Simplified Frame Design of Type PR Construction

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Design of unbraced building frames using simple connections between beams and columns has been based historically on Type 2 Construction assumptions. Extensive research on the performance of this type of frame has led to the recommendation that Type 2 Construction should be limited to frames less than 10 stories tall, when the lateral load is resisted entirely by the unbraced frame.[1]

Moreover, it was shown that the governing limit state in such frames is overall frame instability under combined wind and gravity loads, initiated by extensive plastification of the leeward column stack (Fig. 1). Two common features of this limit state are that girders are loaded to about half their bending capacity and the exterior columns are heavily overloaded in bending. The reason for this inappropriate apportionment of steel to the girders and exterior columns is that the simple connections *do* possess rotational stiffness and tend to decrease the in-span girder gravity moments by developing a fraction of the fixed-end moments, referred to herein as flexible-end moments.

This paper describes a more appropriate model for gravity analysis of frames with flexible beam-to-column connections that enables a simplified design procedure for Type PR Construction.

Behavior of Type PR Frames

Extensive analytical studies of the fully nonlinear response of flexibly connected frames indicates the overall frame deformations are well explained as a superposition of the two deformed shapes (Fig. 2). The deformations induced by wind are well predicted by the portal method of analysis. The deformations induced by gravity consist of beam bending and reverse curvatures of exterior columns due to the one-sided flexible-end moment from the girders.

Simplified Models for Frame Behavior

While the portal method is suitable for computing member forces due to wind, one needs to devise a more appropriate model for predicting member forces due to gravity. If one assumes inflection points at mid-heights of exterior columns and no flexure of interior columns (Fig. 3), then one can model interior girders as beams connected through rotational springs into fixed supports. One can model exterior girders as beams connected through a rotational spring to a fixed support at interior ends and to the column stack segment at exterior ends. Note, we only need to develop a math model for the latter subassemblage, because in the limit as we let the columns become infinitely stiff, we obtain the support conditions of the interior girder.

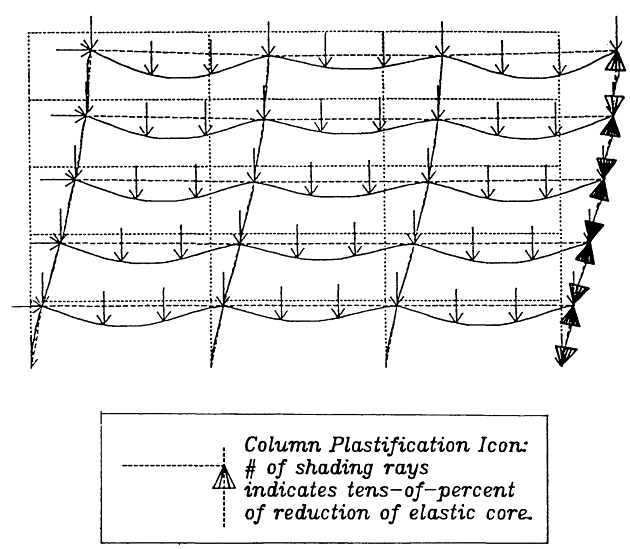

Fig. 1. Typical limit-state mode for simply connected frames

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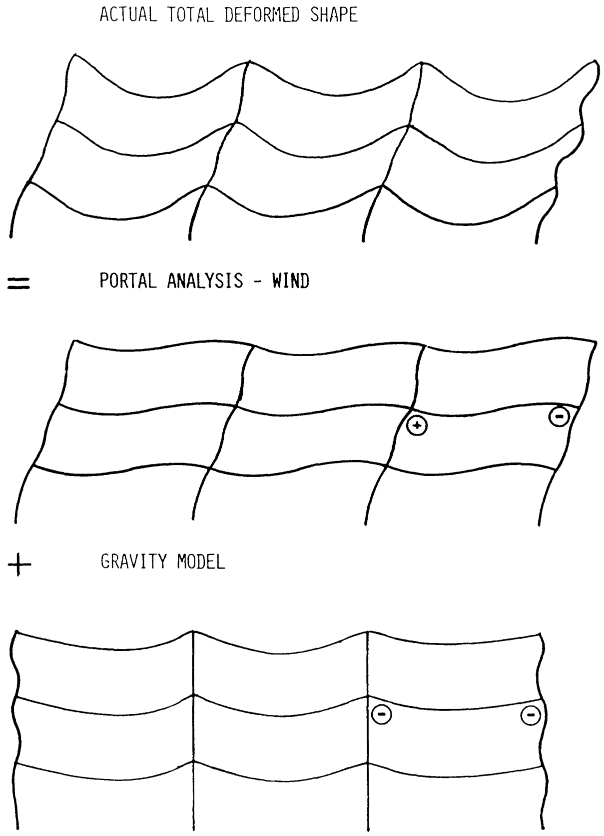


Fig. 2. Component deformed shapes

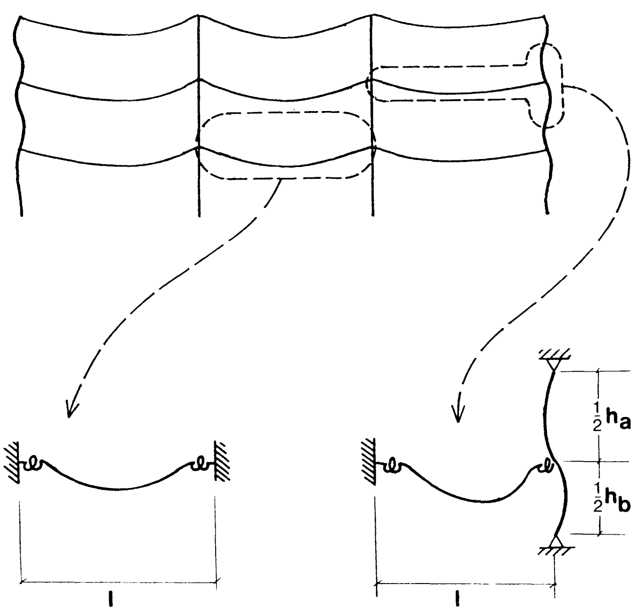


Fig. 3. Flexibly connected girder models

Flexible-end Moment Calculations

Analysis of the exterior girder model for symmetric gravity loads on the girder[1,5] leads to simple expressions for the flexible-end moments at the interior end of the girder M_i and at the exterior end M_e as a fraction of the usual fixed end moment M_f:

$$M_i = \frac{M_f}{C_i}$$

$$M_e = \frac{M_f}{C_e}$$

where the coefficients C_i and C_e depend on connection stiffness k on the bending stiffness of the girder EI_g/l and on the relative flexibility of the girder to the columns $(I_g/l)/(\Sigma I_c/h)$:

$$C_i = (1 + 2a) - \frac{g}{3(g + 1 + 6a)}$$

$$C_e = (1 + 2a) + \frac{2g(1 + 3a)}{3(1 + 6a)}$$

and the parameters, a and g are defined as:

$$\text{Connection flexibility parameter } a = \frac{EI_g}{kl}$$

$$\text{Relative flexibility factor } g = \frac{I_g/l}{\Sigma I_c/h}$$

The above expressions are easily automated on a programmable calculator. Alternatively, Fig. 4 gives a design aid for obtaining values for C_i and C_e for realistic frame configurations.

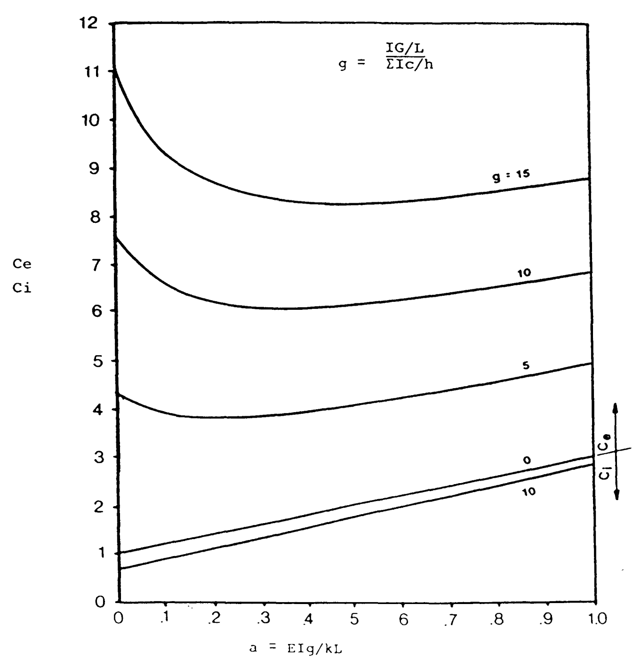


Fig. 4. Flexible end-moment coefficients

SIMPLIFIED DESIGN OF FLEXIBLY-CONNECTED FRAMES

The above method for calculating flexible end moments can be used to predict gravity moments in a frame design if appropriate accounting of connection nonlinearity and column second-order effects can be made, based on observations of these nonlinear effects from analytic studies.[1,5]

Appropriate Connection Stiffness Values

Close study of the results of extensive analyses of frames with nonlinear connections indicate the secant stiffnesses of beam-to-column connections near ultimate frame capacity were typically 20% of the initial stiffness k_i at leeward ends of girders and 80% of k_i at the windward ends of girders, when combined gravity and wind loading was applied. So, it is reasonable to assume an average connection stiffness of, say, 0.5 k_i when computing the flexible end moments for design.

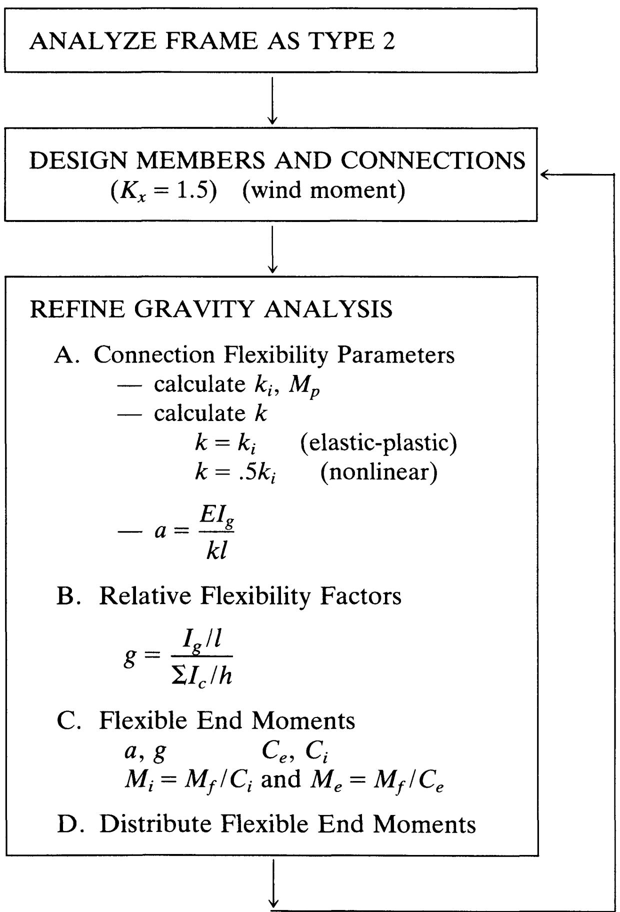


Fig. 5. Simplified PR design procedure

For frames with elastic-plastic connections, it is more appropriate to use the calculated value of k_i directly in the flexible end moment computations. If this moment exceeds the plastic capacity of the connection M_p, then simply use M_p as the flexible-end moment.

Appropriate Effective Length Factors for Column Design

From the extensive analytic studies,[1,2,5] all frame failures were in an overall frame sway mode of instability, caused by plastification of the leeward column stack. No frames failed in the classical story shear mode presumed in the popular Jackson-and-Moreland nomographs. Therefore, based on engineering judgement, it is recommended to use an effective length factor of 1.5 for in-plane buckling effects. Use of the nomographs is *not* recommended. The value of 1.5 recommended here has been tested by analyzing a full range of buildings based on these design assumptions and the ensuing design procedure: all buildings attained their intended load factors without failure.

Proposed Design Procedure

The proposed design procedure starts with the same steps used in a Type 2 design, improves the estimates of moments due to gravity as influenced by connection stiffness and results typically in frames with lighter girders but heavier exterior columns. The steps are outlined in Fig. 5. A detailed example follows:

Detailed Example

An example frame is designed here to illustrate the application of the proposed Design Procedure.

Step 1: Frame Definition

The frame to be designed is three stories tall, with 14 ft per story. It has three bays, with 39 ft per bay. The frame is assumed to be a Type 2 wind bent with 30-ft tributary width of building.

Step 2: Loading Definition

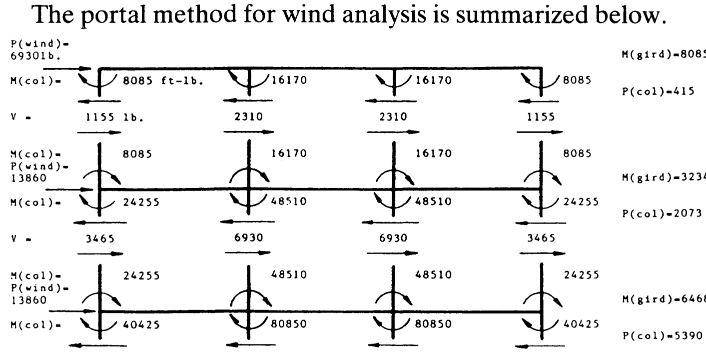
Gravity loading is assumed to be a total deck load of 125 psf with a curtain wall load of 30 psf, vertical area. Wind loads are assumed to be 33 psf uniform over the height of the building.

Step 3: Assumptions

For design, it is assumed girders are braced sufficiently against lateral torsional buckling that allowable bending stress is $F_b = 0.6\ F_y$. A36 steel is to be used throughout. Effective length factor of columns for in-plane, second-order effects is taken as $K_x = 1.5$. Out-of-plane bracing of columns is assumed, so that $K_y = 1.0$. Only W14 series rolled shapes will be considered for design.

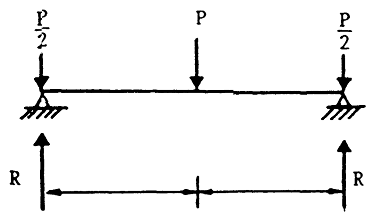
Step 4: Analysis

The portal method for wind analysis is summarized below.



Simple framing assumptions are used for the initial gravity analysis:

Girders:



$P = (125\text{ psf})\ (30')\ (19.5') = 73{,}125\text{ lb.} = R$

M (@ mid-span) $= Pl/4 = 712{,}969$ ft-lb.

Curtain Walls:

P (concentrated) $= (30\text{ psf})\ (30')\ (14')$
$= 12{,}600$ lb.

Exterior Columns:

at each story,

$P = R + P$ (curtain wall)
$= 12{,}600 + 73{,}125$
$= 85{,}725$ lb.

story	P (gravity)
3	85,725 lb.
2	171,450
1	257,175

Interior Columns:

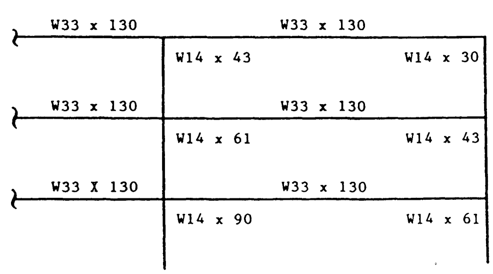
at each story,

$P = 2R = 146{,}250$ lb.

story	P (gravity)
3	146,250 lb.
2	292,500
1	438,750

Step 5: Design

The design resulted in the member sizes shown below, based on AISC Specifications, Part 1.[4]



Initial Type 2 Design

Flange plate connections were used for beam-to-column connections. It was decided to use the same plate size throughout all W33 beams, to carry a design wind moment of 363.8 in.-kips:

Axial load in plate = 363.8 in.-k/(33.09 + t) in.
Compression plate, (governs plate area)
assume width, $b < 11$ in. ($W33 \times 130$ has $b_f = 11.5$)
assume length, $l = 1.5 \times$ width
< 16.5 in.
$r = \text{sqrt}(I/A)$ and assume $K = 1.0$
then $Kl/r = 1.5\ \text{sqrt}(12)\ b/t$
Try $b = 8$ in., $t = 0.25$ in.
Then $P = 10.9$ kips
$Kl/r = 167$,
$F_a = 5.35$ ksi,
allow. $P = 10.7$ kips **o.k.**

Detail calculations for connections, welds, etc. are not shown here for conciseness.

Step 6: Gravity Re-Analysis

Connection calculations (see Appendix for derivations)

Stiffness

$$k = \frac{Etd^2}{3} = \frac{(29{,}000\text{ ksi})(0.25\text{ in.})(33.09 + .25\text{ in.})^2}{3}$$

$= 2.686$ E06 in.-kip/rad.

Moment capacity of flange plates

$$\begin{aligned} M_p &= b \times t \times F_y \times d \\ &= (8'')(.25'')(35\text{ ksi})(33'') \\ &= 2{,}376\text{ in.-kips} \\ &= 198\text{ ft-kips} \end{aligned}$$

Connection flexibility parameter

$$a = EI_g/kl = \frac{(29{,}000\text{ kip})(6{,}710\text{ in.}^4)}{(2.686\text{ E06 in.-}k)(39 \times 12\text{ in.})}$$

$= 0.155$

Relative flexibility factors

$$\text{story 3, } g = \frac{(6{,}710/39)}{(291/14)} = 8.277$$

$$\text{story 2, } g = \frac{(6{,}710/39)}{(291/14 + 428/14)} = 3.350$$

$$\text{story 1, } g = \frac{(6{,}710/39)}{(428/14 + 640/14)} = 2.255$$

Flexible-end moments

interior girders, $C_i = C_e = 1 + 2a = 1.310$

exterior girders,	C_i	C_e
story 3	1.039	5.499
story 2	1.098	3.005
story 1	1.130	2.451

Girder reference moments

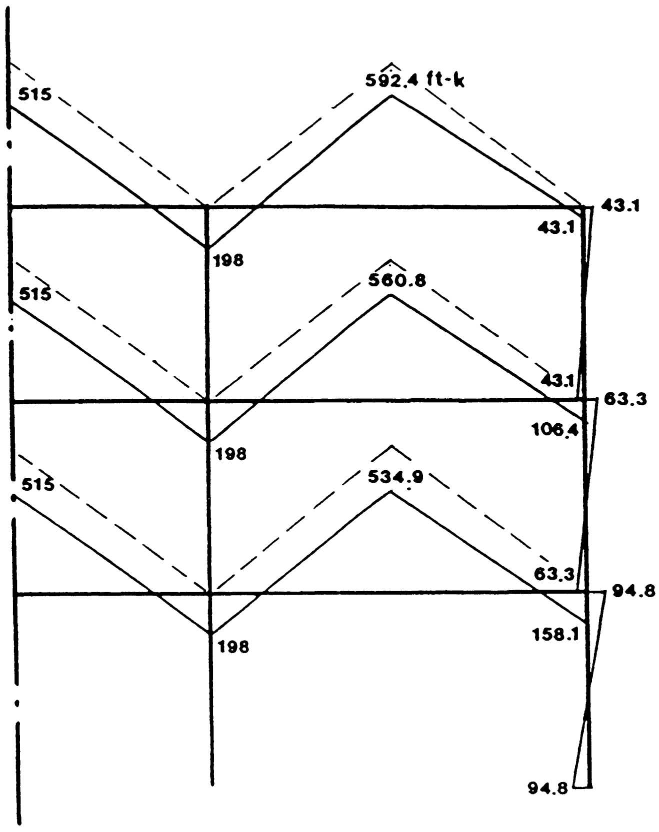
fixed-end moments,

$M_f = Pl/8 = (0.125\text{ ksf})(30')(0.5 \times 39')(39')/8$
$= 356.5$ ft-kips

simple, mid-span moment,

$M_{ss} = Pl/4 = 713.0$ ft-kips

M_i and M_e values are shown directly on the sketch. Note, though, the amount of flexible end moment that could be generated at all interior connections was limited to the upper bound of M_p of the flange plate capacity M_p = 198 ft-k. Adjusted moment diagrams also shown directly on sketch.



Distribution of Moments Accounting for Connection Stiffness

These new moment diagrams are now combined with the original portal analysis results to re-design members with changes in applied forces. The table summarizes the cycles of Steps 5 and 6 for this example. As can be seen, interior columns are not affected in this re-design, interior girders achieve the greatest gains in economy, the uppermost exterior girder does not benefit as much as the other girders because it receives the least rotational restraint from the exterior top column and exterior columns need to be increased in capacity because quite long girders are framed in from one side only, resulting in non-trivial gravity moments.

For these design cycles, the total weight of the frame decreased from one iteration to the next. But essentially all of the economy was achieved in the first re-design cycle, as shown in this table:

Total Weight of Framing Members, Lbs.
(numbers in parentheses are ratios to Type 2)

	Type 2	Cycle-1	Cycle-2	Cycle-3
Ext. Cols.	3,752	5,432	6,300	6,300
	(1.000)	(1.448)	(1.679)	(1.679)
Int. Cols.	5,432	5,432	5,432	5,432
	(1.000)	(1.000)	(1.000)	(1.000)
Girders	45,630	39,156	38,220	38,220
	(1.000)	(0.858)	(0.838)	(0.838)
Total	54,814	50,020	49,952	49,952
	(1.000)	(0.913)	(0.911)	(0.911)
Difference	0	−4,790	−4,862	−4,862

Total weight reduction for this frame was 4,862 pounds, or 8.9%. Note, this represents a true savings, since the connection designs did not change at all. Weight reductions in the range of 4% to 11% have been found for typical building configurations.[1,5]

PR Frames with Composite Girders

When considering the overall sway stability of a frame, one would anticipate that by designing girders as composite girders the overall capacity would be increased because of the increases in girder stiffnesses. A study of a range of Type 2 frame designs was made to assess this possibility.[7] Results showed, however, that while the *EIg* of the girders did increase girder stiffnesses initially, two nonlinear phenomena negated the potential benefits. First, because partial composite construction was used, slip along the concrete-steel interface resulted in a gradual softening of the girders. Second, and more pronounced, because the depth of the steel beam in the composite beam was significantly smaller than the depth of its all-steel counterpart, the steel-beam-to-column connection was significantly smaller than the connection in the all-steel counterpart. The net effect was that the stiffening tendency of the composite girder was nullified essentially by the reduction in connection stiffness. Unless one takes explicit steps to increase the connection stiffness in composite construction, one cannot expect to improve the load-carrying capacity of PR frames using composite girders over that of their all-steel counterparts.

REFERENCES

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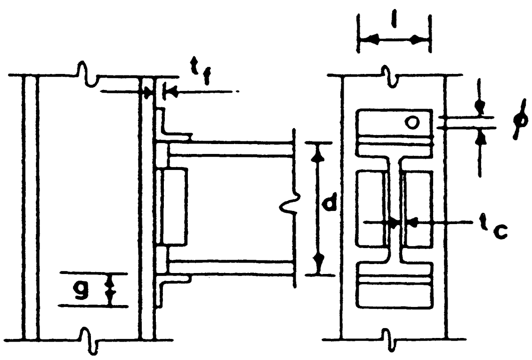
2. *Ackroyd, M. H.* Limit States of Type 2 Steel Building Frames *Research Report to American Iron and Steel Institute, Project No. 311, August 1981.*

3. *Altman, W. G., Jr., A. Azizinamini, J. H. Bradburn and J. B. Radziminski* Moment-Rotation Characteristics of Semi-Rigid Steel Beam-Column Connections *Report to NSF for Grant No. PFR-7923520, University of South Carolina, June 1982, Columbia, S.C.*
4. *American Institute of Steel Construction, Inc.* Specification for the Design, Fabrication, and Erection of Structural Steel Buildings *Effective Nov. 1, 1978.*
5. *Cronembold, J.* Evaluation and Design of Type 2 Steel Building Frames *Master of Architecture Thesis, Rensselaer Polytechnic Institute, May 1986, Troy, N.Y.*
6. *Richard, R. M. and B. J. Abbott* Versatile Elastic-Plastic Stress-Strain Formula *ASCE Journal of the Engineering Mechanics Division, Vol. 101, No. EM4, August 1975 (pp. 511–515).*
7. *Zaremba, C. J.* Limit State Analysis of Partial Composite Girders and Application to Building Frames *Master of Science Thesis, Rensselaer Polytechnic Institute, August 1986, Troy, N.Y.*

APPENDIX

The model used for top and seat angle wind connections was that proposed by Altman, et al.,[3] which used a Frye-and-Morris polynomial moment-rotation curve, as summarized:



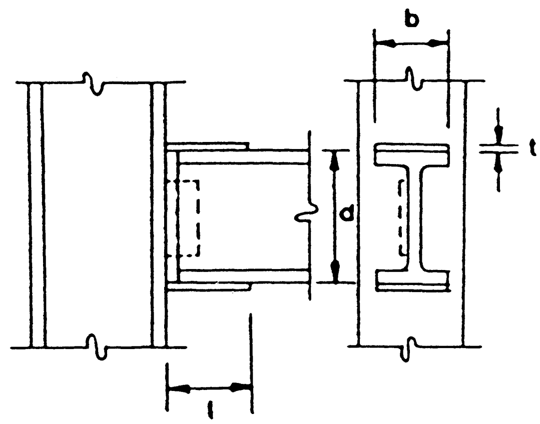
$$P_e = g - \frac{\phi}{2}$$

$$K = t_f^{-1.128} d^{-1.287} t_c^{-0.4145} l^{-0.6941} P_e^{1.35}$$

$$\theta = 0.2234 \times 10^{-4}\ (KM) + 0.18507 \times 10^{-7}\ (KM)^3 + 0.318898 \times 10^{-11}\ (KM)^5$$

$$k_i = .22324 \times 10^{-4} t_f^{-1.128} d^{-1.287} t_c^{-.4145} l^{-.6941} P_e^{1.35}$$

The model used for flange plate connections was derived from strength of materials considerations as shown below, neglecting any rotational stiffness contributed by the shear clip on the web. The resulting elastic-plastic moment-rotation curve was then represented by the Richard-Abbot formulation[6] using $n = 15.5$.



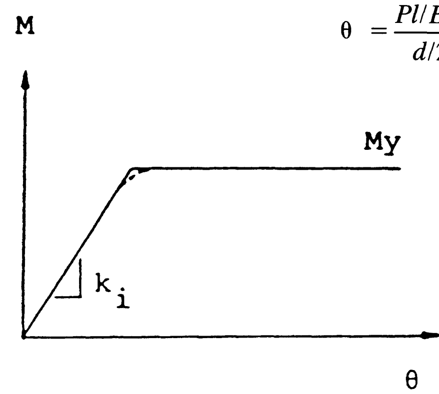
$$k_i = \frac{M}{\theta} = \frac{Ebtd^2}{2l}$$

$$\text{if } l = 1.5b,\ k_i = \frac{Etd^2}{3}$$

$$M_y = (btd)F_y$$

$$M = Pd$$

$$\theta = \frac{Pl/Ebt}{d/2}$$



$$\text{Richard-Abbott Equation: } M = \frac{k_i\theta}{\left[1 + \left(\frac{k_i\theta}{M_y}\right)^{15.5}\right]^{1/15.5}}$$