

# Effect of Connector Spacing and Flexural-torsional Buckling on Double-angle Compressive Strength

C.J. ZAHN and GEERHARD HAAIJER

The 1986 AISC *Load and Resistance Factor Design (LRFD) Specification for Structural Steel Buildings*<sup>1</sup> includes new rules affecting the design of built-up members composed of rolled shapes. These rules are based on the result of recent studies which indicate connector spacing has a direct effect on the strength of a built-up member buckling out of the plane separating the individual members; for example, about the Y-Y axis for two back-to-back angles (Fig. 1). Various test results have provided the necessary information to develop a formula for a "modified" slenderness ratio,  $K\ell/r_m$ . The application of  $K\ell/r_m$  is straightforward, as illustrated in the 1st Edition of the *Load and Resistance Factor Design Manual of Steel Construction*<sup>1</sup> where new design aids are available to assist in determining the number of connectors and the accompanying design strength for double angles. The LRFD Specification also includes formulas for flexural-torsional buckling, which are used to determine the final design strength.

## BACKGROUND

The LRFD Specification provisions for the modified slenderness ratio were developed from recent research by Zandonini<sup>2</sup> and verified with studies by Astaneh, Goel and Hanson<sup>3</sup> and Libove.<sup>4</sup> A study of these research results is presented in Ref. 5. The following is a summary of the conclusions:

Zandonini performed two series of compression tests on built-up struts consisting of two back-to-back channels with intermediate welded or snug-tight bolted filler plates (see Figs. 2 and 3). All of the end connections were made with fully tightened bolts. The test results, summarized in Tables A and B, were compared to the theoretical predictions made by the Structural Stability Research Council (SSRC).

Geerhard Haaijer is Vice President, Technology and Research, AISC, Chicago, Illinois.

C.J. Zahn is Staff Engineer-Structures, AISC, Chicago, Illinois.

Table A gives results for the specimens with welded interconnectors. As indicated in Col. 11 for local slenderness  $a/r_i$  not exceeding 50, test results were at least 98% of the predicted values. Based on a formula used in Euro-

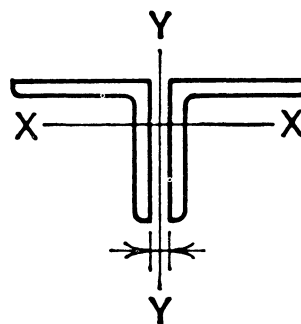


Fig. 1 Orientation of axes

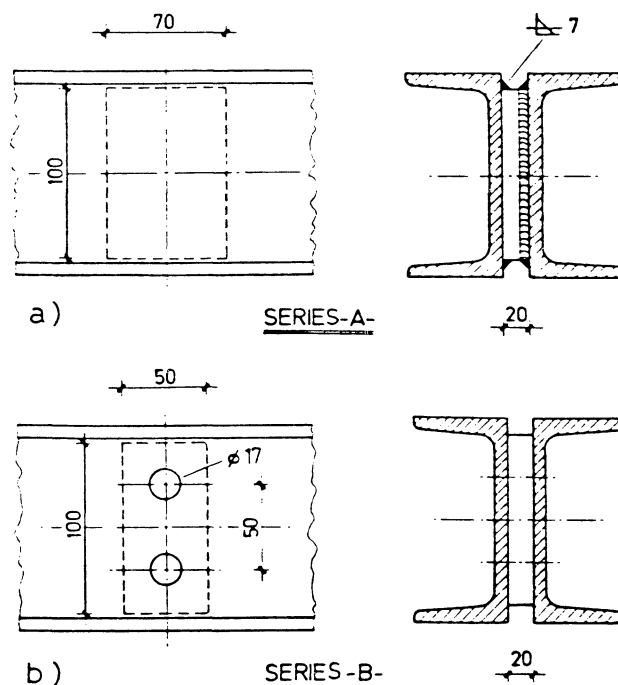


Fig. 2. Connection details of Zandonini tests

**Table A. Analysis of Zandonini Test of Welded Built-up Columns**

| Specimen    | $n_o$         | $L_o$<br>(mm) | $L_o/r$ | $a/r_i$ | Col. 5<br>Col. 4 | $v_o/L_o$        | $P_{c,s}$<br>(kN) | SSRC 2         |              |                   | $\lambda_c$ | $(L_o/r_m)$ | $\lambda_{cm}$ |
|-------------|---------------|---------------|---------|---------|------------------|------------------|-------------------|----------------|--------------|-------------------|-------------|-------------|----------------|
|             |               |               |         |         |                  |                  |                   | $P_{c,s}/P_y$  | $P_{cr}/P_y$ | Col. 9<br>Col. 10 |             |             |                |
| (1)         | Panels<br>(2) | (3)           | (4)     | (5)     | (6)              | (7)              | (8)               | (9)            | (10)         | (11)              | (12)        | (13)        | (14)           |
| A1-R*<br>-N | 3             | 2349          | 76      | 40      | 0.53             | .00084<br>.00119 | 710.6<br>722.0    | 0.602<br>0.612 | 0.615        | 0.98<br>1.00      | 0.993       | 76          | 0.993          |
| A2-R<br>-N* | 3             | 3282          | 107     | 60      | 0.56             | .00083<br>.00071 | 484.5<br>460.9    | 0.411<br>0.391 | 0.388        | 1.06<br>1.01      | 1.399       | 107         | 1.399          |
| A3-R*<br>-N | 3             | 4144          | 135     | 80      | 0.59             | .00061<br>.00088 | 295.8<br>298.9    | 0.251<br>0.253 | 0.277        | 0.91<br>0.91      | 1.765       | 138         | 1.804          |
| A4-R*<br>-N | 3             | 5032          | 164     | 100     | 0.61             | .00088<br>.00091 | 200.4<br>206.6    | 0.170<br>0.175 | 0.200        | 0.85<br>0.88      | 2.144       | 171         | 2.236          |
| A5-R*<br>-N | 5             | 3282          | 107     | 36      | 0.34             | .00092<br>.00099 | 500.9<br>485.5    | 0.424<br>0.411 | 0.388        | 1.08<br>1.05      | 1.399       | 107         | 1.399          |
| A6-R*<br>-N | 5             | 4144          | 135     | 48      | 0.36             | .00098<br>.00088 | 321.5<br>330.4    | 0.272<br>0.280 | 0.277        | 0.98<br>1.01      | 1.765       | 135         | 1.765          |
| A7-R*<br>-N | 5             | 5032          | 164     | 60      | 0.37             | .00069<br>.00095 | 226.6<br>226.4    | 0.192<br>0.192 | 0.200        | 0.96<br>0.96      | 2.144       | 164         | 2.143          |

\* Specimens equipped with strain gages

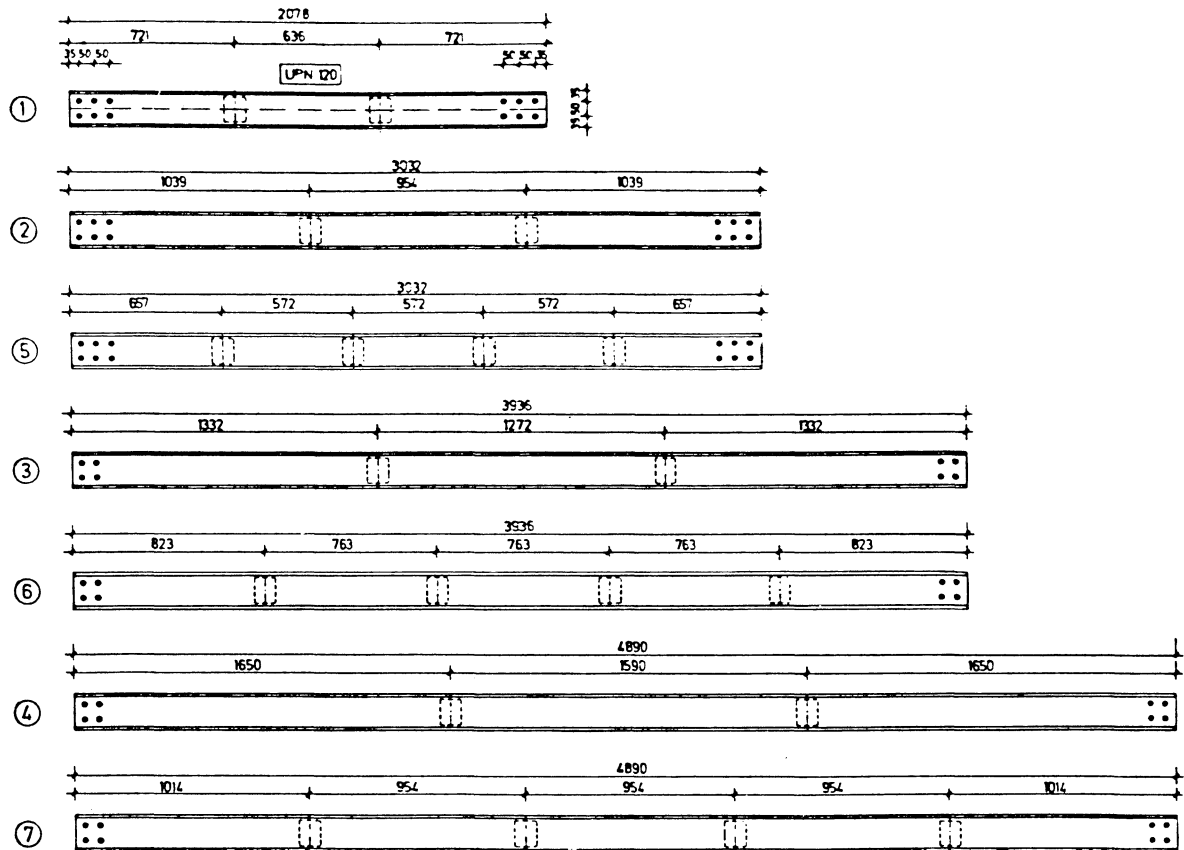


Fig. 3. Test specimens for Zandonini tests

**Table B. Analysis of Zandonini Test of Bolted Built-up Columns**

| Specimen | $n_o$ | $L_o$ | $L_o/r$ | $a/r_i$ | $\frac{\text{Col. 5}}{\text{Col. 4}}$ | $v_o/L_o$ | $P_{c,s}$ | $P_{c,s}/P_y$ | $P_{cr}/P_y$ | SSRC 2                                 |                                        | $\lambda_c$ | $(L_o/r)_m$ | $\lambda_{cm}$ | SSRC 2           |                                        |
|----------|-------|-------|---------|---------|---------------------------------------|-----------|-----------|---------------|--------------|----------------------------------------|----------------------------------------|-------------|-------------|----------------|------------------|----------------------------------------|
|          |       |       |         |         |                                       |           |           |               |              | $\frac{\text{Col. 9}}{\text{Col. 10}}$ | $\frac{\text{Col. 9}}{\text{Col. 10}}$ |             |             |                | $(P_{cr}/P_y)_m$ | $\frac{\text{Col. 9}}{\text{Col. 15}}$ |
| (1)      | (2)   | (3)   | (4)     | (5)     | (6)                                   | (7)       | (8)       | (9)           | (10)         | (11)                                   | (12)                                   | (13)        | (14)        | (15)           | (16)             | (16)                                   |
| A1-R     | 3     | 2349  | 76      | 40      | 0.53                                  | .00079    | 730.6     | 0.633         | 0.615        | 1.03                                   | 0.993                                  | 86          | 1.124       | 0.525          | 1.21             | 1.21                                   |
| -N       |       |       |         |         |                                       | .00135    | 692.2     | 0.601         |              | 0.98                                   |                                        |             |             |                | 1.14             |                                        |
| A2-R     | 3     | 3282  | 107     | 60      | 0.56                                  | .00104    | 368.3     | 0.307         | 0.388        | 0.79                                   | 1.399                                  | 123         | 1.608       | 0.319          | 0.96             | 0.96                                   |
| -N       |       |       |         |         |                                       | .00099    | 421.3     | 0.351         |              | 0.90                                   |                                        |             |             |                | 1.10             |                                        |
| A3-R     | 3     | 4144  | 135     | 80      | 0.59                                  | .00094    | 259.7     | 0.227         | 0.277        | 0.82                                   | 1.765                                  | 157         | 2.053       | 0.217          | 1.05             | 1.05                                   |
| -N       |       |       |         |         |                                       | .00091    | 285.8     | 0.250         |              | 0.90                                   |                                        |             |             |                | 1.15             |                                        |
| A4-R     | 3     | 5032  | 164     | 100     | 0.61                                  | .00101    | 173.4     | 0.149         | 0.200        | 0.74                                   | 2.144                                  | 192         | 2.510       | 0.149          | 1.00             | 1.00                                   |
| -N       |       |       |         |         |                                       | .00120    | 171.1     | 0.147         |              | 0.74                                   |                                        |             |             |                | 0.99             |                                        |
| A5-R     | 5     | 3282  | 107     | 36      | 0.34                                  | .00099    | 445.0     | 0.358         | 0.388        | 0.91                                   | 1.399                                  | 113         | 1.477       | 0.360          | 0.99             | 0.99                                   |
| -N       |       |       |         |         |                                       | .00082    | 416.3     | 0.334         |              | 0.85                                   |                                        |             |             |                | 0.93             |                                        |
| A6-R     | 5     | 4144  | 135     | 48      | 0.36                                  | .00092    | 290.2     | 0.252         | 0.277        | 0.91                                   | 1.765                                  | 143         | 1.870       | 0.253          | 1.00             | 1.00                                   |
| -N       |       |       |         |         |                                       | .00120    | 264.9     | 0.230         |              | 0.83                                   |                                        |             |             |                | 0.91             |                                        |
| A7-R     | 5     | 5032  | 164     | 60      | 0.37                                  | .00099    | 197.5     | 0.170         | 0.200        | 0.85                                   | 2.144                                  | 175         | 2.289       | 0.176          | 0.97             | 0.97                                   |
| -N       |       |       |         |         |                                       | .00093    | 190.1     | 0.164         |              | 0.82                                   |                                        |             |             |                | 0.94             |                                        |

pean codes, the LRFD modified slenderness for  $a/r_i > 50$  can be expressed as:

$$\left(\frac{KL}{r}\right)_m = \left(\frac{KL}{r}\right)_o^2 + \left(\frac{a}{r_i} - 50\right)^2 \quad (1)$$

Figure 4 is a plot of  $\lambda_c$ , where

$$\lambda_c = \frac{L_o}{r \pi} \sqrt{\frac{F_y}{E}}, \quad (2)$$

vs.  $P_{cr}/P_y$ , a normalized version of the test results. The shift from  $\lambda_c$  to  $\lambda_{cm}$  is indicated by the arrows.

The results for the snug-tight bolted specimens are given in Table B. Again, the ratio of test results to predictions is given in Col. 11. In this case, the increased flexibility resulting from slip of the snug-tight bolted joints results in ratios as low as 0.74. For this reason, the more

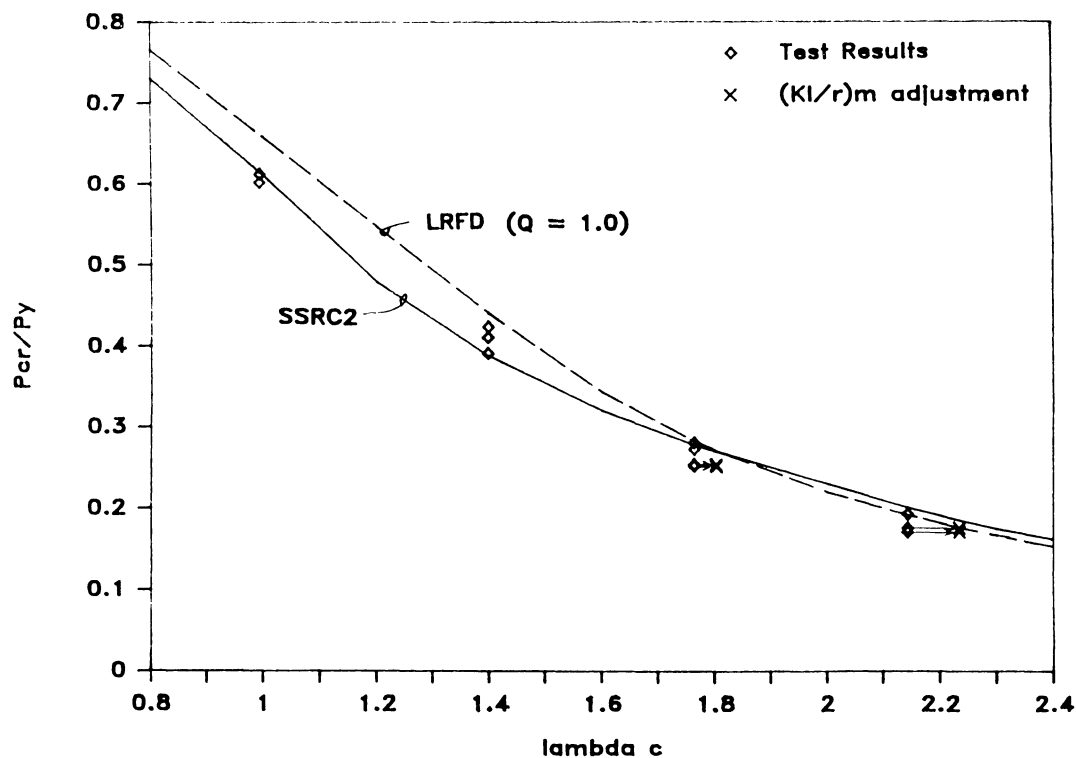


Fig. 4. Zandonini welded specimens comparison of test results

penalizing European code formula is used for all values of  $a/r_i$ ,

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{a}{r_i}\right)^2} \quad (3)$$

As Fig. 5 shows, this modification brings the test points much closer to the predictions of the SSRC2 curve.

From the Zandonini research, it can be concluded a reduction in column strength based on a modified slenderness ratio may be necessary for back-to-back channels. For welded interconnectors, the modified slenderness is determined from Eq. 1. This equation also applies to fully tightened bolts as required for slip-critical connections. Eq. 3 is used for snug-tight bolted interconnectors.

#### Astaneh, Goel, Hanson Tests

The intent of this study was to investigate the cyclic behavior of double-angle struts. However, the first load cycle provides important data for checking the validity of the modified slenderness equations introduced in the LRFD Specification for built-up members.

These tests were performed on double angles with snug-tight-bolted, fully tightened-bolted and welded connec-

tions. The attached Table C gives the pertinent test results for comparison with the LRFD predictions for basic column strength. Applying the appropriate equations to the test results demonstrates a fairly uniform degree of conservatism. Figure 6 indicates the shift that occurs because of modified slenderness adjustment.

#### Libove Analysis

In this paper, Libove questions the actual strength available when only one interconnector is provided in built-up columns, based on the rule that the local slenderness ratio cannot exceed the overall slenderness ratio. The results of an elastic analysis to obtain the initial buckling load (see Ref. 6 also) for this case, reveal that the integral buckling strength may not be an accurate representation of the actual strength available. Using the Libove formulation, a comparative study of double-angle members requiring only one intermediate connector indicates the LRFD method with  $(KL/r)_m$  gives strength values consistent with Libove's.

#### SPECIFICATION PROVISIONS

The above discussion of recent test data demonstrates it is

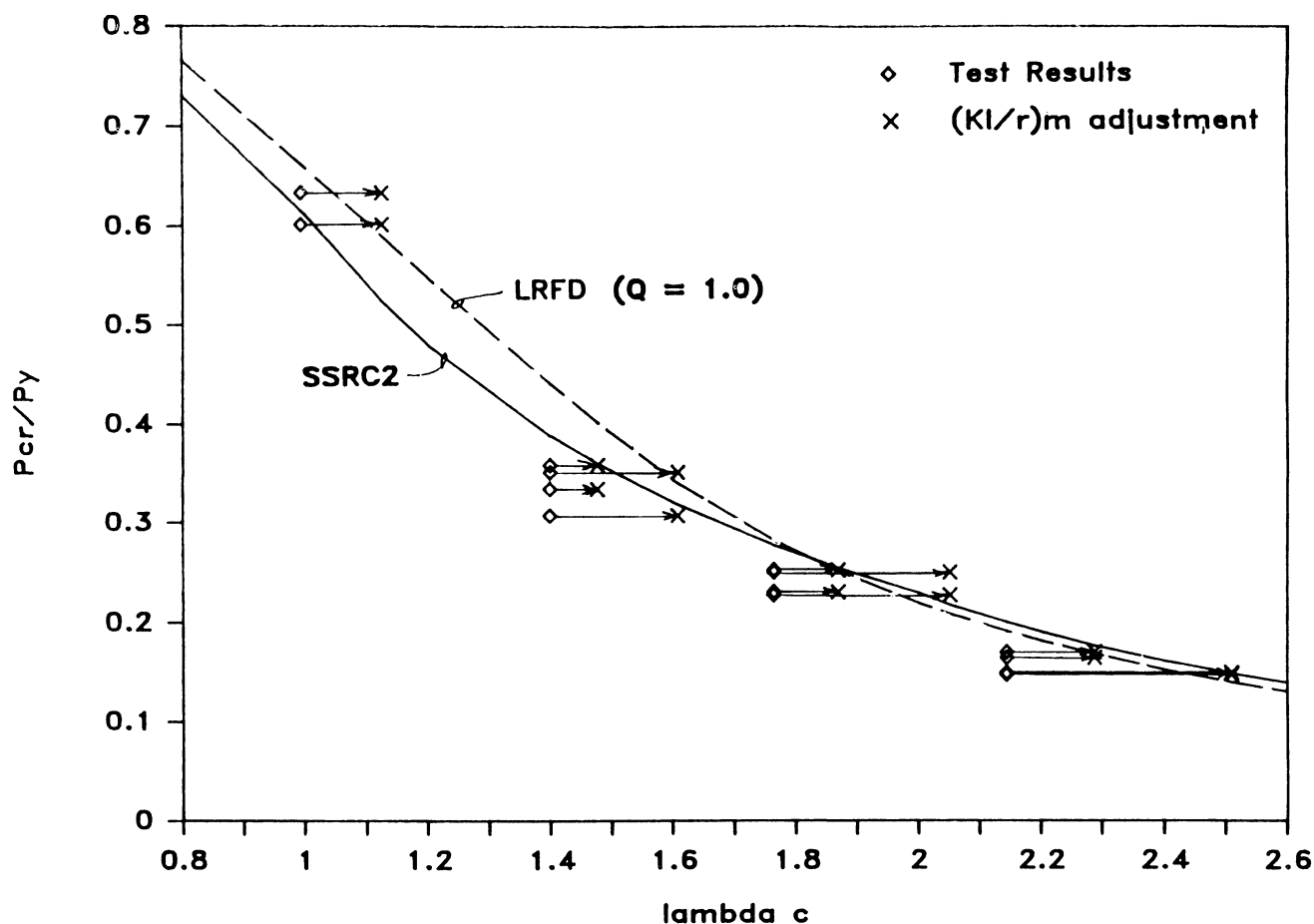


Fig. 5. Zandonini bolted specimens comparison of test results

Table C. Analysis of University of Michigan Tests of Double-angle Bracing

| Specimen<br>(1) | Section <sup>a</sup><br>(2) | Condition<br>(3)            | No. of<br>Stitches<br>(4) | $a/r_z$<br>(5) | $K$<br>(6) | $L/r_y$<br>(7) | $\frac{a/r_z}{KL/r_y}$<br>(8) | $F_y$<br>ksi<br>(9) | EXP. LRFD                     |                              | Col. 10<br>Col. 11<br>(12) | $\lambda_c$<br>(13) | $(KL/r)_m$<br>(14) | $\lambda_{cm}$<br>(15) |
|-----------------|-----------------------------|-----------------------------|---------------------------|----------------|------------|----------------|-------------------------------|---------------------|-------------------------------|------------------------------|----------------------------|---------------------|--------------------|------------------------|
|                 |                             |                             |                           |                |            |                |                               |                     | $\frac{P_{max}}{P_y}$<br>(10) | $\frac{P_{cr}}{P_y}$<br>(11) |                            |                     |                    |                        |
| AB 2            | 2L-5×3×¼                    | Snug-tight bolts            | 1                         | 98             | 0.94       | 114            | 0.91                          | 42                  | 0.347                         | 0.493                        | 0.782                      | 1.298               | 145                | 1.756                  |
| AB 4            | 2L-3×2×½                    | Snug-tight bolts            | 2                         | 79             | 0.88       | 143            | 0.63                          | 42                  | 0.337                         | 0.378                        | 0.892                      | 1.524               | 149                | 1.805                  |
| AB 6            | 2L-3×2×½                    | Slip critical bolted joints | 2                         | 79             | 0.88       | 143            | 0.63                          | 42                  | 0.477                         | 0.378                        | 1.272                      | 1.524               | 129                | 1.563                  |
| AW 8            | 2L-3½×2½×¼                  | Welded                      | 2                         | 71             | 0.90       | 110            | 0.72                          | 42                  | 0.735                         | 0.548                        | 1.341                      | 1.199               | 101                | 1.223                  |
| AW 10           | 2L-3×2×½                    | Welded                      | 2                         | 99             | 0.81       | 151            | 0.81                          | 47                  | 0.473                         | 0.357                        | 1.325                      | 1.567               | 132                | 1.692                  |
| AW 12           | 2L-2½×1½×⅝                  | Welded                      | 2                         | 132            | 0.80       | 186            | 0.89                          | 42                  | 0.290                         | 0.270                        | 1.062                      | 1.803               | 170                | 2.045                  |
| AW 14           | 2L-3½×2½×½                  | Welded                      | 2                         | 74             | 0.86       | 110            | 0.78                          | 42                  | 0.690                         | 0.577                        | 1.194                      | 1.146               | 98                 | 1.187                  |
| AW 16           | 2L-3½×2½×½                  | Welded                      | 2                         | 76             | 0.82       | 110            | 0.84                          | 47                  | 0.595                         | 0.572                        | 1.044                      | 1.156               | 94                 | 1.205                  |
| AW 18           | 2L-2½×1½×¼                  | Welded                      | 2                         | 130            | 0.80       | 189            | 0.86                          | 47                  | 0.261                         | 0.234                        | 1.115                      | 1.938               | 171                | 2.191                  |

<sup>a</sup> Long legs back-to-back

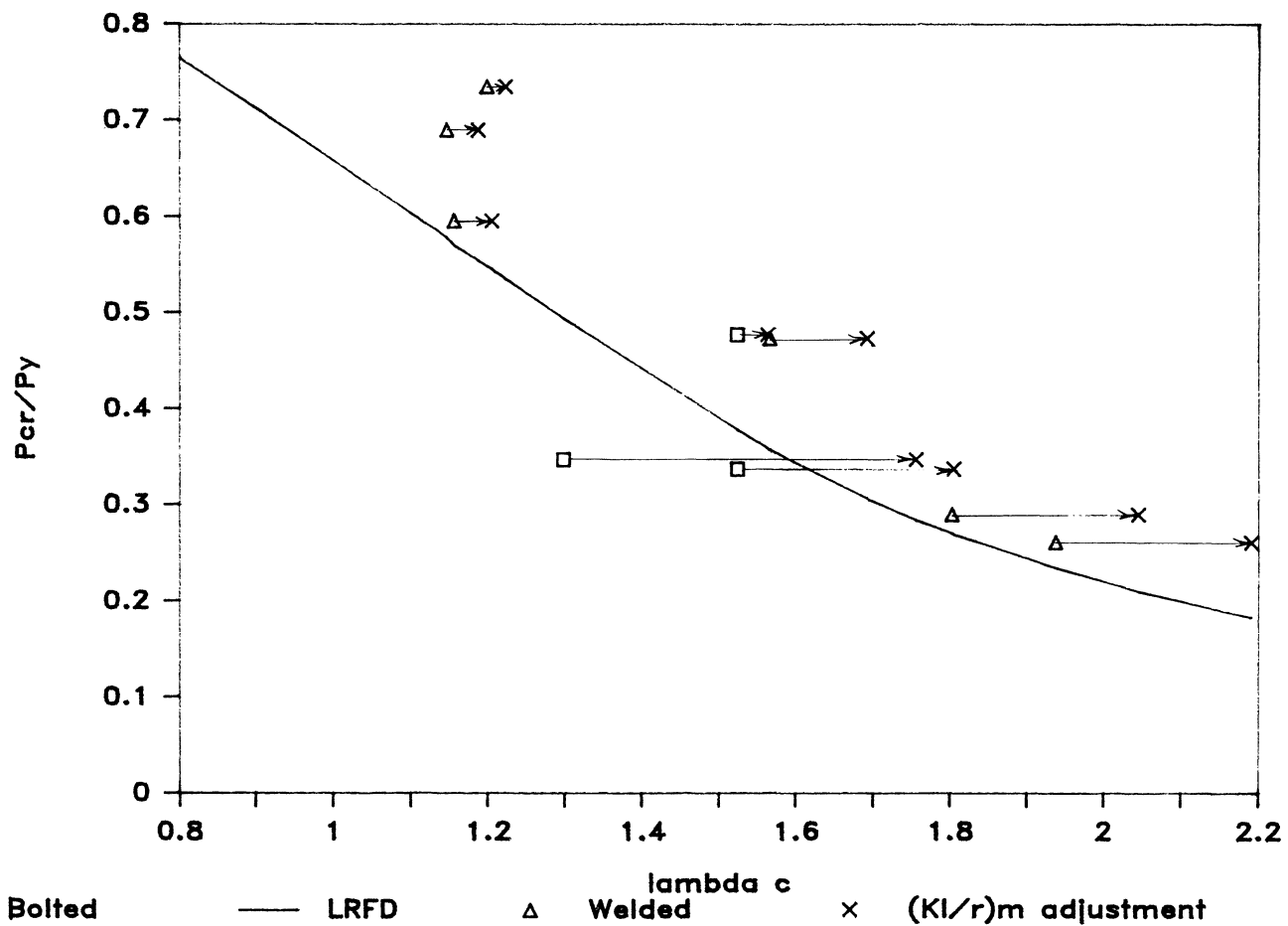


Fig. 6. Univ. of Michigan double-angle test results

appropriate to use a modified slenderness ratio for built-up members in which the individual members are not in continuous contact. If the buckling mode involves relative deformation between two members, i.e. Y-Y axis buckling, the modified slenderness ratio must be employed in determining the compressive strength. The formulas for  $K\ell/r_m$  are summarized below.

For snug-tight bolted connectors:

$$\left(\frac{K\ell}{r}\right)_m = \sqrt{\left(\frac{K\ell}{r}\right)_o^2 + \left(\frac{a}{r_i}\right)^2} \quad (\text{E4-1})$$

For welded and fully tightened bolted connectors as required for slip-critical joints:

with  $\frac{a}{r_i} > 50$

$$\left(\frac{K\ell}{r}\right)_m = \sqrt{\left(\frac{K\ell}{r}\right)_o^2 + \left(\frac{a}{r_i} - 50\right)^2} \quad (\text{E4-2})$$

with  $\frac{a}{r_i} \leq 50$

$$\left(\frac{K\ell}{r}\right)_m = \left(\frac{K\ell}{r}\right)_o \quad (\text{E4-3})$$

where

$\left(\frac{K\ell}{r}\right)_o$  = column slenderness of built-up member acting as a unit

$\frac{a}{r_i}$  = largest column slenderness of individual components

$\left(\frac{K\ell}{r}\right)_m$  = modified column slenderness of built-up member

$a$  = distance between connectors

$r_i$  = minimum radius of gyration of individual component.

The modified slenderness replaces  $K\ell/r_y$  in the appropriate strength formula.

LRFD acknowledges two possible buckling modes for double angles: X-X axis flexural and Y-Y axis flexural-torsional buckling. Considering X-X axis buckling first, the angles move parallel to each other so the number of connectors is not related to the design strength. The only criterion affecting the number of connectors is, the local slenderness must not exceed the overall slenderness. The compressive design strength can be calculated according to either Sect. E2 or Appendix B5.3. The design strength is  $\phi_c P_n$ , where  $\phi_c$  is 0.85 and  $P_n$  is the product of  $A_g$  and  $F_{cr}$ . Using the comprehensive formulas of Appendix B5.3, the critical stress  $F_{cr}$  is :

$$\text{for } \lambda_c \sqrt{Q} \leq 1.5, F_{cr} = (0.658^{Q_{\lambda}^2}) F_y \quad (\text{A-B5-11})$$

$$\text{for } \lambda_c \sqrt{Q} > 1.5, F_{cr} = \left(\frac{0.877}{\lambda_c^2}\right) F_y \quad (\text{A-B5-13})$$

$$\text{where } \lambda_c = \frac{K\ell}{r\pi} \sqrt{\frac{F_y}{E}} \quad (\text{E2-4})$$

The value  $Q$  is a reduction factor unique to Appendix B5.3 and is involved only when slender compression elements are present. For double angles, this means  $b/t$  of one angle exceeds  $76/\sqrt{F_y}$ . The formula for  $Q$  is  $Q_s$  times  $Q_a$ , where  $Q_a = 1.0$  for members with unstiffened elements and  $Q_s$  is calculated as follows.

When  $76.0 / \sqrt{F_y} < b/t < 155 / \sqrt{F_y}$ ,

$$Q_s = 1.340 - 0.00447 (b/t) \sqrt{F_y} \quad (\text{A-B5-1})$$

When  $b/t \geq 155 / \sqrt{F_y}$ ,

$$Q_s = 15,500 / [F_y (b/t)^2] \quad (\text{A-B5-2})$$

These formulas are all applicable to X-X axis buckling for double angles. For Y-Y axis buckling, the effects of flexural-torsional buckling must be included.

The inclusion of the flexural-torsional and torsional buckling limit states in column design is new in LRFD. Usually, these modes of buckling are not critical in the design of doubly symmetric, hot-rolled columns. For unsymmetric or singly symmetric shapes such as double angles, flexural-torsional buckling may be critical. Appendix E3 gives the applicable formulas. In general, both Appendix B5.3 (or Sect. E2) and Appendix E3 must be checked for Y-Y axis buckling, using the modified slenderness ratio. For double angles, the shear center and centroid do not coincide so bending about the Y-Y axis is always accompanied by torsion. Thus, the limit state is flexural-torsional buckling. The applicable formulas from Appendix E3 are

for  $\lambda_e \sqrt{Q} \leq 1.5$

$$F_{cr} = Q (0.658^{Q_{\lambda}^2}) F_y \quad (\text{A-E3-2})$$

for  $\lambda_e \sqrt{Q} > 1.5$

$$F_{cr} = \left(\frac{0.877}{\lambda_e^2}\right) F_y \quad (\text{A-E3-3})$$

These formulas resemble Formulas A-B5-1 and A-B5-2, except for the equivalent slenderness factor  $\lambda_e$ . The equation for flexural-torsional buckling are applicable only to elastic buckling for a perfectly straight column. The equivalent slenderness factor makes the adjustment required for an actual column and is defined as follows:

$$\lambda_e = \sqrt{\frac{F_y}{F_e}} \quad (\text{A-E3-4})$$

The critical flexural-torsional elastic buckling stress for singly symmetric shapes, where  $y$  is the axis of symmetry, is

$$F_e = \frac{F_{ey} + F_{ez}}{2H} \left( 1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}} \right) \quad (\text{A-E3-6})$$

$$\text{where } H = 1 - \left( \frac{x_o^2 + y_o^2}{\bar{r}_o^2} \right) \quad (\text{A-E3-9})$$

$$F_{ey} = \frac{\pi^2 E}{(K\ell / r_m)^2} \quad \text{from (A-E3-11)}$$

$$F_{ez} = \left( \frac{\pi^2 EC_w}{(K_z L)^2} + GJ \right) \frac{1}{A\bar{r}_o^2} \quad (\text{A-E3-12})$$

$$r_o^2 = x_o^2 + y_o^2 + \frac{I_x + I_y}{A} \quad (\text{A-E3-8})$$

Formula A-E3-9 defines a torsional property called the flexural constant, tabulated in Part 1 of the LRFD Manual.  $F_{ey}$  represents the elastic buckling stress at which flexural buckling about the Y-Y axis occurs. The torsional buckling term  $F_{ez}$  is the elastic buckling stress where torsional buckling occurs. The first term in parentheses is for warping torsion and the second is the St. Venant (or pure twisting) term. These formulas introduce some properties that may not have been readily available to the designer, previously. The polar radius of gyration about the shear center  $\bar{r}_o$  and  $H$  are tabulated in Part 1 of the LRFD Manual for double angles. The properties  $J$  and  $C_w$  can be obtained for double angles simply by doubling the values given in Part 1 for single angles. With these values and using the formulas above, the compressive design strength can be calculated for double angle column members. Notice that the modified slenderness ratio is inserted into Formula A-E3-11, replacing  $K\ell/r_y$ .

The design procedure for built-up double angle compression members is summarized as follows. Select a trial section and calculate  $K\ell/r_x$  and  $K\ell/r_y$ . Based on the larger of the two, calculate the connector spacing,  $a$ , required from the relationship  $a/r_z \leq K\ell/r$ . A viable number of connectors can then be determined. If  $K\ell/r_m$  governs, then the design strength is determined from Appendix E3. The other case is when  $K\ell/r_x$  is the larger value, then Sect. E2 or Appendix B5.3 (depending on member slenderness) is applicable. In this situation, the effective slenderness due to flexural-torsional buckling should also be checked to see that it does not govern. This is accomplished by inserting  $K\ell/r_m$  into the flexural-torsional buckling formulas and then solving for  $K\ell/r_{eff}$  from

$$\lambda_{ceff} = \left( \frac{K\ell}{r} \right)_{eff} \frac{1}{\pi} \frac{F_y}{E}$$

The result will be a  $K\ell/r_{eff}$  slightly greater than  $K\ell/r_m$ . The only instance when this check is important is when  $K\ell/r_x \approx K\ell/r_m$ . Figures 7a and 7b outline pictorially the process of finding the governing slenderness ratio and its corresponding design strength, with a flow chart. To simplify the design strength determination, tables are also provided in the LRFD Manual.

## DESIGN AIDS

With the addition of the flexural-torsional buckling limit state, the design method for double-angle compression members has become slightly more complex in LRFD. Double-angle Column Tables, included in the 1st Edition *LRFD Manual of Steel Construction*, give the compressive design strength for 36- and 50-ksi double angles spaced 3/8 in. back-to-back. No values are given beyond a  $K\ell/r$  value of 200. Although not mandatory, the LRFD Specification suggests this limit (Sect. B7) for practical reasons.

In addition to the design strength, the corresponding number of intermediate connectors is given in the far right-hand column of the tables, as shown in Fig. 8. A value is given only for the Y-Y axis part of the table, since this strength is dependent on the number of connectors. In developing the table, an initial number of connectors was determined from the rule  $a/r_z \leq K\ell/r_y$  for the Y-Y axis. As the overall length increased, a significant drop in strength occurred. For this reason, another connector was added when the design strength dropped below 90% of the strength that would be calculated if the angles were assumed to act as a single unit and  $K\ell/r_m$  was not used. This 90% rule is an arbitrary cut-off point where the design strength was reduced to such a point that it is practical to add another connector. If the designer chooses to use a different number of connectors than given in the table, the design strength must be recalculated based on the revised connector spacing. It is also important to note that values given in the tables apply to welded or fully tightened bolted connection, assuming no slip occurs at the ends or at intermediate connectors.

## CONCLUSION

Results of recent research indicate an adjustment to the method of designing double-angle compression members is appropriate. Tests by Zandonini on back-to-back channels under compression were used to develop the formulas for  $K\ell/r_m$ . The proposed procedure was then verified by applying it to the tests on double-angle struts by Astaneh, Goel and Hanson.

For Y-Y axis strength, this modified slenderness can be inserted into the applicable formulas for load and resistance factor design. The LRFD Specification also includes flexural-torsional buckling formulas that must be checked in addition to the basic column strength formulas. However, for double angles, flexural-torsional buckling will always control. The  $K\ell/r_m$  does not enter into the X-X axis strength calculation.

To facilitate the design of double angle compression members in LRFD, tables are included giving the design strength, and for the Y-Y axis, the corresponding number of intermediate connectors. This design aid will be helpful in a new aspect of double-angle column design never before addressed.

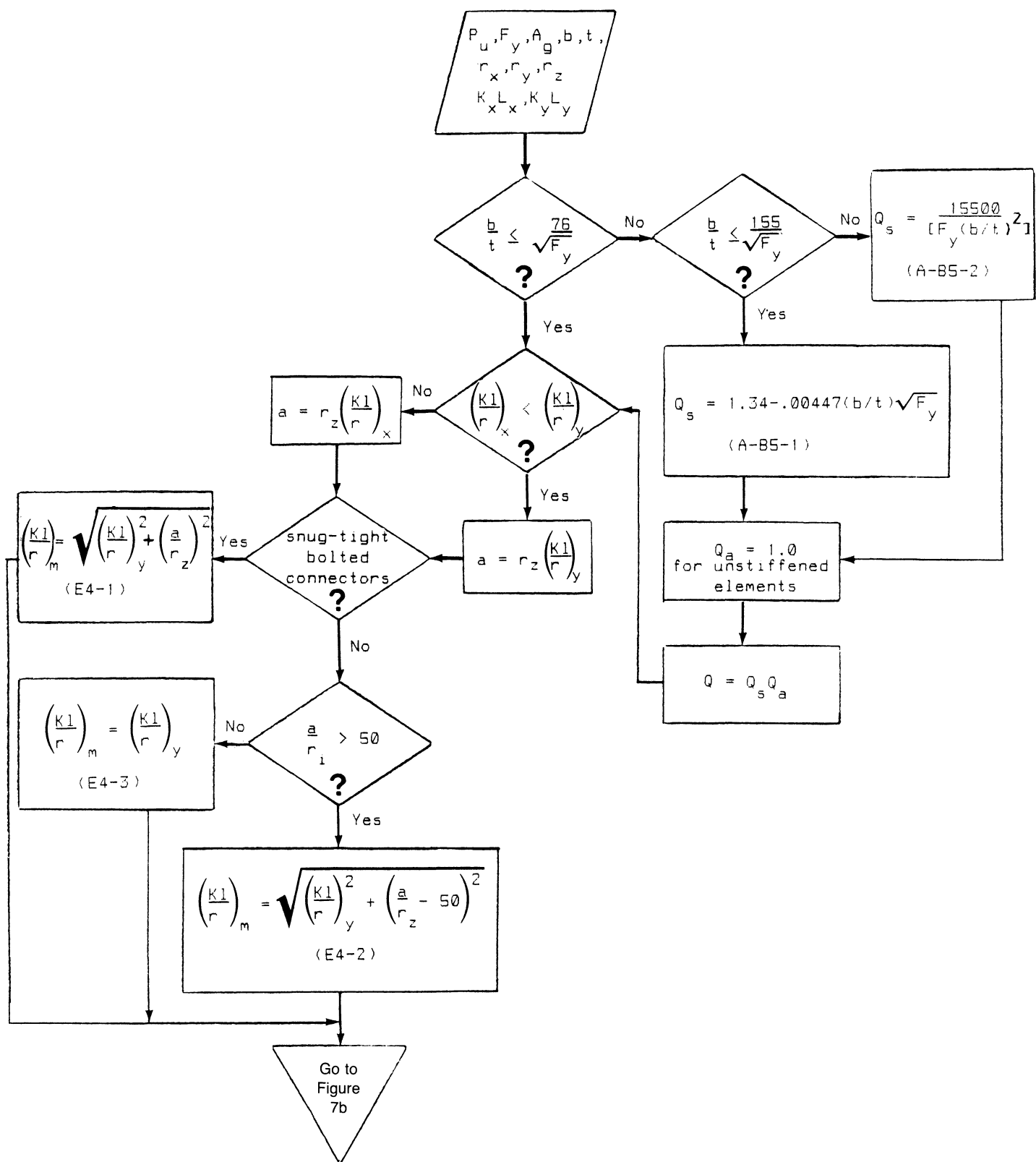


Fig. 7a. LRFD flow chart for double-angle compression members

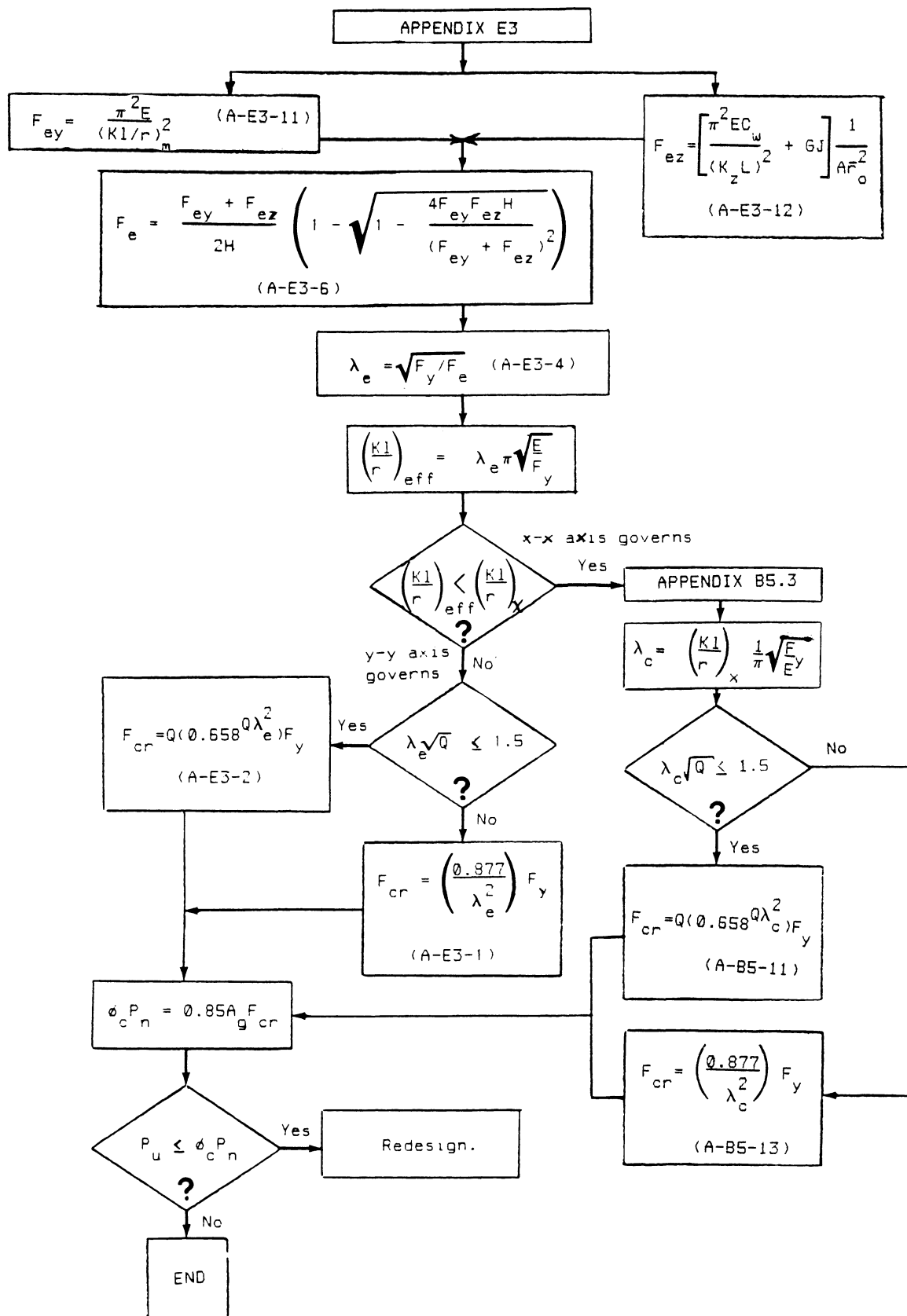


Fig. 7b. Page 2 of LRFD flow chart

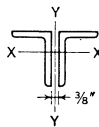
|                                                                                                                                                           |          |    |                                                                                                                                                                                                                                                                                                           |     |      |     |      |     |      |     |                                   |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------|----------|----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|------|-----|------|-----|------|-----|-----------------------------------|
| $F_y = 36 \text{ ksi}$                                                                                                                                    |          |    | <div style="text-align: center;"><b>COLUMNS</b><br/>Double angles<br/>Design axial strength in kips (<math>\phi = 0.85</math>)<br/>Unequal legs<br/>Long legs <math>\frac{3}{8}</math> in. back to back of angles</div>  |     |      |     |      |     |      |     |                                   |
| $F_y = 50 \text{ ksi}$                                                                                                                                    |          |    |                                                                                                                                                                                                                                                                                                           |     |      |     |      |     |      |     |                                   |
|                                                                                                                                                           |          |    |                                                                                                                                                                                                                                                                                                           |     |      |     |      |     |      |     |                                   |
| Size                                                                                                                                                      |          |    | 4 x 3 1/2                                                                                                                                                                                                                                                                                                 |     |      |     |      |     |      |     | No. of<br>Connectors <sup>a</sup> |
| Thickness                                                                                                                                                 |          |    | 1/2                                                                                                                                                                                                                                                                                                       |     | 3/8  |     | 5/16 |     | 1/4  |     |                                   |
| Wt. ft                                                                                                                                                    |          |    | 23.8                                                                                                                                                                                                                                                                                                      |     | 18.2 |     | 15.4 |     | 12.4 |     |                                   |
| $F_y$                                                                                                                                                     |          |    | 36                                                                                                                                                                                                                                                                                                        | 50  | 36   | 50  | 36   | 50  | 36   | 50  |                                   |
| Effective length in ft. KL with respect to indicated axis                                                                                                 | X-X AXIS | 0  | 214                                                                                                                                                                                                                                                                                                       | 298 | 163  | 227 | 137  | 179 | 101  | 129 | b                                 |
|                                                                                                                                                           |          | 2  | 210                                                                                                                                                                                                                                                                                                       | 289 | 160  | 221 | 134  | 174 | 99   | 126 |                                   |
|                                                                                                                                                           |          | 4  | 198                                                                                                                                                                                                                                                                                                       | 266 | 151  | 204 | 127  | 162 | 94   | 118 |                                   |
|                                                                                                                                                           |          | 6  | 179                                                                                                                                                                                                                                                                                                       | 232 | 137  | 178 | 115  | 143 | 87   | 106 |                                   |
|                                                                                                                                                           |          | 8  | 155                                                                                                                                                                                                                                                                                                       | 190 | 120  | 147 | 101  | 120 | 77   | 91  |                                   |
|                                                                                                                                                           |          | 10 | 130                                                                                                                                                                                                                                                                                                       | 148 | 101  | 116 | 85   | 96  | 66   | 75  |                                   |
|                                                                                                                                                           |          | 12 | 104                                                                                                                                                                                                                                                                                                       | 109 | 81   | 86  | 69   | 73  | 55   | 59  |                                   |
|                                                                                                                                                           |          | 14 | 80                                                                                                                                                                                                                                                                                                        | 80  | 63   | 63  | 54   | 54  | 44   | 44  |                                   |
|                                                                                                                                                           |          | 16 | 61                                                                                                                                                                                                                                                                                                        | 61  | 48   | 48  | 41   | 41  | 34   | 34  |                                   |
|                                                                                                                                                           |          | 18 | 48                                                                                                                                                                                                                                                                                                        | 48  | 38   | 38  | 33   | 33  | 27   | 27  |                                   |
|                                                                                                                                                           |          | 20 | 39                                                                                                                                                                                                                                                                                                        | 39  | 31   | 31  | 26   | 26  | 22   | 22  |                                   |
|                                                                                                                                                           |          | 21 |                                                                                                                                                                                                                                                                                                           |     |      |     | 24   | 24  | 20   | 20  |                                   |
|                                                                                                                                                           | Y-Y AXIS | 0  | 214                                                                                                                                                                                                                                                                                                       | 298 | 163  | 227 | 137  | 179 | 101  | 129 | 2                                 |
|                                                                                                                                                           |          | 2  | 198                                                                                                                                                                                                                                                                                                       | 267 | 143  | 189 | 113  | 140 | 78   | 92  |                                   |
|                                                                                                                                                           |          | 4  | 195                                                                                                                                                                                                                                                                                                       | 261 | 141  | 184 | 112  | 137 | 77   | 90  |                                   |
|                                                                                                                                                           |          | 6  | 187                                                                                                                                                                                                                                                                                                       | 246 | 136  | 176 | 109  | 132 | 75   | 88  |                                   |
|                                                                                                                                                           |          | 8  | 172                                                                                                                                                                                                                                                                                                       | 220 | 127  | 160 | 103  | 123 | 72   | 83  |                                   |
|                                                                                                                                                           |          | 10 | 155                                                                                                                                                                                                                                                                                                       | 189 | 115  | 139 | 93   | 108 | 67   | 76  |                                   |
|                                                                                                                                                           |          | 12 | 134                                                                                                                                                                                                                                                                                                       | 155 | 99   | 114 | 81   | 90  | 60   | 66  |                                   |
|                                                                                                                                                           |          | 14 | 111                                                                                                                                                                                                                                                                                                       | 120 | 82   | 88  | 68   | 71  | 51   | 54  |                                   |
|                                                                                                                                                           |          | 16 | 89                                                                                                                                                                                                                                                                                                        | 90  | 66   | 66  | 54   | 54  | 42   | 42  |                                   |
|                                                                                                                                                           |          | 18 | 76                                                                                                                                                                                                                                                                                                        | 76  | 56   | 56  | 46   | 46  | 33   | 33  |                                   |
|                                                                                                                                                           |          | 20 | 61                                                                                                                                                                                                                                                                                                        | 61  | 45   | 45  | 37   | 37  | 29   | 29  |                                   |
|                                                                                                                                                           |          | 22 | 50                                                                                                                                                                                                                                                                                                        | 50  | 37   | 37  | 30   | 30  | 24   | 24  | 3                                 |
|                                                                                                                                                           |          | 24 | 41                                                                                                                                                                                                                                                                                                        | 41  | 31   | 31  | 25   | 25  | 20   | 20  |                                   |
|                                                                                                                                                           |          | 25 | 38                                                                                                                                                                                                                                                                                                        | 38  | 28   | 28  | 23   | 23  | 18   | 18  |                                   |
|                                                                                                                                                           |          | 26 | 35                                                                                                                                                                                                                                                                                                        | 35  | 26   | 26  |      |     |      |     |                                   |
|                                                                                                                                                           |          |    |                                                                                                                                                                                                                                                                                                           |     |      |     |      |     |      |     |                                   |
| Properties of 2 angles— $\frac{3}{8}$ in. back to back                                                                                                    |          |    |                                                                                                                                                                                                                                                                                                           |     |      |     |      |     |      |     |                                   |
| $A \text{ (in}^2\text{)}$                                                                                                                                 |          |    | 7.00                                                                                                                                                                                                                                                                                                      |     | 5.34 |     | 4.49 |     | 3.63 |     |                                   |
| $r_x \text{ (in)}$                                                                                                                                        |          |    | 1.23                                                                                                                                                                                                                                                                                                      |     | 1.25 |     | 1.26 |     | 1.27 |     |                                   |
| $r_y \text{ (in)}$                                                                                                                                        |          |    | 1.58                                                                                                                                                                                                                                                                                                      |     | 1.56 |     | 1.55 |     | 1.54 |     |                                   |
| <sup>a</sup> For Y-Y axis, welded or fully tightened bolted connectors only.<br><sup>b</sup> For number of connectors, see double angle column discussion |          |    |                                                                                                                                                                                                                                                                                                           |     |      |     |      |     |      |     |                                   |

Fig. 8. Double-angle column table

## NOMENCLATURE

|          |                                                                               |
|----------|-------------------------------------------------------------------------------|
| $A, A_g$ | = Gross area of member, in. <sup>2</sup>                                      |
| $C_w$    | = Warping constant, in. <sup>6</sup>                                          |
| $E$      | = Modulus of elasticity (29,000), ksi                                         |
| $F_{cr}$ | = Critical Stress, ksi                                                        |
| $F_e$    | = Elastic buckling stress, ksi                                                |
| $F_{ey}$ | = Elastic flexural buckling stress about the minor axis, Formula A-E3-11, ksi |
| $F_{ez}$ | = Elastic torsional buckling stress, Formula A-E3-12, ksi                     |
| $F_y$    | = Specified yield stress, ksi                                                 |
| $G$      | = Shear modulus of elasticity of steel (11,200), ksi                          |
| $H$      | = Flexural constant, Formula A-E3-9                                           |
| $J$      | = Torsional constant, in. <sup>4</sup>                                        |

|             |                                                                     |
|-------------|---------------------------------------------------------------------|
| $K$         | = Effective length factor for prismatic member                      |
| $L$         | = Unbraced length of member, in.                                    |
| $L_o$       | = Unbraced length for the built-up member overall, in.              |
| $P_n$       | = Nominal compressive strength, kips                                |
| $Q$         | = Full reduction factor for slender compression elements            |
| $Q_a$       | = Reduction factor for slender stiffened compression elements       |
| $Q_s$       | = Reduction factor for slender unstiffened compression elements     |
| $a$         | = Distance between connectors in a built-up member, in.             |
| $b$         | = Compression element width, in.                                    |
| $\ell$      | = Unbraced length of member, in.                                    |
| $r$         | = Governing radius of gyration, in.                                 |
| $r_i$       | = Minimum radius of gyration of individual component, in.           |
| $\bar{r}_o$ | = Polar radius of gyration of about the shear center, in.           |
| $r_z$       | = Radius of gyration about the Z-Z axis, in.                        |
| $t$         | = Compression element thickness, in.                                |
| $x_o, y_o$  | = Coordinates of the shear center with respect to the centroid, in. |
| $\lambda_c$ | = Column slenderness parameter                                      |
| $\lambda_e$ | = Equivalent slenderness parameter                                  |
| $\phi_c$    | = Resistance factor for compression                                 |

## REFERENCES

1. American Institute of Steel Construction, Inc. Load and Resistance Factor Design Manual of Steel Construction 1st Ed., 1986, Chicago, Ill.
2. Zandonini, R. Stability of Compact Built-Up Struts: Experimental Investigation and Numerical Simulation *Costruzione Metalliche*, No. 4, 1985.
3. Astanteh, A., S.C. Goel and R. D. Hanson Cyclic Out-of-Plane Buckling of Double-Angle Bracing *Journal of Structural Engineering*, Vol. III, No. 5, May 1985.
4. Libove, C. Sparsely Connected Built-Up Columns *Journal of Structural Engineering*, Vol. III, No. 3, March 1985.
5. Zahn, C. J. and G. Haaijer Effect of Connector Spacing on Double Angle Compressive Strength *Materials and Member Behavior, Proceeding of sessions at 1987 ASCE Structures Congress*.
6. Johnston, B. G., Ed. Guide to Design Criteria for Metal Compression Members, 3rd. Ed., *Structural Stability Research Council, John Wiley and Sons*, 1976, New York, N.Y.