# Design of 8-bolt Stiffened Moment End Plates

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This paper presents two design procedures for the 8-tension bolt, stiffened, extended, moment end plate shown in Fig. 1. The resulting end-plate design will be satisfactory for use in Type 1 construction (rigid-frame) as defined in Ref. 1 or, with appropriate modifications, as a FR type (fully restrained) connection as defined in the LRFD design notation.<sup>2</sup> With bolt size limited to a maximum of  $1\frac{1}{2}$  in. dia., the configuration is capable of developing the full moment capacity of most available hotrolled beam sections. The two design procedures are limited to use with A36 steel and A325 bolts.

## **OVERVIEW OF ANALYTICAL AND EXPERIMENTAL STUDIES**

The design procedures presented here are the result of extensive analytical and experimental studies on the behavior of 8-bolt, stiffened, moment end plates. $3$  First, a hybrid 2D/3D finite element model was developed.<sup>4,5,6</sup> It was assumed the beam tension flange and a symmetrical portion of the end plate acts as a stiffened tee hanger, as shown in Fig. 2. One quarter of this section was then analyzed using the finite element model in Fig. 3. The end plate and the beam flange-to-end-plate welds were modeled using three dimensional (3D) subparametric elements. The stiffener and beam flange were modeled using two dimensional (2D) elements. Both the bolt heads and shanks were modeled with 3D elements. The nodes for the shank and center portion of the bolt head were kept separate to more accurately model actual behavior. Nonlinear behavior of both the plate material (assumed to be elasticperfectly plastic) and bolt material (bi-linear) were included in the analysis. Tension at support nodes on the

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back of the end-plate was not permitted; nodes showing tension forces were released and new analyses conducted.

The accuracy of the finite element model was then verified with two series of tests. First, six stiffened tee-hanger tests were conducted. Each tee-hanger specimen consisted of two A36 steel tee-stubs connected by four rows of two  $\frac{5}{8}$ -in. dia. A325 bolts. The specimens were loaded using a 200-kip capacity universal testing machine by applying load to the tee stems. Measurements were made to determine strains on and separation of the tee-stub flanges and bolt shank strains. Experimental strains and displacements were compared to predictions from the finite element model and were found to be in close agreement. $4,5,6$ 



*Fig. L 8-tension bolt stiffened moment end-plate* 





*Fig. 3. Configuration of stiffened tee-hanger finite element model* 

To further evaluate the finite element model, eight endplate connection tests were conducted. Each test specimen consisted of two beam sections with end plates at each end. The sections were bolted together and tested under pure moment, developed by a symmetric two-point loading applied using a spreader beam. Separation of the end plates near the beam tension flanges, tension bolt strains

and vertical deflections were measured. The connections were also analyzed using the finite element model. Figure 4 shows selected results for one of these tests. These curves show that the finite element tee-hanger model gives results that compare well with full connection test results. A similar conclusion was reached for each of the other full connection tests.



Fig. 4. Typical experimental and analytical results

## DEVELOPMENT OF THE BASIC DESIGN PROCEDURE

To develop equations which predict end-plate deflection, end-plate strains and bolt force response to loading, sensitivity and parametric studies were first conducted. The sensitivity study was used to determine the most significant geometric and force related variables that govern connection behavior. The following six geometric and two force variables were found to be significant:  $t_p$  = end-plate thickness  $p_f$  = distance from the face of the beam flange to the centerline of the nearer bolts  $d_b$ = nominal bolt diameter  $t_s$  = end-plate stiffener thickness  $g$  = gage of vertical bolt lines  $b_p$  = end-plate width  $F_u$  = factored beam flange force and  $P_T$  = bolt pretension force. All other dimensions necessary to define an end-plate configuration were determined from these six independent geometric param-







eters. Only A36 steel end-plate material and A325 bolts were considered. The six independent geometric variables were then reduced to five dimensionless parameters. The normalizing variable was chosen as the end plate width  $b_n$ .

Once the five independent dimensionless parameters were established, ranges for each of the six geometric parameters were selected based on usual detailing practice, Table 1. Also, limitations were placed on the combinations of bolt diameter and end-plate thickness as in Table 2. Based on these ranges, "low," "intermediate" and "high" values of each of the dimensionless parameters were calculated. Using combinations of the three ranges of the dimensionless parameters, 21 end-plate configurations were selected. In addition, four special and extreme cases were formulated using the smallest and largest values of flange width. Thus, the resulting 25 cases bracketed all reasonable end-plate design configurations.

Next, the 25 cases were analyzed using the previously described finite element model. The following criteria was used to complete the detailing: (1) All edge distances were 1.75  $d_b$ . (2) The end-plate width equaled the beam-flange width. (3) The distance between bolt rows on the same side of the flange was set at  $2\frac{3}{4}$ . And (4), the length of the 45° profile stiffener was made to extend beyond the far bolt centerline by a distance equal to the bolt diameter. In the finite element analyses, the flange force was apphed (after pretensioning of the bolts) in increments of  $\frac{1}{2}$ oth of

the ultimate capacity of the eight bolts until failure occurred. Failure was defined to occur when the ratio of the secant modulus and the elastic modulus of the plate material became equal to or less than 0.1 or when a bolt strain reached a value of 0.00693 in./in.

Regression analyses, using results from the finite element analyses together with the five non-dimensional parameter terms for each of the 25 cases, were then conducted to generate prediction equations for maximum plate separation, maximum end-plate strain and maximum bolt force. The three best fit equations (with least square fit values of 0.961, 0.979 and 0.988) were then rearranged for design use as follows:

$$
t_{p1} = \frac{0.00553 \ p_f^{0.873} \ g^{0.577} \ F_u^{0.917}}{d_b^{0.924} \ t_s^{0.112} \ b_p^{0.682}} \tag{1}
$$

$$
t_{p2} = \frac{0.00371 \, p_f^{0.257} \, g^{0.148} \, F_u^{1.017}}{d_b^{0.719} \, t_s^{0.162} \, b_p^{0.319}}
$$
\n(2)

$$
T_u = \frac{2.305 \times 10^{-5} p_f^{0.591} F_u^{2.583}}{t_p^{0.885} d_b^{1.909} t_s^{0.327} b_p^{0.965}} + P_T
$$
 (3)

where  $t_{p1}$  and  $t_{p2}$  are required end-plate thicknesses for a maximum (factored) flange force  $F_{\mu}$ ,  $T_{\mu}$  is the corresponding bolt force, and other variables are as previously defined. Equation 1 was developed from the end-plate displacement prediction equation using a limit of 0.02 in. to ensure sufficient stiffness for use in Type 1 or FR construction.<sup>4</sup> Equation 2 was developed from the prediction equation for maximum end-plate strain and Eq. 3 from the prediction equation for maximum bolt force. The maximum bolt force  $T_u$  includes prying action effects and bolt pretension  $P_T$ . Obviously, the larger of  $t_{p1}$  and  $t_{p2}$  would be used to select a final end-plate thickness. The specified minimum tensile strength of the bolt material (88 ksi for A325 bolts based on nominal bolt area) and  $T_u$  are used to determine the required bolt diameter.

For allowable stress design use, a factor of safety of 1.67 is introduced into Eqs. 1 and 2 by substituting  $1.67F$  for  $F_u$ , where  $F =$  beam tension flange force (unfactored). Similarly, a factor of safety of 2.0 is introduced into Eq. 3. The resulting allowable stress design equations are:

$$
t_{p1} = \frac{0.00885 \ p_f^{0.873} \ g^{0.577} \ F^{0.917}}{d_b^{0.924} \ t_s^{0.112} \ b_p^{0.682}}
$$
 (4)

$$
t_{p2} = \frac{0.00625 \ p \, \text{p}^{0.257} \ g^{0.148} \ F^{1.017}}{d_b^{0.719} \ t \, \text{s}^{0.162} \ b \, \text{p}^{0.319}} \tag{5}
$$

$$
T_u = \frac{1.381 \times 10^{-4} p_f^{0.591} F^{2.583}}{t_p^{0.885} d_b^{1.909} t_s^{0.327} b_p^{0.965}} + P_T
$$
 (6)

In the application of Eqs. 4 and 5, a preliminary bolt diameter is selected assuming that 6.8 of the 8 tension bolts are effective. This ratio must often be decreased depending on the results of Eq. 6.



*Fig. 5. End-plate tension flange area geometry* 

# **BASIC DESIGN PROCEDURE AND EXAMPLE**

This *allowable stress design* procedure is only valid for *A36 steel end-plate material* and *A325 bolts.* Figure 5 defines the geometry.

#### **Allowable Stress Design Procedure:**

- 1. Select beam size.
- 2. Compute beam flange force *F\*   $F = M/(d_{\tau}t_{\tau})$ ,

where

- *M =* beam end moment
- $d =$  beam depth
- $t_f$  = beam flange thickness
- 3. Determine single bolt force *T* assuming 6.8 bolts are effective:
	- $T = F/6.8$
- 4. Determine the required A325 bolt diameter and select bolt size:

 $d_b = \sqrt{(4T/(\pi F_t)}$  with  $F_t = 44$  ksi

Or select bolt diameter from Table 1-A, p. 4-3, of the AISC Manual.<sup>7</sup>

5. Select gage g, pitch  $p_f$ , end-plate width  $b_p$  and stiffener plate thickness  $t_s$ . The actual end-plate can be of any width, but the width  $b<sub>p</sub>$  must not exceed the beam flange width plus 1 in. or the actual width. The gage  $g$ must not exceed the beam flange width. The stiffener plate thickness should be approximately the same thickness as the beam web.

6. Determine  $t_{p1}$  from stiffness criterion (Eq. 4):

$$
t_{p1} = \frac{0.00885 \ p_f^{0.873} \ g^{0.577} \ F^{0.917}}{d_b^{0.924} t_v^{0.112} b_p^{0.682}}
$$

7. Determine  $t_{p2}$  from strength criterion (Eq. 5):

$$
t_{p2} = \frac{0.00625 \ p_f^{0.257} \ g^{0.148} \ F^{1.017}}{d_b^{0.719} \ t_s^{0.162} \ b_p^{0.319}}
$$

8. Select end-plate thickness,  $t_p$ :

$$
t_p > \max.
$$

9. Calculate ultimate bolt force (Eq. 6):

$$
T_u = \frac{1.381 \times 10^{-4} p_f^{0.591} F^{2.583}}{t_p^{0.885} d_b^{1.909} t_s^{0.327} b_p^{0.965}} + P_T
$$

 $t_{p1}$ 

 $t_{p2}$ 

10. Check bolt size

$$
T_u < 2 \times T_{\text{allow}}
$$

where  $T_{\text{allow}}$  = tension allowable load from Table 1-A, p. 4-3, of the AISC Manual.<sup>7</sup>

11. Redesign if necessary

Two possibilities exist. Either the end-plate thickness can be increased which reduces  $T_u$  or a larger bolt diameter can be selected.

#### **Example:**

Design an 8-tension bolt moment end plate to develop the allowable stress design moment capacity of a W24  $\times$  94, A36 steel, beam under gravity loading

W24 × 94 A36 steel 
$$
b_f = 9.065
$$
 in.  $d = 24.31$  in.  
\n $t_f = 0.875$  in.  $t_w = 0.515$  in.  
\n $M = M_r = 444$  ft/kips (AISC Manual,<sup>7</sup> p. 2-7)

*Flange force: F =*  $(444 \times 12)/(24.31 - 0.875)$  $= 227.4$  kips

*Estimate single-bolt force:*  $T = 227.4/6.8 = 33.4$  kips

*Trial bolt size:* Select 1-in. diameter A325 bolts

 $T_{\text{allow}} = 34.6 \text{ kips (AISC Manual},^7 \text{ p. } 4-3) > 33.4 \text{ kips}$ 

*Select end-plate geometry:* 

 $d_b = 1$  in.  $p_f = 1 + \frac{1}{2} = 1\frac{1}{2}$  in.  $g = 5\frac{1}{2}$  in.  $b_p = 9$  in.<br> $p_b = 3$  in.  $t_s = \frac{1}{2}$  in.  $p_b = 3$  in.

(Note: stiffener thickness is approximately the beam web thickness, 0.515 in.)

*Calculate*  $t_{p1}$ *:* 

$$
t_{p1} = \frac{0.00885 (1.5)^{0.873} (5.5)^{0.577} (227.4)^{0.917}}{(1.0)^{0.924} (0.5)^{0.112} (9.0)^{0.682}}
$$
  
= 1.18 in.

*Calculate*  $t_{p2}$ *:* 

$$
t_{p2} = \frac{0.00625 (1.5)^{0.257} (5.5)^{0.148} (227.4)^{1.017}}{(1.0)^{0.719} (0.5)^{0.162} (9.0)^{0.319}}
$$

 $= 1.24$  in.

*Select end-plate thickness:* 

$$
t_p = 1\frac{1}{4} \text{ in.} > \text{max.}
$$

$$
1.24~\mathrm{in.}
$$

*Calculate ultimate bolt force:* 

$$
T_u = \frac{1.381 \times 10^{-4} (1.5)^{0.591} (227.4)^{2.583}}{(1.25)^{0.885} (1.0)^{1.909} (0.5)^{0.327} (9.0)^{0.965}} + 51
$$
  
= 26.53 + 51 = 77.53 kips

*Check bolt size:* 

 $T_u = 77.53 \text{ kips} > 2 \times 34.6 = 69.2 \text{ kips}.$  n.g. Redesign is necessary.

*Redesign* 

Increase bolt size to 1½-in. dia.  
\n
$$
T_{\text{allow}} = 43.7 \text{ kips}
$$
  
\n $p_f = 1\frac{1}{8} + \frac{1}{2} = 1\frac{5}{8} \text{ in. } p_b = 3 \times 1\frac{1}{8} = 3\frac{3}{8} \text{ in.}$   
\nAs previous  $t_{p1} = 1.13 \text{ in. } t_{p2} = 1.16 \text{ in.}$   
\nUse  $t_p = 1\frac{1}{4} \text{ in.}$ 

*Check bolt capacity i* 

$$
T_u = \frac{1.381 \times 10^{-4} (1.625)^{0.591} (227.4)^{2.583}}{(1.25)^{0.885} (1.125)^{1.909} (0.5)^{0.327} (9.0)^{0.965}} + 56
$$
  
= 22.21 + 56 = 78.21 kips < 2 × 43.7  
= 87.40 kips

Therefore,  $1\frac{1}{8}$  in. dia. bolt is satisfactory

*Final design* 

 $W24 \times 94$  A36 M = 444 ft/kips End plate:  $1\frac{1}{4} \times 9$  in. Stiffener:  $\frac{1}{2}$  in. Bolts: 8-11/8 in. dia. A325  $g = 5\frac{1}{2}$  in.  $p_f = 1\frac{5}{8}$  in.  $p_b = 3\frac{3}{8}$  in.

# **DEVELOPMENT OF SIMPLIFIED DESIGN PROCEDURE**

Because of the difficulty of using Eqs. 4, 5 and 6, except for completely computerized designs, an additional effort was made to develop a simplified allowable stress design procedure. First, end-plate connection designs were generated using Eqs. 1, 2 and 3 for all hot-rolled A36 steel beam sections at 100%, 75% and 50% of full moment capacity ( $M = 0.66 F_v S_x$ ). The effective number of tension bolts was then computed from

$$
N_{\text{eff}} = \frac{2F/8}{T_u} N \tag{7}
$$

where  $N_{\text{eff}}$  = the effective number of tension bolts,  $F =$ unfactored beam flange force,  $T_u$  = maximum bolt force calculated using Equation 3 with  $F_u = 2F$  and  $N = 8$ . From this procedure, it was found a conservative value for the effective number of bolts is six. Note a factor of safety of 2.0 was used to determine  $N_{\text{eff}}$ .

Next, an equivalent pitch  $p_{\text{eff}}$  was found so classical teestub type calculations could be used for determining required end-plate thickness rather than Eqs. 1 and 2. From Fig. 6, assuming an inflection point at  $p\ll 2$  and two bolts per row, the tee-stub flange moment is

$$
M_{PL} = 2T(p_f/2) = Tp_f \tag{8}
$$

where  $T =$  force per bolt based on six effective bolts. The required section modulus and tee-stub flange thickness can then be calculated. To determine the corresponding required end-plate thickness  $p_{\text{eff}}$  is substituted for  $p_f$  in Eq. 8.

To determine  $p_{\text{eff}}$ , numerous expressions were developed and evaluated as follows. First, the required endplate thickness was determined for all hot-rolled beam section capacities (subject to limitations included in the following design procedure) using the tee-stub analogy (Fig. 5) and  $p_{\text{eff}}$ . The required end-plate thickness was then rounded to the next highest 1/<sub>8</sub>th in. and the capacity of the connection determined from the minimum  $F_u$  calculated by rearranging Eqs. 1, 2 and 3 and using factors of safety of 1.67 for Eqs. 1 and 2 and 2.0 for Eq. 3. Bolt size was determined using  $N_{\text{eff}} = 6$ . The resultant allowable stress design capacity  $F_{\text{can}}$  was then compared to the required unfactored flange force *F.* The following expression for  $p_{\text{eff}}$  resulted in values of  $F/F_{cap}$  less than 1.05 for all but 19 of the 726 cases examined. The maximum  $F/F_{cap}$  ratio was 1.10. Note that a value of  $F/F_{cap}$  greater than 1.0 is unconservative.

$$
P_{\text{eff}} = \frac{\sqrt{g^2 + p_f^2}}{5} p_f \tag{6}
$$

The final result is the following simplified procedure for determining bolt diameter and end-plate thickness of 8-tension bolt, extended, stiffened moment end-plates.

#### **SIMPLIFIED DESIGN PROCEDURE AND EXAMPLE**

This *allowable stress design* procedure is only valid for *A36 steel end-plate material* and *A325 bolts* and is subject to the following limitations. Figure 5 defines the geometry.

- 1. The connected beam section must be hot-rolled and included in the Allowable Stress Design Selection Table in the 8th ed. AISC Manual.<sup>9</sup>
- 2. The effective end-plate width, e.g. the end-plate width used in the design calculations, must not be greater than the beam flange width plus 1 inch.
- 3. The pitch  $p_f$  from the face of the beam tension flange to the first row of bolts must not exceed  $2\frac{1}{2}$  in. The recommended pitch is bolt diameter plus  $\frac{1}{2}$  in.
- 4. The spacing  $p<sub>b</sub>$  must not exceed  $3d<sub>b</sub>$ .
- 5. The gage g must not be less than  $3\frac{1}{2}$  in. nor greater than *IV2* in.
- 6. Stiffener thickness  $t<sub>s</sub>$  must be approximately equal to the beam web thickness.
- 7. Bolt diameter must not be less than *VA* in. nor greater than  $1\frac{1}{2}$  in.

#### **Simplified Allowable Stress Design Procedure:**

- 1. Select beam size.
- 2. Compute beam flange force *F:*

$$
F = M/(d-t_f),
$$

where

- *M=* beam end moment
- *d =* beam depth
- $t_f$  = beam flange thickness
- 3. Determine single bolt force *T:*

$$
T = F/6
$$

4. Determine the required A325 bolt diameter and select bolt size:

$$
d_b = \sqrt{(4T/(\pi F_t))} \text{ with } F_t = 44 \text{ ks}
$$

Or select bolt diameter from Table 1-A, p. 4-3, of the AISC Manual.<sup>7</sup>

5. Select gage g (not to exceed the beam flange width), pitch  $p_f$  and compute effective pitch,  $p_{eff}$ .

$$
P_{\text{eff}} = \frac{\sqrt{g^2 + p_f^2}}{5} p_f
$$

- 6. Compute effective tee-stub analogy moment *M^.* 
	- $M_e$  = effective moment in plate caused by two bolts with inflection point at  $p_{\text{eff}}/2$  (see Fig. 6).

$$
=2T (p_{\text{eff}}/2) = T p_{\text{eff}}
$$

7. Select effective end-plate width  $b_p$  (not to exceed beam flange width plus 1 in.) and determine required endplate thickness  $t_p$ :

$$
S_R = M_e/(0.75F_y) \text{ with } F_y = 36 \text{ ksi}
$$
  

$$
t_p = \sqrt{6S_R/b_p}
$$

#### **Example:**

Using the simplified procedure, design an 8-tension bolt moment end-plate to develop the allowable stress design moment capacity of a W24  $\times$  94, A36 steel, beam under gravity loading. (Same as previous example).

W24 × 94 A36 steel 
$$
b_f = 9.065
$$
 in.  $d = 24.31$  in.  
\n $t_f = 0.875$  in.  $t_w = 0.515$  in.  
\n $M = M_r = 444$  ft./kips (AISC Manual,<sup>9</sup> p. 2-7)



*Fig. 6. Tee-stub analogy moments* 

*Flange force: F =*  $(444 \times 12)/(24.31 - 0.875)$  $= 227.4$  kips

*Single-bolt force:*  $T = 227.4/6.0 = 37.9$  kips

*Bolt size:* Select 1<sup>1</sup>/<sub>8</sub> in. dia. A325 bolts

 $T_{\text{allow}} = 43.7 \text{ kips (AISC Manual}, ^{9} \text{p.4-3}) > 37.9 \text{ kips}$ 

*Select end-plate geometry:* 

$$
d_b = 1\frac{1}{8} \text{ in.} \quad p_f = 1\frac{1}{8} + \frac{1}{2} = 1\frac{5}{8} \text{ in.} \\
 g = 5\frac{1}{2} \text{ in.} \quad b_p = 9 \text{ in.} \\
 p_b = 3d_b = 3\frac{3}{8} \text{ in.}
$$

*Effective pitch:* 

$$
p_{\text{eff}} = \frac{\sqrt{g^2 + p_f^2}}{5} p_f = \frac{\sqrt{5.5^2 + 1.625^2}}{5} (1.625)
$$
  
= 1.86 in.

*Effective plate moment:* 

$$
M_e = T p_{eff} = (37.9) (1.86)
$$
  
= 70.49 in./kips

*Required end-plate thickness:* 

$$
S_R = M_e/(0.75F_y) = 70.49/(0.75 \times 36.0)
$$
  
= 2.61 in.<sup>3</sup>  

$$
t_p = \sqrt{6S_R/b_p} = \sqrt{6(2.61)/9}
$$
  
= 1.32 in. Use 1% in. plate

*Final design:* 

```
W24 \times 94 A36 M = 444 ft/kips
End plate: 1\frac{3}{8} in. \times 9 in.
Stiffener: \frac{1}{2} in.
Bolts: 8-11/8 in. dia. A325
g = 5\frac{1}{2} in. p_f = 1\frac{5}{8} in.
                  p_h = 3\frac{3}{8} in.
```
The selected stiffener thickness is approximately equal to the beam web thickness (0.515 in.). The simplified design procedure resulted in a slightly thicker end plate *(Vs* in.), but the same bolt diameter.

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#### **NOMENCLATURE**

- $b_p$  = End-plate width.
- $d =$  Beam depth.
- $d_h$  = Nominal A325 bolt diameter.
- *F =* Unfactored beam-flange force.
- $F_{cap}$  = Allowable stress end-plate connection design capacity.
- $F_t$  = Allowable tension stress of A325 bolt material, 44 ksi.
- $F_{\mu}$  = Factored beam-flange force.
- $F_y$  = Yield stress of A36 end-plate material, 36 ksi.
- *g =* Gage of vertical bolt lines on end-plate.
- *N*  = Number of tension bolts, 8.
- $N_{\text{eff}}$  = Effective number of tension bolts.
- *M*  = Allowable stress design beam end moment.
- *M,*  = Effective moment in plate caused by two bolts with inflection point at  $P_{\text{eff}}/2$ .
- $M_{PL}$  = Tee-stub flange plate moment.
- *Pb*  = Spacing of bolt rows on same side of flange.
- *Peff*  = Effective pitch.
- *Pf*  = Distance from the face of the beam flange to the centerline of the nearer bolts.
- *Pr*  = Bolt pretension force.
- $S_R$ = Required section modulus of end-plate.
- $S_x$ = Strong axis beam section modulus.
- = Beam-flange thickness.  $t_f$
- = End-plate thickness.  $t_{\boldsymbol{p}}$
- $t_{p1}$  $=$  Required end-plate thickness to satisfy maximum allowed end-plate separation criterion.
- $t_{p2}$ = Required end-plate thickness to satisfy maximum allowed end-plate strain criterion.
- = End-plate stiffener thickness.  $t_{s}$
- $\check{T}$ = Force per tension bolt.
- $T_u$ = Bolt force at factored load including prying force.