Lateral Support for Tier Building Frames

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UNLESS PREVENTED, sidesway buckling will preclude the most efficient use of a column. A better utilization of these members can be realized by providing diagonal bracing (as shown in Fig. 1) or shear walls. In many instances non-structural building elements, curtain walls for example, can also provide the necessary stiffness against sidesway buckling. In Sect. 1.8 of the 1963 Specification of the American Institute of Steel Construction a braced frame is defined as a structure where lateral stability is provided "by diagonal bracing, shear walls, attachment to an adjacent structure having adequate lateral stability, or by floor slabs or roof decks secured horizontally by walls or bracing systems parallel to the plane of the frame." In contrast, an unbraced frame is a structure which depends upon its own bending stiffness to furnish the necessary lateral stability.

Whereas the specification defines the general conditions by which a structural frame may be considered to be braced, it does not provide precise criteria for determining the amount of stiffness necessary to prevent sidesway buckling. To begin with, a precise evaluation of the lateral support contributed by the non-structural elements of the building is not possible. The exercise of considerable engineering judgment, based on past experience, is inevitable.

In this short article a method will be presented which can be of assistance in tempering this judgment.

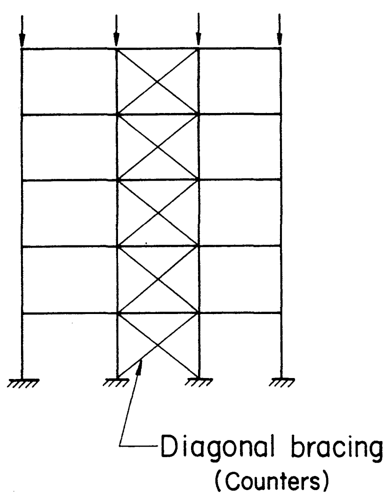

Fig. 1. Lateral stability provided by diagonal bracing

REQUIRED LATERAL SUPPORT

The buckling strength of a simple braced frame, such as that shown in Fig. 2, can be determined by the classical methods of frame stability analysis.[1, 2] However, these procedures are lengthy and complicated and it will be found that the area of a tension diagonal brace needed to prevent sidesway buckling is only a small fraction of the area of the columns to which it is connected.

A simpler solution, erring somewhat on the safe side, can be developed based upon the following assumptions:

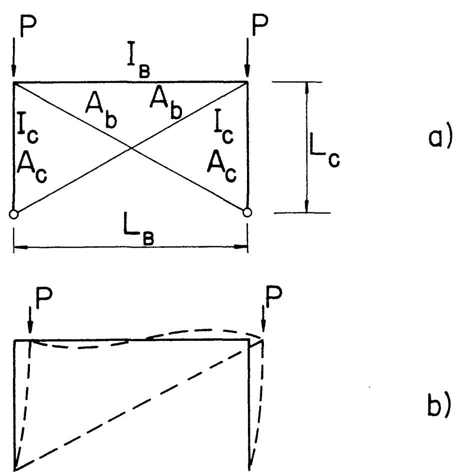

Fig. 2. Buckling of a simple braced frame

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1. *Bleich, F.* The Buckling Strength of Metal Structures, *McGraw-Hill, New York, 1952.*
2. *Timoshenko, S., Gere, J. M.* The Theory of Elastic Stability, *McGraw-Hill, New York, 1961.*

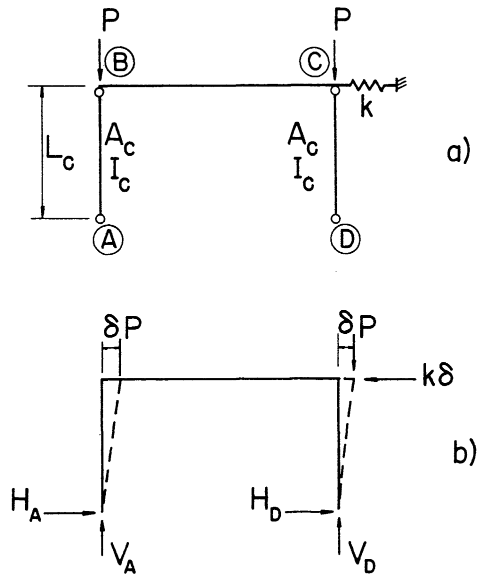

Fig. 3. Idealized bracing arrangement

1. Neglect any participation of the columns in resisting sidesway.
2. Treat columns as pinned-end members.
3. Assume that the bracing alone acts independently as a spring to resist lateral displacement at the top of the columns.

These assumptions are illustrated in Fig. 3 for one story of a single-bay, multi-story frame. Fig. 3a shows the simplified system and Fig. 3b represents the "buckled" shape of this system.

Summation of moments due to the forces acting on the frame (Fig. 3b) leads to the following expressions for the vertical reactions:

$$V_D = P + \frac{2P\delta}{L_B} - \frac{kL_c\delta}{L_B} \qquad (1)$$

$$V_A = P - \frac{2P\delta}{L_B} + \frac{kL_c\delta}{L_B} \qquad (2)$$

Summing moments about the column tops:

$$\Sigma M_B = 0 = V_A\delta - H_A L_C \qquad (3)$$

$$\Sigma M_C = 0 = V_D\delta - H_D L_C \qquad (4)$$

From Eqs. (3) and (4)

$$H_A = \frac{V_A\delta}{L_C} \qquad (5)$$

$$H_D = \frac{V_D\delta}{L_C} \qquad (6)$$

Summing forces in the horizontal direction:

$$H_A + H_D - k\delta = 0 \qquad (7)$$

Substitution of Eqs. (1), (2), (5), and (6) into Eq. (7) results in the following buckling equation:

$$\frac{\delta}{L_C}(2P - kL_C) = 0 \qquad (8)$$

Since at buckling $\delta \neq 0$, the critical load is

$$P_{cr} = \frac{kL_C}{2} \qquad (9)$$

Equation (9) permits determination of the sidesway buckling load if the spring constant k of the bracing is known. The value of k to fully develop the column as a laterally-supported pin-end member is determined by letting

$$P_{cr} = \frac{\pi^2 E_t I_C}{L_C^2} \qquad (10)$$

where E_t is the tangent modulus. Solving for k by equating (9) and (10),

$$k = \frac{2\pi^2 E_t I_C}{L_C^3} \qquad (11)$$

In reality the axial loads on the columns will be less than P_{cr} defined by Eq. (10), when, as usually is the case, these members are also called upon to resist concurrent bending. Thus a more realistic value for P_{cr} is

$$P_{cr} = P \times (\text{F.S.}) \qquad (12)$$

where P is the actual axial load in the columns, and F.S. is a suitable factor of safety. Taking F.S. conservatively as 2.0, equating Eqs. (9) and (12), and solving for k_{min}:

$$k_{min} = \frac{4P}{L_C} \qquad (13)$$

The required lăteral bracing stiffness according to Eq. (13) is valid for a single-bay structure. By the same reasoning as used before, the formula can be extended to a multi-bay frame (Fig. 4). For each individual column

$$H = \frac{V\delta}{L_C} \qquad (14)$$

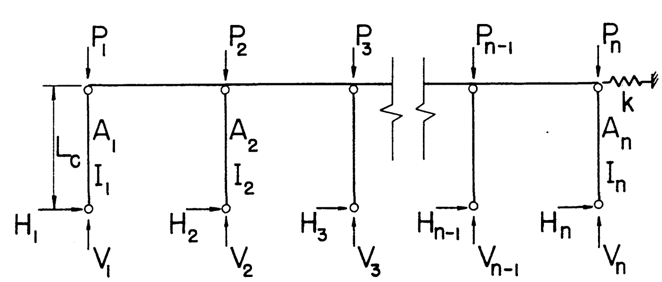

Fig. 4. Multi-bay frame

The buckling condition results from setting

$$H_1 + H_2 + \ldots + H_n - k\delta = 0 \qquad (15)$$

or

$$\frac{\delta}{L_C}\,[V_1 + V_2 + \ldots + V_n - kL_C] = 0 \qquad (16)$$

Since $V_1 + V_2 + \ldots + V_n = P_1 + P_2 + \ldots + P_n = \Sigma P$, and $\frac{\delta}{L_C} \neq 0$

$$k = \frac{\Sigma P}{L_C} \qquad (17)$$

If ΣP in Eq. (17) is the sum of the design loads, the required bracing stiffness is obtained by multiplying this sum by a factor of safety of 2.

$$k_{min} = \frac{2\Sigma P}{L_C} \qquad (18)$$

Equation (18) defines the required stiffness for the bracing of multi-bay structures if bracing is provided in only one of the bays, as is usually the case. If bracing is not present in every frame, but proper shear transfer between parallel frames is provided horizontally by floor slabs or roof decks, then ΣP in Eq. (18) must include the axial loads in all the columns braced by the brace being designed.

SUPPORT BY DIAGONAL BRACING

Refer to Fig. 5. Elongation of the tension diagonal can be expressed as

$$\Delta = \sqrt{L_C^2 + (L_B + \delta)^2} - \sqrt{L_C^2 + L_B^2} \qquad (19)$$

Since δ is small compared to the width and height of the frame, Eq. (19) can be simplified to

$$\Delta = \frac{L_B\delta}{\sqrt{L_C^2 + L_B^2}} \qquad (20)$$

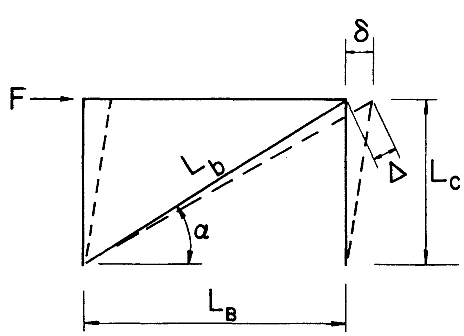

Fig. 5. Deformation of the bracing member

The corresponding force in the diagonal can be expressed as

$$\bar{F} = \frac{F}{\cos\alpha} = \frac{F\sqrt{L_C^2 + L_B^2}}{L_B} = \frac{A_bE\Delta}{\sqrt{L_C^2 + L_B^2}} \qquad (21)$$

Substituting for Δ the value from Eq. (20) and simplifying,

$$F = \frac{A_bE\,L_B^2\delta}{(L_C^2 + L_B^2)^{3/2}} \qquad (22)$$

Since by definition

$$F = k\delta \qquad (23)$$

the value of k for diagonal bracing is

$$k = \frac{A_bE\,L_B^2}{(L_C^2 + L_B^2)^{3/2}} \qquad (24)$$

Substitution of k from Eq. (24) into Eq. (18) leads to the following equation for the required area, allowing a factor of safety equal to 2.0.

$$A_b = \frac{2[L_C^2 + L_B^2]^{3/2}\,\Sigma P}{L_CL_B^2E} \qquad (25)$$

or, in a more convenient rearrangement of the terms:

$$A_b = \frac{2\left[1 + \left(\frac{L_B}{L_C}\right)^2\right]^{3/2}\Sigma P}{\left(\frac{L_B}{L_C}\right)^2 E} \qquad (26)$$

The required bracing area computed from Eq. (26) is conservative since it is assumed that the columns do not contribute any stiffness to resist sidesway buckling, and that the compression diagonal may buckle and hence be unable to provide any lateral stiffness.

To afford an idea of the magnitude of the bracing area, an example of a three-bay frame is shown in Fig. 6. From Eq. (26)

$$A_b = \frac{2\left[1 + \left(\frac{20}{12}\right)^2\right]^{3/2}[600 + 700 + 800 + 800]}{\left(\frac{20}{12}\right)^2(29{,}000)}$$

$$= 0.53 \text{ sq. in}$$

The required area of bracing is less than one-sixtieth of the area of the lightest column in the frame.

SUPPORT BY BRICK WALLS

Because of the non-homogeneous character of brick walls, it is possible to derive analytical expressions for their lateral stiffness only on the basis of experimental results obtained from large scale tests. The following formula is based on tests performed by Benjamin[3] on brick panels enclosed in steel and in concrete frames.

$$k = \frac{L_B t G}{1.2 L_C} \tag{27}$$

where t is the wall thickness, expressed in inches, and G is the shearing modulus of the brick and mortar wall. The value of G for brick walls enclosed in a steel frame can be roughly approximated as 20 ksi. Only two panel tests were performed in steel frames and therefore this value should be taken with some reservations. It is somewhat lower than what one could expect. However, using this value the required brick wall thickness can be computed from Eqs. (27) and (18) as follows:

$$k = \frac{L_B t G}{1.2 L_C} = \frac{2\Sigma P}{L_C} \tag{28}$$

Where P is in kips and L_B is in inches

$$t_{req'd.} = \frac{0.12\Sigma P}{L_B} \tag{29}$$

If the center bay alone contained a 12 inch wall in the example of Fig. 6, the stiffness provided would be five times that required to prevent sidesway buckling of the 4-column frame.

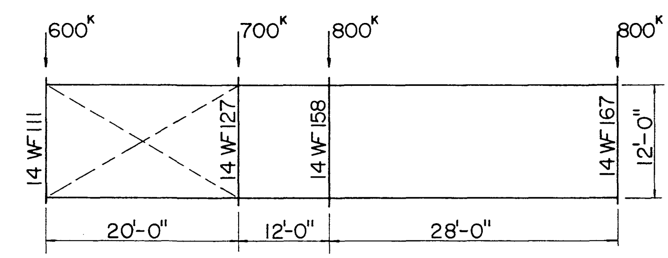

Fig. 6. Example of a multi-bay frame

If usual drift limitations are observed in the design of a frame, the effective column lengths can be computed on the basis of sidesway inhibited provided the *stiffness* of the masonry walls meets the requirements of Eq. (29). This is true even if these walls lack the *strength* to resist the horizontal design loads and these loads are taken by the frame in bending.

3. *Benjamin, J. R., Williams, H. A.* The Behavior of One Story Brick Shear Walls, *ASCE Proceedings, Vol. 84, ST4, July, 1958.*

SUMMARY

An approximate method has been presented for the evaluation of the lateral support required by free-standing steel rigid frames. The criterion used for required bracing stiffness was that the bracing must insure full buckling strength of each column functioning as a laterally-supported, pin-end member (that is, one having an effective length factor $K = 1.0$). Since no contribution to lateral stiffness is derived from the frame, the design based on the proposed method is somewhat conservative.

Procedures are discussed for the design of diagonal bracing and evaluation of the stiffness of brick walls. These are by no means the only way the problem of bracing can be approached. Another approach would be to design the bracing to resist a certain small percentage of the vertical loads (say 5%) applied as horizontal loads at the panel points.

In case the diagonal bracing must take care of both the side loads and the prevention of frame instability, the required area computed by Eq. (26) must be added to the area needed to resist the horizontal forces.

Whereas brick walls will usually provide the necessary stiffness against sidesway buckling, they should be called upon to resist the horizontal forces only in relatively low buildings. In all other cases these forces should be taken care of by frame action or by some form of positive bracing. Under usual drift limitations the stiffness of brick walls will not be impaired by frame deformations when the steel frame is designed to resist the given horizontal loads in bending.

The examples worked in this article, as well as problems worked by others[4] indicate very clearly that a small amount of bracing permits a considerably more efficient utilization of the main rigid frame than could be achieved by adding the same amount of material to the frame itself. The designer should assess the stiffness furnished by the non-structural elements, such as the walls and, if these alone are insufficient, use diagonal bracing wherever possible.

4. Multi-Story Frames *by G. C. Driscoll and V. Levi, Chapts. 16 and 17.* Structural Steel Design Seminar Notes *Fritz Eng. Rep. No. 354.3. The approach presented in this article was adapted from the ideas contained in this reference.*