# Design for Eccentric and Inclined Loads on Bolt and Weld Groups

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 ${f T}$ he 8th Edition AISC Manual of Steel Construction introduced the ultimate strength, instantaneous center of rotation method for computation of the tabulated bolt and weld group eccentricity coefficients C. As discussed in detail by Tide<sup>1</sup> and Brandt,<sup>2,3</sup> this marked a major departure from the traditional and conservative elastic analysis employed in the 7th Edition AISC Manual (and previous editions) and which many design textbook authors still emphasize. The reason for the change was the ultimate strength method better reflected the ductility and load redistribution capability within the group by explicit consideration of each connector's characteristic load-deformation curve.<sup>4,5</sup> The ultimate strength method's connector shape factor effect, somewhat analogous to the plastic section modulus to elastic section modulus ratio for steel beams, consequently accounts for this increase in connection strength. Unfortunately, this type of nonlinear analysis is certainly more complex, it involves trial iterations until satisfactory numerical convergence, thus, is probably only suitable for computerized solutions for those cases not covered by the Manual.

The AISC Manual eccentricity C tables have always been limited to only vertical loads acting on certain connector patterns. A frequent question arises on how to treat inclined eccentric loads resulting from a combination of vertical shear and horizontal axial force. As a partial answer to this possibility, the new 1st Edition AISC Load and Resistance Factor Design (LRFD) Manual has included C coefficients for 45° and 75° eccentric loads in addition to the usual vertical  $(0^{\circ})$  loads. More inclined load tables were calculated, but the new Manual space restrictions precluded their publication. However, in retrospect, these few additional tables may be of little practical value, since linear interpolation between load angles for both bolts and welds may be significantly unconservative and is not recommended in the LRFD Manual, particularly for angles larger than about 45°. Essentially, this leaves the engineer or

Nestor R. Iwankiw is AISC Director, Research and Codes, Chicago, Illinois. detailer in a quandary when faced with a load angle other than 0°, 45° or 75° for LRFD design. For the Allowable Stress Design (ASD) 8th Edition Manual, any load angle besides 0° presents a design difficulty which is further compounded by this warning published in the AISC *Engineering for Steel Construction*:<sup>6</sup>

> Tables XIX to XXVI, inclusive, Manual Part 4, have been prepared to simplify design. These tables, based on the ultimate strength method, tabulate coefficients, the use of which will be explained later in the text. Since these coefficients were derived on the basis of physical tests with loading at the ultimate strength level, they should only be used for the weld pattern indicated in the tables, and not in combination with any additional loading. For these cases, a special ultimate strength analysis is required.

More recently, an AISC Engineering Journal article<sup>7</sup> has provided some ASD design aids consistent with the ultimate strength method for inclined loads on common C, Land line weld groups used in framing connections. Nevertheless, the general problem remains, i.e., how does one efficiently figure or design for an inclined eccentric load in a situation that is not covered by listed C coefficients? This subject is addressed in this paper, and a few possible design solutions, both old and new, will be reviewed and compared. The final goal is to provide some practical guidance and options on this topic for both ASD and LRFD applications.

# Method 1—Ultimate Strength/Instantaneous Center of Rotation

As previously described, the nonlinear instantaneous center of rotation model is the basis for both the 8th Edition AISC Manual and the 1st Edition LRFD Manual connector eccentricity tables  $(C_u)$ . It is definitely the preferred and most accurate method for this type of design problem.

Briefly, a maximum connector deformation establishes a linear strain distribution, which in turn, determines the corresponding connector forces from an empirical loaddeformation relationship. Finally, moment and force equilibrium is checked relative to an assumed center of rotation (Fig. 1). If a balance is not achieved, the center of rotation  $r_o$  is adjusted and the connector forces correspondingly recalculated until equilibrium is satisfied within an acceptable numerical tolerance. Unless one has access to a computer and the resources to program this detailed solution procedure, it does not realistically present a viable option for a special analysis or for design by hand. However, the coefficients  $C_u$  calculated by this state-of-the-art method represent the best estimates of connection strength and will serve, for the purposes of this paper, as an upper-bound benchmark to the alternate and less accurate, but much quicker solutions.

#### Method 2—Elastic

Prior to the 8th Edition Manual, a pure elastic analysis was the standard way to calculate the eccentricity coefficients that herein will be designated  $C_e$ . This linear elastic solution always provides a conservative lower bound since additional loads can be redistributed to the connectors in a manner similar to additional simple beam flexural capacity beyond initial yield. However, the elastic method also results in an inconsistent factor of safety relative to failure. Hence, it has been superseded by the ultimate strength method in the AISC Manuals.

Since most elastic derivations have been restricted to only the applied vertical load case, the effects of the load horizontal component must be added to represent the general situation for a bolt group shown in Fig. 1. Note  $x_o$  is merely the horizontal distance from the connector centroid to the applied force line of action; the true load eccentricity



Fig. 1. Inclined and eccentric load on bolt group

 $\ell$  perpendicular to the resultant force line of action can be calculated from geometry ( $x_o = \ell$  only for vertical loads).

From Fig. 1, the elastic vectorial superposition of fastener stresses for a group of identical bolts results in this almost familiar equation:

$$R_{max} = \left[ \left( \frac{P \cos \theta}{n} + \frac{P x_o(\cos \theta) r_{x, max}}{\Sigma r_i^2} \right)^2 + \left( \frac{P \sin \theta}{n} + \frac{P x_o(\cos \theta) r_{y, max}}{\Sigma r_i^2} \right)^2 \right]^{1/2}$$
(1)

which after algebraic re-arrangement produces:

$$P = \frac{R_{max}}{\left\{\cos^{2}\theta \left[\frac{1}{n^{2}} + \frac{2x_{o}r_{x,max}}{n\Sigma r_{i}^{2}} + \frac{x_{o}^{2}(r_{x,max}^{2} + r_{y,max}^{2})}{(\Sigma r_{i}^{2})^{2}}\right] + \sin^{2}\theta \frac{1}{n^{2}} + \sin\theta\cos\theta \left(\frac{2x_{o}r_{y,max}}{n\Sigma r_{i}^{2}}\right)\right\}^{1/2}}$$
(2)

The consideration of the applied force horizontal component has added the two  $\sin\theta$  terms under the radical to the basic vertical load solution (first  $\cos^2\theta$  term). By definition, the elastic  $C_e$  coefficient is simply one divided by the square root factor, so that

$$P = C_e r_v \tag{3}$$

 $r_{v}$ , the appropriate single fastener design strength (allowable load in ASD or  $\phi R_n$  in LRFD), replaces  $R_{max}$  consistent with the Manual's format. Note, this bolt coefficient  $C_e$ is only a function of the fastener pattern geometry and applied load angle and is independent of the selected design method or bolt design strength  $r_v$ . As a check,  $C_e$  by Eq. 2 will reduce to its expected maximum, i.e., the total number of fasteners n, when  $\theta = 0^\circ$  and  $x_o = 0$  (concentric vertical load), or  $x_o = 0$  (inclined concentric load), or  $\theta = 90^\circ$ (concentric horizontal load).

In general, the bolt eccentricity coefficients are uncoupled from the design strength regardless whether derived from an elastic analysis, as here, or from the previously described ultimate strength center of rotation method. Hence, the bolt coefficients from the 8th Edition Manual and the LRFD Manual are identical for identical situations.

For a similar elastic analysis of C-shaped or line weld groups, simply replace n by (1 + 2k),  $\sum r_i^2$  by  $I_p$  (weld polar moment of inertia about centroid) and  $x_o$  by a (Fig. 2).

However, since the standard AISC Manual format assumes an E70 electrode strength per  $\frac{1}{16}$ th in. of weld size per lineal inch as a basis, Eq. 3 is altered so that

$$P = C_e C_1 D\ell \tag{4}$$

 $C_1$  = ratio of actual electrode strength to E70

D = number of sixteenths of weld size

 $\ell$  = characteristic vertical length

$$C_{e} = \frac{f_{D}}{\left\{\cos^{2}\theta \left[\frac{1}{(1+2k)^{2}} + \frac{2ar_{x, max}}{(1+2k)I_{p}} + \frac{a^{2}(r_{x, max}^{2} + r_{y, max}^{2})}{I_{p}^{2}}\right] + \sin^{2}\theta \frac{1}{(1+2k)^{2}} + \sin\theta\cos\theta \left(\frac{2ar_{y, max}}{(1+2k)I_{p}}\right)\right\}^{1/2}}$$

where

- $f_D$  = design strength of E70 weld per sixteenth inch of weld size per lineal in., kips
  - $= .3(70)(\frac{1}{16})(.707) = 0.928 \text{ k/in}./\frac{1}{16} \text{ in.}$  (ASD)
  - $= .75(.6)(70)(\frac{1}{16})(.707) = 1.392 \text{ k/in}./\frac{1}{16} \text{ in. (LRFD)}$

Again, a check of the default concentric load cases ( $a = 0, \theta = 0^{\circ}$ , or a = 0, or  $\theta = 90^{\circ}$ ) simplifies the weld  $C_e$  by Eq. 5 to  $f_D$  (1 + 2k), its maximum value for use in Eq. 4. In contrast to bolts, because the  $f_D$  design strength is included in the eccentricity coefficient for Eq. 4, the comparable weld C values are numerically different between the 8th Edition and LRFD Manuals for identical cases.

In summary, the derived general elastic  $C_e$  coefficients for bolts and welds, as represented in Eqs. 2 and 5, respectively, can provide a rather quick, closed-form and lower bound answer for any inclined eccentric load applied to any connector pattern.

## Method 3—Rotation of Inclined Load to Vertical

By this simple approach, any inclined load is just rotated to the vertical position and subsequently evaluated from the AISC Manual table (Fig. 3). It is a very quick and direct method and usually results in conservative answers. All angular orientations of the applied load thereby reduce to the same answer.

For high  $\theta$ 's and large  $x_o$  distance, this approximation will be unduly punishing as the exaggerated vertical load component and its increased eccentric moment will then have most effect. For an extreme example of an eccentric load that is almost horizontal (high  $\theta$ ), the actual eccentric moment ( $P\ell$ ) would be negligible and the load could practically be considered concentric. However, blind adherence to this rule would produce a high calculated eccentric moment of  $Px_o$ . It is advisable to use another solution for such a situation.

In the case of smaller  $\theta$ 's and  $x_o$ 's, this approximation is reasonable and is often within a few percent of the actual  $C_u$ , sometimes on the low side. The reason for this minor anomaly is the nature of the change in the trigonometric sin $\theta$  and cos $\theta$  functions for  $\theta \le 45^\circ$ , wherein the horizontal component related to sin $\theta$  increases proportionately more than the vertical component related to cos $\theta$  decreases. For design purposes, this small deviation may usually be ignored (see later examples) with the overall conclusion that such a rotation of the inclined load through small to moderate angles will always result in a safe approximate answer.

# Method 4—Algebraic Addition of Direct Horizontal Shear and Eccentric Moment Resistances

This novel approximation developed by the author can be considered a refinement of Method 3. Instead of merely assuming that the full applied load is rotated to the vertical



position, only its actual vertical component is used to identify an initial connector resistance to the applied eccentric moment by means of the conventional Manual C tables. This looked-up value is called  $C_o$ . The horizontal component of the applied load acts concentrically to the centroid of the bolt or weld group, by definition; thus, its effect can be independently and directly evaluated.

Finally, these two effects are added algebraically, rather than vectorially, to provide a total estimate of connector strength (Fig. 4).

Theoretically, true superposition is only valid in the elastic response range, as the previously referenced *Engineering for Steel Construction* statement warns. However, this shortcut approach to the inelastic connector analysis simply adds the distributed horizontal load component effect to the pre-analyzed vertical load effect, regardless of the actual maximum connector force orientation due to the vertical component and its eccentricity. Since algebraic additions always result in a larger answer than by vectorial superposition, Method 4 should consistently be conservative.

To put this possible design approach into mathematical form compatible with the AISC Manual C tables, one starts with the simple trigonometric relationship between the horizontal  $P_H$  and vertical  $P_v$  applied load components as shown in Fig. 1.

$$\frac{P_H}{P_v} = \frac{P \sin \theta}{P \cos \theta} = \tan \theta \tag{6}$$

A part of the total connector capacity  $C_o$  will resist  $P_v$  and the remainder will resist  $P_H$  in direct shear according to the algebraic addition simplification. First, for bolt groups let

$$P_{\nu} = C' r_{\nu}$$

$$P_H = \left(\frac{C_o - C'}{C_o}\right) nr_v$$

where

and

n = total number of fasteners (different definition than given in AISC Manuals)

 $r_{v}$  = single fastener design strength



Fig. 4. Algebraic vs. vectorial addition

- $C_o$  = AISC Manual tabulated C value  $C_u$  for a given vertical load case
- C' = derived eccentricity coefficient for only the vertical load component  $\leq C$ .

Eq. 8 represents the remainder of the bolt group capacity that can resist in direct shear the applied horizontal component.

From Eq. 6, 
$$\left(\frac{C_o - C'}{C_o}\right)nr_v = C'r_v \tan\theta$$

which may be solved for the intermediate unknown C':

$$C' = \frac{C_o n}{C_o \tan\theta + n} \tag{9}$$

If the applied load is truly vertical  $(\tan \theta = 0)$ , Eq. 9 will correctly reduce to its maximum— $C' = C_o$  and  $P_H = 0$ ; also, for a concentric horizontal load  $(\tan 90 = \infty)$ , C' = 0,  $P_v = 0$  and  $P_H = nr_v$ . Since the resistance components have now been identified by geometry, the resultant applied load P may be expressed as

$$P = C_a r_v = \sqrt{P_v^2 + P_H^2}$$

or, equivalently,

$$C_a = \sqrt{(C')^2 + \left(1 - \frac{C'}{C_o}\right)^2 n^2}$$
(10)

where

(7)

(8)

 $C_a$  = approximate eccentricity coefficient for total applied load by Method 4

Thus,  $C_a$  could be used to estimate the design eccentricity coefficient in the usual Manual format (for any load orientation that the Manual tabulates corresponding  $C_o$ values). Again, as a check,  $C_a = C_o$  for  $\theta = 0$  and  $C_a = n$ for  $\theta = 90^\circ$ . If one accepts the Method 3 premise that  $C_o$  is a true minimum value and subsequently combines this with the realization that *n* is the maximum value for concentric loading, the following practical working limits for bolt groups may be placed on  $C_a$ :

$$n \ge C_a \ge C_o \tag{11}$$

This corrects the unnecessarily low  $C_a$  coefficients that result from small  $x_o$  distances and small to moderate load angle configurations. For example, for a concentric load at  $\theta = 45^\circ$ ,  $C_a$  by Eq. 10 would be 0.707*n*, whereas the proper answer is 1.0*n*. This maximum error (on the safe side) is due exclusively to the algebraic addition simplification. Imposing  $C_o = n$  for this case as the minimum  $C_a$  value rectifies this gross error.

For C-shaped or line weld groups, again a simple substitution of variables in the previous equations will provide a comparable  $C_a$  approximation: replace n with  $f_D(1 + 2k)$ ;  $\Sigma r_i^2$  with  $I_p$ , and  $x_o$  by a (Fig. 2). Thus, the similar weld  $C_a$ algorithm will be as follows:

1. Compute C' for given weld configuration and applied load orientation

$$C' = \frac{C_o f_D (1+2k)}{C_o \tan\theta + f_D (1+2k)}$$
(12)

2. Determine  $C_a$ 

$$C_a = \sqrt{(C')^2 + \left(1 - \frac{C'}{C_o}\right)^2 f_D^2 (1 + 2k)^2}$$
(13)

3. Finally, check  $C_a$  limits

$$f_D(1+2k) \ge C_a \ge C_o \tag{14}$$

With such assumptions and derivation, only Eqs. 9, 10 and 11 for bolts and 12, 13 and 14 for welds are needed to compute the approximate  $C_a$ . Again, note bolt C' and  $C_a$  equations are independent of  $r_v$ , while their weld counterparts are a function of  $f_D$ . Similar to the elastic Method 2 solution, the calculations involved are very straightforward and direct.

 $C_a$  usually results in higher design coefficients than  $C_e$  because one of its components,  $C_o$ , is based on the ultimate strength Method 1 of the AISC Manual tables. Also, because of the conservatism of arithmetic addition of connector resistances,  $C_a$  is also assured of being lower than the maximum  $C_u$  values by Method 1. Thus,  $C_a$  captures both the computational ease of  $C_e$  and some of the redistribution benefits of  $C_u$  without their respective previously discussed shortcomings. The  $C_a$  method will generally be most effective for larger angles and larger eccentricities. Otherwise, the minimum  $C_a$  limit will prevent any major inaccuracies.

Method 4 clearly disputes the previously quoted statement in the Introduction from the AISC Engineering for Steel Construction.<sup>6</sup> It demonstrates AISC Manual ultimate strength coefficients ( $C_o = C_u$ ) may be used in combination with an additional imposed horizontal load component, as explained here, to provide a satisfactory and quick solution to any inclined eccentric load problem.

#### Method 5—Full Plastic Analysis

Another alternate procedure possible for non-tabulated cases is the fully plastic approach.<sup>8,9</sup>

For a load  $P_u$ , acting at an eccentricity  $\ell$ , the elastic center of rotation at a distance  $r_o$  from the centroid of the bolt group (Fig. 1) is given by:

$$r_o = k_o^2 / \ell$$

- $k_o$  = polar radius of gyration of the bolt group relative to the centroid =  $\sqrt{\Sigma r_i^2/n}$
- $r_i$  = distance from centroid to the *i*th bolt
- n =total number of bolts

Using this center of rotation, the applied ultimate moment is  $P_u(\ell + r_o)$  and the resisting moment, if all bolts are assumed to have reached their ultimate capacity  $R_{ult}$  at complete redistribution is:

$$M_u = R_{ult} \sum_{i=1}^n d_i \tag{16}$$

(15)

in which  $d_i$  = distance from the center of rotation to the *i*th bolt. This approximation predicts failure when:

$$C_p = \frac{P_u}{R_u} = \frac{\sum_{i=1}^{n} d_i}{\ell + r_o}$$
(17)

where  $C_p$  = plastic eccentricity coefficient

A check for transverse force equilibrium is not necessary.

Because of the simplified assumption all bolts are at their ultimate load  $R_{ult}$  this analysis provides unconservative results, compared to the most accurate Method 1- $C_u$  values, on the order of about 15%. Thus, it is recommended, if this fully plastic analysis method is used, the computed capacity  $C_p$  coefficient must be reduced appropriately, since this plastic method provides an unreachable upper bound. For this reason and because of complications introduced by weld load-deformation dependence on the resistance force angle relative to the weld axis, this bolt plastic analysis will not be extended any further.

#### **Comparisons and Evaluation**

All but two (Methods 3 and 4) of the previously discussed eccentricity design procedures are well documented and do not require any additional justification. Method 5 is a mere extension of the usual steel plastic design theory. Methods 1 and 2 have been developed rigorously, they have served as standard procedures for years and are widely accepted. However, the extra complications caused by the horizontal component of an inclined load warrant the additional elaborations of this article and the introduction of approximate Methods 3 and 4 solutions which, in actuality, can be condensed through Eqs. 11 or 14 into basically one new procedure.

To give a better illustration of the similarities, differences and parameter sensitivity of the new approximate answers relative to the other more established methods, a few sample problems have been analyzed and the results tabulated.





Load	C <sup>*</sup> <sub>u</sub>	C <sub>e</sub>	C <sub>o</sub>	C <sub>a</sub>	C <sub>p</sub>	
Angle, θ	(Method 1)	(Method 2)	(Method 3)	(Method 4)	(Method 5)	
0 15 30 45 60 75 90	$ \begin{array}{c} 0.17\\ 0.19\\ 0.23\\ 0.33\\ 0.62\\ 3.0 \end{array} $	0.17 0.17 0.186 0.222 0.303 0.532 3.0	0.17	$\begin{array}{c} 0.17 \\ 0.17 \\ 0.186 \\ 0.224 \\ 0.305 \\ 0.534 \\ 3.0 \end{array}$	$\begin{array}{c} 0.17 \\ 0.18 \\ 0.20 \\ 0.24 \\ 0.35 \\ 0.67 \\ 3.0 \end{array}$	

\* Preferred and most accurate.

Values from LRFD Manual Table X

For this large eccentricity bolt problem, the baseline  $C_o$  value of 0.17 can be read directly from either the 8th Edition or the LRFD Manual Table X. The bracketing of the approximate  $C_a$  by  $C_o = 0.17$  and n = 3.0 is evident throughout the entire load angle range. Furthermore, complete reliance on only the Method 3 rule would yield terribly conservative answers for the larger angles and is not recommended. As expected, the elastic Method 2  $C_e$ 's are consistently below the Method 1  $C_u$ 's, which, in turn, are below the Method 5  $C_p$ 's. The  $C_a$ 's closely track the corresponding  $C_e$ 's for this problem and, thus, are definitely reasonable. At the extremes,  $\theta = 0$  and 90, all the methods give essentially identical answers.

The LRFD Manual caution against linear interpolation between angles will now be demonstrated. Using only the preferred Method 1, interpolation between 60° and 90° for a desired  $C_u$  value at 75° will produce 1.67, whereas the actual  $C_{\mu}$  is a much lower 0.62 (only 37% of interpolated value). The largest possible interpolation errors ordinarily will occur in this higher angle range where the eccentricity coefficient increases very rapidly as the horizontal concentric load position ( $\theta = 90^{\circ}$ ) is approached. This nature of the  $C_{\mu}$  versus  $\theta$  function is clearly illustrated in Fig. 5. Nevertheless, even straight-line interpolations between 45° and 75°  $C_{\mu}$ 's (now given in the LRFD Manual) for 60° will yield 0.425, compared to the actual  $C_{\mu}$  of 0.33 (about 78%) of interpolated value). Even though the unconservative interpolation error will decrease for load angles less than 45° from the vertical, this type of linear interpolation is strongly discouraged in favor of one of the rational methods presented in this paper. Alternately, as suggested in the LRFD Manual, it is safe to use the  $C_u$  values tabulated for the next lower angle, i.e., for the latter example of a load angle of  $60^\circ$ , simply use the  $45^\circ$  coefficient of 0.23. Such an approximation, much like employed for Method 3, can be rather conservative in many cases.

Another configuration with 9 total bolts in a single row was analyzed and the results presented in Table 2 for a small (Case A) and large (Case B) load offset distance  $(x_o)$ :





Load Angle, θ	<i>C</i> <sup>*</sup> <sub><i>u</i></sub> (Method 1)	C <sub>e</sub> (Method 2)	C <sub>o</sub> (Method 3)	<i>C<sub>a</sub></i> by Eq. 10 (Method 4)	C <sub>p</sub> (Method 5)			
0 15 30 45 60 75 90	8.52 8.47 8.44 8.48 8.59 8.73 9.0	8.356 7.748 7.432 7.397 7.644 8.179 9.0	8.52 8.52 8.52 8.52 8.52 8.52 8.52	8.52 -7.04 -6.36 -6.19 -6.16 -7. <u>26</u> 9.0	8.71 8.71 8.72 8.78 8.87 8.96 9.0			
Case B— $x_o = 36$ in.								
Load Angle, θ	<i>C<sub>u</sub></i> (Method 1)	C <sub>e</sub> (Method 2)	C <sub>o</sub> (Method 3)	<i>C<sub>a</sub></i> by Eq. 10 (Method 4)	C <sub>p</sub> (Method 5)			
0 15 30 45 60 75 90	$     \begin{array}{r}         \hline             1.54 \\             1.59 \\             1.74 \\             2.07 \\             2.75 \\             \hline             4.30 \\             9.0 \\         \end{array}     $	1.238 1.236 1.325 1.54 2.0 3.17 9.0	<u>1.54</u> 1.54	1.54 1.52 1.62 1.86 2.38 3.63 9.0	1.69 1.74 1.93 2.32 3.11 4.84 9.0			

\* Preferred and most accurate.

Values from LRFD Manual Table X

More observations can be made and trends noted from the Table 2 data. It is obvious that the case B,  $x_o = 36$  large load distance from the bolt group centroid severely reduces all the eccentricity coefficients until the load position approaches a horizontal orientation with little real eccentricity. In contrast, the almost concentric load position of case A,  $x_o = 2$  results in rather large C coefficients for all inclinations. This demonstrates the powerful effect of eccentric moment on connector group capacity.

As previously mentioned in the five methods of analysis, the Method 4 algebraic combination will produce unnecessarily low  $C_a$  values by Eq. 10 for small  $x_o$  distances and small to moderate load angles. Case A demonstrates this problem and its suggested quick resolution by simple repeated use of the Method 3- $C_o$  as the minimum limit given in Eq. 11. One may argue against this approach by pointing to the slight decrease in the baseline Method 1- $C_u$ ,  $\theta = 0$ value for Case A through about 45°. However, this maximum reduction is only on the order of 1-2%, which is within the numerical tolerance for this solution method, and is hardly noteworthy for design purposes. This calculated



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Case	A	 a	=	0	.2.	k	=	0.	5

Load	<i>C</i> <sup>*</sup>	C <sub>e</sub>	C <sub>o</sub>	C <sub>a</sub>	
Angle, θ	(Method 1)	(Method 2)	(Method 3)	(Method 4)	
0	2.494	1.88	2.494	2.494	
15	2.50	1.81	2.494	2.08	
30	2.53	1.82	2.494	1.90	
45	2.59	1.91	2.494	1.95	
60	2.67	2.11	2.494	1.95	
75	2.75	2.41	2.494	2.22	
90	2.784	2.784	2.494	2.784	
	Case E	B-a = 2.0,	k = 0.5	<u>.</u>	
0 15 30 45 60 75 90	0.538 0.556 0.614 0.735 0.977 1.504 2.784	0.39 0.393 0.425 0.499 0.656 1.057 2.784	0.538 0.538	0.538 0.530 0.559 0.638 0.806 1.208 2.784	

\* Preferred and most accurate.

Values from LRFD Manual Table XXII.

reduction admittedly is more pronounced for the elastic  $C_e$ 's (maximum drop from  $\theta = 0^\circ$  of about 15%), but this does not represent a real strength decrease according to the Method 1 ultimate strength theory based on tests. Hence, there should be little qualms about accepting  $C_o$  as the minimum design coefficient for any load angle.

Case B further shows the design advantages of  $C_a$  over the lower  $C_e$  values while remaining on the safe side of the upper bound  $C_u$ 's. As predicted before, the Method 5- $C_p$ 's are always above the  $C_u$ 's and can be used only with due care to maintain the intended factor of safety for ASD or the reliability index for LRFD.

The bolt C coefficients shown in Tables 1 and 2 may be used for either LRFD or ASD, since they are uncoupled from any design parameters.

Analysis of weld groups will follow the same trends as noted for bolts. For example, a general C-shaped weld with an inside eccentric load (Table XXIII in 8th Edition Manual; Table XXII in LRFD Manual) produce the results tabulated in Table 3 for LRFD design.

First of all, the maximum capacity for a concentric loading, such as  $\theta = 90^{\circ}$ , is simply  $f_D (1 + 2k) = 1.392(2) = 2.784 \text{ k/in}./\frac{1}{16}$  in. Again, for the small load offset Case A, all the eccentricity coefficients are much larger than for the large load offset Case B. For those load angles wherein  $C_a$  is less than  $C_o$ , use  $C_o$ . The  $C_a$  values will then be bounded by the  $C_u$  and  $C_e$  numbers, as expected. Similar to bolt usage, do not use linear interpolation between load angles for tabulated weld C values.

A reminder that one major difference between bolts and welds lies in the inclusion of the E70 weld design strength  $f_D$ in the weld eccentricity coefficient itself. Thus, since the ratio of 1.392/0.928 = 1.5, the LRFD weld eccentricity coefficients will be about this much greater than comparable ASD values. Consequently, Table 3 values are intended exclusively for LRFD design.

Two simple design problems in the Appendix illustrate the easy applicability of the algebraic  $C_a$  method for inclined loads.

#### **Summary and Conclusions**

This article has reviewed and critically evaluated five rational methods for the general analysis/design of eccentric and inclined loads on bolt and weld groups. The strengths and weaknesses of each have been discussed and demonstrated. Relative comparisons were made for a few sample problems:

- 1. Method 1. Ultimate strength/instantaneous center of rotation is recognized as the best predictor of connector group strength. Because of its demanding computational requirements, however, it may be difficult to implement for non-routine special applications.
- 2. Method 2. The elastic approach  $C_e$  has been extended in this paper to include horizontal load components. This still presents a rather conservative but expedient option for analysis/design.
- 3. Method 3. It simply uses the tabulated *C* coefficients for vertical loads in the AISC Manuals as the default safe value for any load angle. While very easy, this will be extremely punishing for certain situations.
- 4. Method 4. A new approximate Method 4 has been derived from algebraic addition of connector strengths and proposed as a convenient alternative for cases not directly covered in the AISC Manual. The algorithm involves the solution of only two short equations, which, in combination with the previous Method 3 minimum  $C_o$  value limit, produces answers bounded by the elastic and ultimate strength values, a clearly acceptable design range.

This latter Method 4 algebraic addition has a valid conservative basis and is, thus, proposed as an attractive design option wherein the favored instantaneous center of rotation solution is not feasible. Readers are encouraged to try the  $C_a$  approach when applicable and execute additional comparisons. A more thorough parameter study, possibly by an engineering intern or a university student, would be useful to explore some different connector configurations or other aspects of the  $C_a$  method, such as new refinements or unforeseen limitations.

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## APPENDIX—DESIGN EXAMPLES

- **Ex. 1.** Two vertical rows of 6 bolts,  $5\frac{1}{2}$  in. apart, standard 3-in. spacing, eccentric load *P* at 16 in. from group centroid inclined at an angle of  $60^{\circ}$  (similar to example on pg. 4-61 8th Edition Manual and pg. 5-62 LRFD Manual). Using  $\frac{7}{8}$ -in. A325-N bolts, what is maximum design capacity  $P_{max}$ ?
- **ASD** 1. From Table XII, 8th Edition Manual:  $C_o = 3.55$

2. Eq. 9 
$$C' = \frac{3.55(12)}{3.55 \tan 60 + 12} = 2.35$$

3. Eq. 10 
$$C_a = \sqrt{2.35^2 + \left(1 - \frac{2.35}{3.55}\right)^2 (12^2)} = 4.69$$
  
3.55 < 4.69 < 12 o.k.

4. Using 
$$r_v = 12.6$$
 kips,  $P_{max} = 4.69 \times 12.6$   
= 59.1 kips

- **LRFD** 1. From Table XII, angle =  $0^{\circ}$ , LRFD Manual:  $C_o = 3.55$ 2. and 3. Identical to ASD,  $C_a = 4.69$ 
  - 4. Using  $\phi r_v = 21.1$  kips,  $P_{max} = 4.69 \times 21.1 = 99$  kips
- **Ex. 2.** C-shape weld, E70xx electrodes, with  $\ell = 10$  in., kl = 5 in., eccentric service load P = 87 kips at 75° with al = 8.75 (Fig. 2). Determine minimum required weld size.
- ASD (Problem similar to example on pg. 4-75, 8th Edition Manual)
  - 1. k = 0.5, a = 0.875, use Table XXIII, 8th Edition Manual,  $C_o = 0.704$

2. Eq. 12 
$$C' = \frac{.704 (.928) [1 + 2(.5)]}{.704 \tan 75 + .928 [1 + 2(.5)]}$$
  
= 0.27

3. Eq. 13 
$$C_a = \sqrt{.27^2 + \left(1 - \frac{.27}{.704}\right)^2 (.928)^2 [1 + 2(.5)]^2}$$
  
 $C_a = 1.176$ 

$$0.704 < 1.176 < .928 [1 + 2(.5)] = 1.856$$
 o.k.

4.  $C_1 = 1.0$   $D = \frac{87}{(1.176)(10)} = 7.4$ , say 8 min. weld size  $= \frac{8}{16} = \frac{1}{2}$  in.

- **LRFD** Same as for ASD, but eccentric factored load  $P_u = 130$  kips (problem similar to example on pg. 5-90, LRFD Manual).
  - 1. k = 0.5, a = 0.875, use Table XXII (angle = 0°),

LRFD Manual, 
$$C_o = 1.136$$
  
2. Eq. 12  $C' = \frac{1.136 (1.392)[1 + 2(.5)]}{1.136 \tan 75 + 1.392[1 + 2(.5)]}$   
 $= 0.45$   
3. Eq. 13  $C_a = \sqrt{.45^2 + (1 - \frac{.45}{1.136})^2 (1.392)^2 [1 + 2(.5)]}$   
 $C_a = 1.74$   
 $1.136 < 1.74 < 1.392 [1 + 2(.5)] = 2.784$  o.k.

(Note—exact  $C_u$  value can be obtained for this case from LRFD Table XXII, angle = 75°, equal to 2.17)

4. Using computed approximate  $C_a = 1.74$ ,  $C_1 = 1.0$ 

$$D = \frac{130}{1.74(10)} = 7.47, \text{ say 8}$$
  
d size =  $\frac{8}{1.74(10)} = \frac{1}{2}$  in.

min. weld size = 
$$\frac{1}{16} = \frac{1}{2}$$
 in.

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