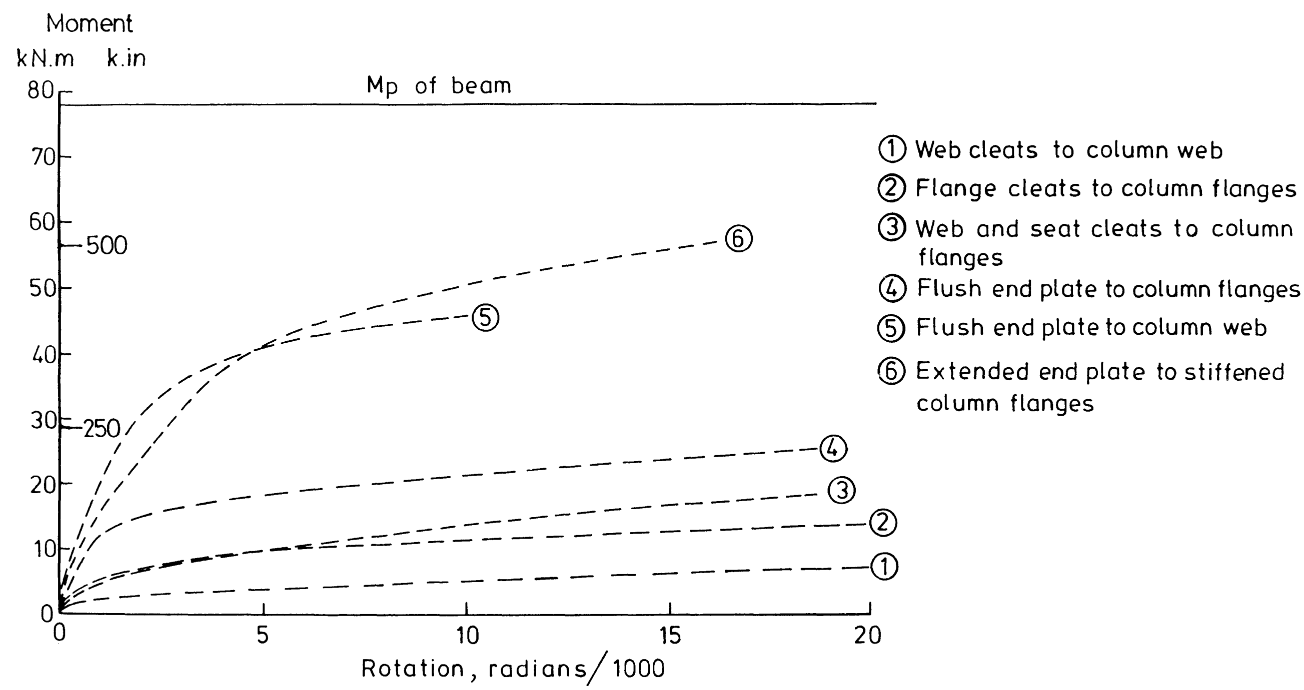

Fig. 2. M–ϕ curves for range of connections

would be to use fully torqued bolts, an approach that has already been shown to have the desired effect for single-angle connections by Richard.[11]

2. For either flush or extended end-plate connections, plate distortion of the type produced during welding had negligible detrimental effect on M–ϕ performance. The use of fully torqued or ordinary bolts made little difference, and efforts to close gaps by excessive bolt tightening in an attempt to improve connection performance would appear to be unwise.

INFLUENCE OF CONNECTION STIFFNESS ON BEAM BEHAVIOR

Figure 3 illustrates the three cases of simply supported, partially restrained and fixed ended beams, showing how both the maximum moment and the central deflection are reduced by end fixity when elastic behavior is assumed. The extent to which similar gains may be achieved when behavior up to failure is considered and the ability of typical forms of beam to column connection to supply sufficient restraint to induce worthwhile gains has been investigated in a limited numerical study.

The problem considered is illustrated in Fig. 4. A 305×127UB48 in steel with a yield strength of 40 ksi ($275 N/mm^2$) with symmetrical end restraints has been considered for span/depth (L/D) ratios of 15,20 and 25. M–ϕ curves for five different connection types, including the upper and lower bounds of fixed and simply supported, have been constructed based on the work of Goverdhan,[10] shown in Fig. 5. The three intermediate types have been selected on the basis that the extended end plate approximates full fixity, double-web angles would be considered as simply supported and that the flush end-plate (which is extremely popular in the UK) would be designed to supply only simple support despite overwhelming evidence[10,16] that it possesses considerable rotational stiffness.

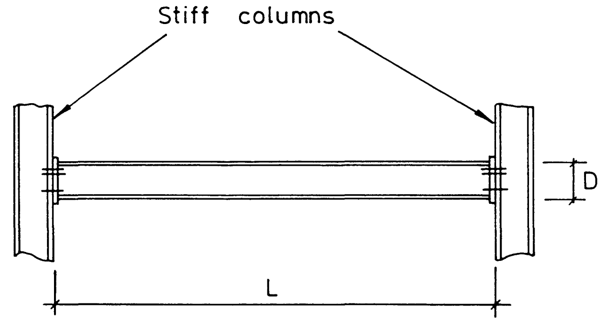

Fig. 4. Arrangement for numerical study of end-restrained beams

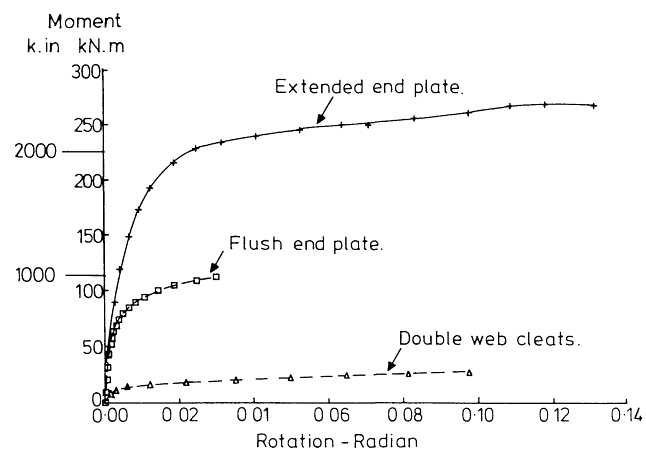

Fig. 5. M–ϕ curves used in numerical study for 305×127UB48

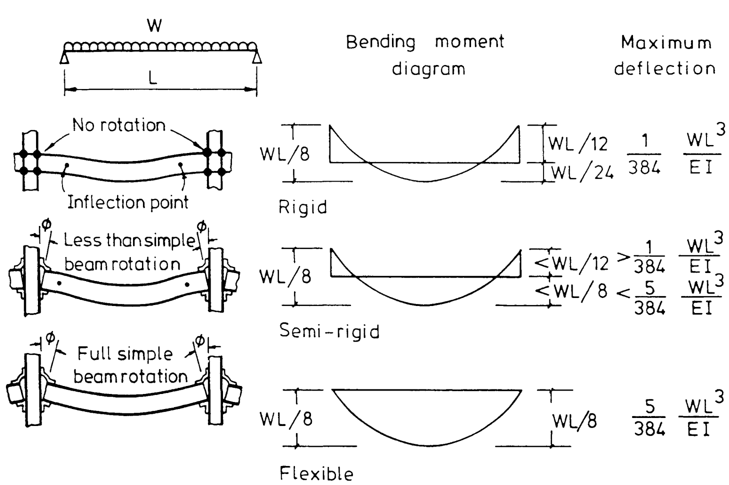

Fig. 3. Effect of end restraint on moments and deflections for elastic response

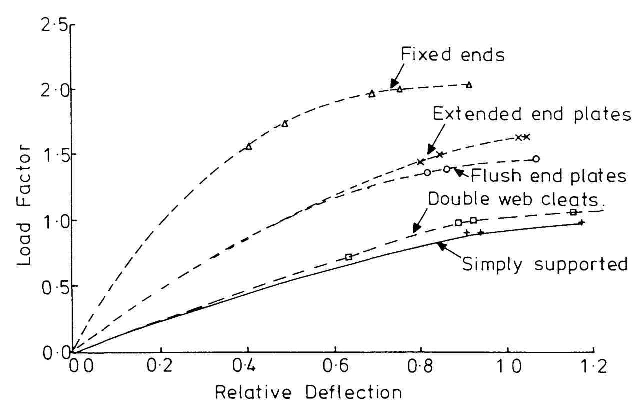

Fig. 6. Effect of connection restraint on beam deflections

A series of numerical analyses have been conducted using a program prepared for frames with semi-rigid joints[26] made available by the Politecnico di Milano. Figure 6 presents a set of five load-deflection curves for one arrangement. From this it is immediately clear that the web angles are only slightly better than simple supports, that the extended end-plate functions as significantly less than rigid and the flush end-plate is almost as effective as the more complex extended variant. Similar behavior was obtained for the other span/depth ratios studied. To check whether absolute size was significant, all results were repeated using a 610×229UB101 with a suitable set of equivalent M–ϕ curves derived from Ref. 10. Figure 7 presents the six load-deflection curves for the extended end-plate case, which exhibited the largest variation between individual results. Clearly some differences will occur as the balance of stiffness between restraint and beam is changed, i.e., longer beams are more flexible. However, viewed in the context of design and noting particularly that the particular M–ϕ curves used are *representative*, i.e., small changes in connection details or even repeat tests on notionally identical joints will cause the M–ϕ curve to shift, the spread is not regarded as large.

The sensitivity of beam behavior to changes in the assumed M–ϕ characteristic of the connections has been studied by repeating selected analyses assuming a 10% shift in the M–ϕ curve. The results presented in Fig. 8 are typical of those obtained in every case. Only in the case of the stiffer connections was any difference noticeable and even then changes in ultimate load were still less than 3%. This suggests behavior will be relatively unaffected by the sort of changes likely to be found between samples of similar connections.

The complete set of results showed average percentage improvements in ultimate load carrying capacity of 4, 41, 61 and 100 for web angles, flush end plates, extended end plates, and fixed-end condition respectively. Clearly, for all practical purposes, double-web angles may be regarded as simple connections. Examination of the rotations at ultimate load (taken as being attained when the load-deflection relationship became horizontal) gave values of approximately 14, 21 and 43 rads × 10^{-3} for L/D ratios of 15, 20 and 25 respectively. Clearly, from Fig. 3, these are sufficiently large for the connection to be operating on the flat portion of its M–ϕ curve with the result that little rotational restraint is available. For both forms of end plate the equivalent rotations of 11, 14 and 21 rads × 10^{-3} and 7, 9 and 16 rads × 10^{-3} correspond to the availability of some stiffness from the connections, particularly in the extended end plate case.

The improved response due to end restraint is based solely on considerations of strength. However, a limit-states approach to design also requires that deflections under serviceability conditions be acceptable. Clearly, a quantitative definition of acceptable will depend upon circumstances and this is consistent with one feature of recent codes being a tendency to provide advisory or recommended serviceability limits. To gain some idea of the extent to which the strength gains due to end restraint may be eroded because of prior attainment of a serviceability deflection limit, a simple check on working load deflections has been applied to the results. Using the criterion $\delta_{max} \not> L/360$, where δ_{max} is due to live load only, and assuming a 1:1 mix of live and dead load and a load factor at ultimate of 1.5, it was found no case was governed by serviceability deflections.

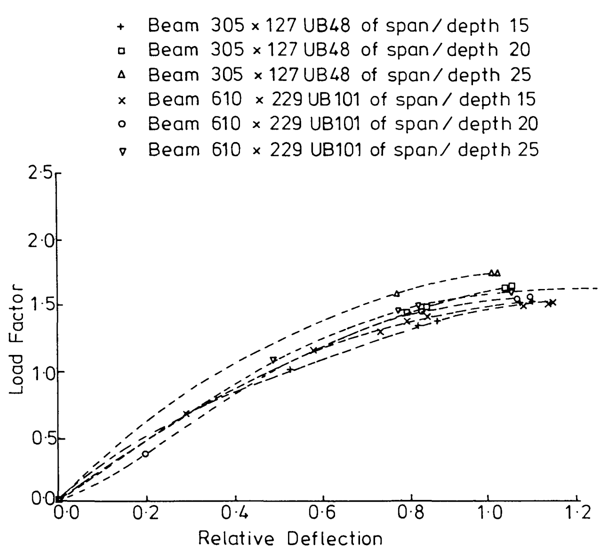

Fig. 7. Variability of beam response due to change in section size and span/depth ratio

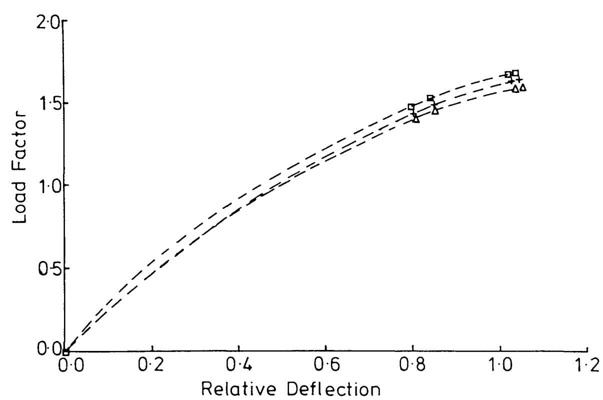

Fig. 8. Effect of 10% shift in M–ϕ characteristic on beam response

Table 1. Lightest beam sections capable of carrying 1.8 ton/ft (60 kN/m) over spans indicated Span in ft (m)

End conditions	Span L ft (m)					
	13 (4)	20 (6)	26 (8)	33 (10)	39 (12)	46 (14)
Simply supported	356×171UB45	406×178UB67	533×210UB101	610×229UB140	762×267UB173	914×305UB210
Double web cleats	356×127UB39	457×152UB60	533×210UB101	686×254UB125	686×254UB170	838×292UB194
Flush end plate	356×127UB33	406×140UB52	457×152UB82	610×229UB101	610×229UB101	686×254UB170
Extended end plate	305×102UB28	406×140UB46	457×152UB74	533×210UB101	610×229UB125	686×254UB152
Fully fixed	305×102UB25	406×140UB25	406×178UB67	533×210UB82	610×219UB113	610×229UB140

Table 2. Percentage weight savings compared with "simply supported" due to inclusion of end fixity in design

End conditions	Span L ft (m)					
	13 (4)	20 (6)	26 (8)	33 (10)	39 (12)	46 (14)
Double web cleats	13	10	0	11	2	3
Flush end plate	27	22	19	28	19	19
Extended end plate	38	31	27	28	28	24
Fully fixed	44	42	34	57	35	30

Based on the above numerical study the following average beam end moments are suggested as representative for each of the connection types studied:

Double-web angles	8% of beam M_p
Flush-end plate	45% of beam M_p
Extended-end plate	60% of beam M_p

Of course these values have been influenced by the types of connection chosen for the M–ϕ data used in the analyses. In the case of the extended end plate, selection of a more heavily stiffened version would probably have led to a greater proportion of M_p being developed, although not necessarily the full value. Assuming such moments be generated, a series of beam designs have been conducted for a standard load of 1.80 ton/ft (60 kN/m) and a range of spans of 13 to 46 ft (4 to 14m). Table 1 gives the appropriate sections for each case, while Table 2 lists percentage benefits in terms of steel weight. Clearly, the results are affected by the availability of a discrete range of sections. Average benefits are:

Double-web angles	6%
Flush end plate	22%
Extended end plate	29%

These suggest the gains from relying on the end restraint provided by very flexible connections such as double-web angles are probably insufficient to warrant being treated as other than simple supports. Flush end plates, which are the most popular form of beam-to-column connection in the UK, however, provide sufficient restraint for its inclusion in beam design to be worthwhile. The ability of extended end plates of the type considered here to function as rigid appears highly questionable (see Fig. 6). If such connections have to be regarded as semi-rigid, the additional fabrication required, together with the inconvenience of using a joint that extends beyond the beam depth, would seem to point to a change to a flush end-plate arrangement.

LATERALLY UNSUPPORTED BEAMS

In the study just described, it was assumed beams were provided with full lateral restraint so only in-plane bending need be considered. In the absence of such support, failure by lateral-torsional buckling is likely.[18] Although this problem has received considerable study, almost no work has been conducted in which the restraining effects of realistic end connections has been included. It is convenient to separate the influence of end restraint into two parts:

1. In-plane rotational restraint—this alters the effective pattern of moments.
2. Out-of-plane restraint, i.e., against lateral bending, twisting and warping—this acts directly to increase stability.

While the second of these is, of course, analogous to the effect of rotational restraint at the ends of a strut, the first is peculiar to the beam buckling problem for which lateral stability is affected significantly by the pattern of moment within the beam segment under consideration.[18]

Some quantitative indication of the importance of the first effect may be obtained by checking the lateral stability of one of the previous examples using the pattern of

Table 3. Effect of end restraint on lateral buckling strength of 305 × 127 UB 48 for L/D = 20 according to UK Code

End conditions	Effective Length Factor	Moment Pattern Factor n	Effective Slenderness	Buckling Moment as Percentage of M_p
Simply supported	1	0.94	119	0.35
Double web cleats	1	0.93	118	0.36
Flush end plate	1	0.89	113	0.38
	0.85	0.89	102	0.44
Extended end plate	1	0.57	72	0.66
	0.7	0.57	58	0.80
Fully fixed	1	0.50	63	0.75
	0.7	0.50	51	0.86

moments at in-plane collapse as produced by the five different forms of end support. To do this, it is necessary to use a design approach for laterally unsupported beams which includes an explicit allowance for the pattern of moments. For convenience, that contained in the new British Code[4] has been selected. This uses a factor n, the value of which depends directly upon the moment diagram, to modify the geometrical slenderness. Table 3 lists the results of this exercise, showing how the development of larger end moments by the stiffer connections progressively reduces the effective slenderness leading to larger load-carrying capacities.

Inclusion of out-of-plane restraint requires a knowledge of the restraining ability of connections. Only one study in which torsional characteristics were measured[2] and one piece of elastic analysis for warping restraint[24] are known. A rigorous study clearly requires that suitable joint data be available, a topic on which research is required. Once again indicative, design-type calculations may be performed to provide some idea of potential benefits. This requires judgements be made on the ability of different connection types to provide either "full" or "partial rotational restraint in plan."[4] In this context, joints involving positive attachment to the beam flanges, e.g., flange cleats, are likely to come within the second category, with full restraint being provided if such joints really inhibit lateral rotation, e.g., extended end plates. In this context it is, of course, important to consider the flexibility of the complete end support arrangement; a heavy connection to a light column will clearly lose much of its effectiveness. Making conservative assessments for each connection type and using the design approach of the British Code[4] to select effective length factors less than 1.0 leads to the beam capacities of Table 3. These show substantial benefits for the heavier forms of connection, e.g., allowing for both forms of restraint provided by extended end plates more than doubles the beam's buckling resistance.

The provision of specific guidance on end restraint effects on laterally unsupported beams requires the topic be considered fully at the fundamental level. Work by Trahair[22,23] was limited to elastic buckling, assuming all restraining actions to be represented by elastic springs. Tests by Hechtmann, et al,[11] on beams provided with web angles or top and seat angles did show improved performance over equivalent specimens with ideal "simple supports," but the range was limited and evaluation of the tests is complicated by a lack of full details. A small study by Lindner and Gietzelt[13] on beams provided with end plates has included both elastic buckling analysis and a few tests. The existing body of knowledge is therefore sparse.

Work at Sheffield has just started on the ultimate strength analysis of laterally unsupported beams provided with end restraint. It represents an extension to the work of El-Khenfas[8] on biaxial bending and torsion of beam-columns. So far, only in-plane rotational restraint (M–ϕ) has been fully incorporated, since only for this characteristic is a proper understanding of joint behavior available. Parallel experimental studies to measure connection out-of-plane stiffness properties are also in hand. The analysis has been applied to the tests of Ref. 24, using the background gained from previous studies to devise M–ϕ curves for the two types of connection employed.

A selection of results is given in Tables 4 and 5. In both cases, four results have been obtained for each test beam: a pair assuming simply supported end conditions in the vertical plane and a second pair allowing for end restraint against in-plane bending. For the out-of-plane end conditions twisting has been assumed to be prevented, no restraint to be present against lateral (horizontal) bending and either complete freedom to warp or prevention of warping. The last option has been included as an attempt to gain some indication of the effect of out-of-plane restraint. In every case, the test result is bracketed by the pair of values determined from the analyses which allow for the presence of in-plane restraint. Moreover, for each case involving flange cleats as well as for the shortest beam provided with web angles both analyses assuming simple supports underestimate the test value.

For the flange angle arrangement, for which a greater degree of warping restraint together with some degree of lateral bending restraint may be expected, the theoretical results which include full warping restraint provide a better estimate, whereas for the web angles, which provide rather less out-of-plane restraint, those which neglect warping restraint are closer to the experimental values.

Table 4. Analytical results for laterally unsupported beams: comparison with tests of Ref. 11 for web cleat connections

Beam No.	L/r_y	Test	Maximum Loads (kips) Simply Supported E.W.F.*	Simply Supported E.W.P.+	With End Restraints E.W.F.*	With End Restraints E.W.P.+
47	110	40.7	31.6	40.4	35.5	44.0
49	147	23.5	18.0	26.9	20.4	29.7
51	221	11.2	8.2	11.8	9.8	13.7
53	294	6.4	4.7	6.7	5.9	8.2

E.W.F.* stands for end warping free
E.W.P.+ stands for end warping prevented

Table 5. Analytical results for laterally unsupported beams: comparison with tests of Ref. 11 for flange cleat connections

Beam No.	L/r_y	Test	Maximum Loads (kips) Simply Supported E.W.F.*	Simply Supported E.W.P.+	With End Restraints E.W.F.*	With End Restraints E.W.P.+
37	110	46.7	33.4	44.6	40.7	53.8
39	147	32.7	19.8	29.5	25.3	36.2
41	221	14.7	8.2	12.4	11.8	16.7
43	294	9.1	4.6	6.6	7.4	9.7

E.W.F.* stands for end warping free
E.W.P.+ stands for end warping prevented

However, really close agreement cannot be expected for either series of tests, as in addition to the needs to estimate the in-plane M–ϕ curve and the present inability to properly model out-of-plane restraint, assumptions were necessary in the analyses for the initial lack of straightness in the beams (a bow of $L/1000$ was used) and for the pattern of residual stress present (a Lehigh-type distribution with a maximum compressive stress at the flange tips of 30% of yield was used). While the sensitivity of the results to reasonable variations in the whole set of input parameters has not been checked, calculations in which the M–ϕ characteristic for each connection type has been displaced horizontally by 10%, i.e., stiffness increased or decreased, has been found to alter the buckling load by no more than 2% in any case.

Bearing these points in mind, the results of Tables 4 and 5 are thought to be most encouraging, confirming the trends from the design-type calculations of Table 3 that inclusion of end restraint effects in lateral buckling strength assessments is worthwhile. The straight comparison between the simply supported and restrained theoretical results for both warping conditions given in Table 6 further supports this view, indicating the in-plane restraint provided by web angles provides between 10 and 20% additional capacity, while the stiffer flange angles produce improvements of between 20 and 50%. These figures actually suggest that the code-based calculations of Table 3 may well underestimate the benefits. The topic is therefore being pursued.

CONCLUSIONS

The restraining characteristics of steel beam-to-column connections have been discussed. And it has been suggested the inclusion of the PR category in the latest AISC Specification represents a correct appreciation of their real behavior. Test data for a range of connections, obtained from a single series, have been presented to show the range of M–ϕ behavior obtainable with several commonly used types. Fabrication induced imperfections have been found to have only a moderate effect on the performance of shear-type connections and very little effect on end-plate connections.

Numerical studies of the effect of end restraint on the behavior of beams as supplied by various connection types have been reported. These show reduced deflections and enhanced maximum loads when compared with the assumption of simple support. Worthwhile economies for laterally supported beams are therefore possible if the

Table 6. Improvements in lateral stability due to in-plane end restraint

Beam No.	L/r_y	Percentage gain in load-carrying capacity due to in-plane restraint E.W.F.*	E.W.P.+
47	110	12.2	8.7
49	147	13.6	10.5
51	221	19.6	15.9
53	294	24.9	20.3
37	110	21.9	20.6
39	147	26.4	21.1
41	221	43.9	35.0
43	294	58.7	46.9

E.W.F.* stands for end warping free
E.W.P.+ stands for end warping prevented

form of end restraint actually present in current forms of construction but normally neglected in design is properly allowed for. Some extensions to cover laterally unsupported beams have also been reported. Despite a lack of full input data to permit rigorous study, it would appear similar benefits may be obtained by allowing for restraint effects presently ignored in design.

ACKNOWLEDGEMENTS

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APPENDIX I—REFERENCES

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