Mill Building Frame Analysis: Distribution of Lateral Loads

HOWARD K. VILAS

In recent years, few papers have covered the subject of mill buildings. In one of the papers,[1] a programmable solution for mill building columns was given using the equations proposed in the Murray-Graham stepped column model. It dealt with a truss-column system with knee braces or parallel chord trusses where both chords were connected to the columns, providing a partially fixed condition at the truss column interface.

Not all mill buildings have knee-braced roof trusses because of the lack of clearances and other restrictions. This paper is an analysis of such mill buildings. Also, it is an attempt to relate the stiffness of trusses and columns in a mill building with knee braces and to develop a method of analyzing such frames. It deals only with the distribution of horizontal loads acting along the transverse direction. The horizontal loads, either external loads or those induced from vertical loads, are carried by the frame or bent along a transverse row (Fig. 1; for sect. A-A showing row 6, see Fig. 3). The analysis, involving trusses and columns, is on the basis of a plane frame, i.e., loads on a frame in a row are carried by that frame only and not shared by adjacent rows. This assumption is more valid for wind loads than for localized crane wheel loads (localized compared to the length of the building). However, in this paper it is assumed bottom chord bracing, which makes it possible for adjacent rows to share crane-induced loads, is either not present for the entire length of the building or not effective enough.

TRUSS SYSTEMS WITHOUT KNEE BRACES

In this system, the column bases are considered fixed and their tops pinned, with translation and rotation being allowed. The truss bottom chord acts as a prop distributing loads from loaded bay to adjacent ones in that row, depending upon flexural stiffness of the columns (in that row). The elasticity of the props can be included in the calculations for a more refined analysis of load distribution. The condition of compatibility imposed is the horizontal deflection of the top of each column in that row at the prop level is the same. The method is illustrated in the following example of a two-bay mill building. In the calculations the elasticity of the props is not included for simplicity. The equations are set up as though prop forces are compressive and imposed deflections at the top of each column are to the right of the paper when they are positive.

Flexibility coefficients d_{ij} for each column, as a cantilever, are required for analysis. The first subscript i is the location on the column where horizontal deflection is being calculated. The second subscript j represents load point where either a unit load or a moment is considered, depending on the actual loading, to obtain the deflection. For example d_{11} is the deflection at location 1 due to a unit load (or moment) acting at location 1 on that column and d_{12} is the deflection at location 1 due to a unit load (or moment) acting at location 2. In this paper the flexibility coefficients are calculated using the moment area method. Consider the case of a propped cantilever with a load P in the middle of span (Fig. 2). In order to find the prop reaction the column is made statically determinate by removing the prop. The cantilever deflection d_1 at the prop location 1 due to load P

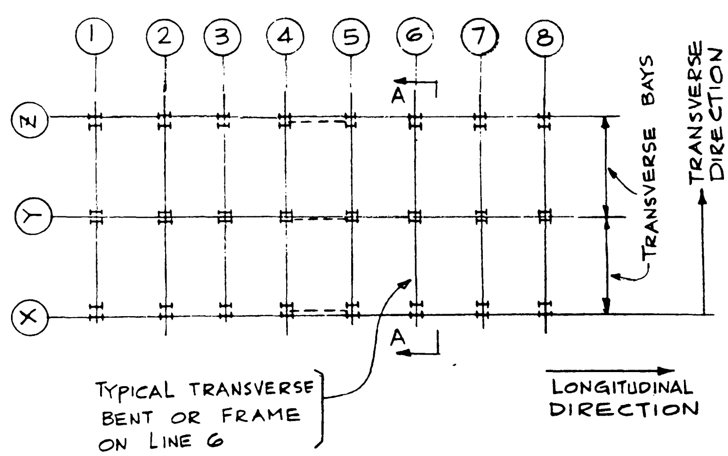


Figure 1

Howard K. Vilas is a Structural Engineer with the firm of Sargent & Lundy Engineers, Chicago, Illinois.

at 2 is $P \times d_{12}$ where $d_{12} = 1 \times (L/2)\ (L/2)\ (1/2)\ ((2/3)\ (L/2) + (L/2))/EI = (5/48)\ (L^3/EI)$. The reaction at the prop (rigid) is such that the deflection d_1 is zero. The opposing deflection due to R is $R \times d_{11}$ where $d_{11} = L^3/(3EI)$. The deflection compatibility is $R \times d_{11} = P \times d_{12}$. Therefore reaction $R = (d_{12}/d_{11}) \times P = (5/16)P$. If the prop is elastic because it is connected to another flexible system, then the value of the reaction will be smaller compared to the above. If the other flexible system to which the above column is connected has its own loads then that system would further influence the value of the prop reaction. The analysis of prop forces that follows is similar to the above except that all the columns in a given row are considered in the deflection compatibility calculations.

Referring to Figs. 3 and 4, the flexibility coefficients for each column in the example are as follows (see Appendix A for derivation; locations 1 through 10 are shown in Fig. 3):

For column X d_{11} = 1490.6 $/EI$ ft/kip at 1 due to unit load at 1
d_{12} = 498.4 $/EI$ ft/kip at 1 due to unit load at 2

For column Y d_{44} = 1800.55$/EI$ ft/kip at 4 due to unit load at 4
d_{45} = 1116.55$/EI$ ft/kip at 4 due to unit load at 5
d_{55} = 745.3 $/EI$ ft/kip at 5 due to unit load at 5
d_{56} = 249.2 $/EI$ ft/kip at 5 due to unit load at 6
d_{46} = 326.8 $/EI$ ft/kip at 4 due to unit load at 6
d_{57} = 5.62$/EI$ ft/kip-ft at 5 due to unit moment at 7
d_{47} = 6.75$/EI$ ft/kip-ft at 4 due to unit moment at 7

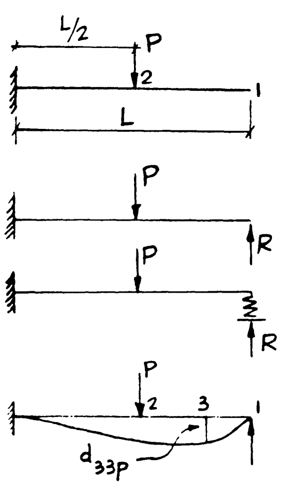


Figure 2

For column Z d_{88} = 2469.3 $/EI$ ft/kip at 8 due to unit load at 8
d_{89} = 477.3 $/EI$ ft/kip at 8 due to unit load at 9
d_{810} = 10.8 $/EI$ ft/kip-ft at 8 due to unit moment at 10

Let d_x and d_z be the deflections at the top of columns X and Z respectively due to loads acting on each of them calculated as a pure cantilever. Similarly let d_{y1} and d_{y2} be the deflections in column Y at lower prop and upper prop levels respectively due to loads acting on that column. Equating the deflections at lower prop level in columns X and Y,

$$d_x - 1490.6\ P_1 = d_{y1} + 745.3\ P_1 - 1116.55\ P_2$$

or
$$2235.9\ P_1 - 1116.55\ P_2 = (d_x - d_{y1}) \quad (1)$$

Similarly at the upper prop level:

$$d_{y2} + 1116.55\ P_1 - 1800.55\ P_2 = d_z + 2469.3\ P_2$$

or
$$1116.55\ P_1 - 4269.85\ P_2 = (d_z - d_{y2}) \quad (2)$$

Equations 1 and 2 in matrix form:

$$\begin{bmatrix} 2235.9 & -1116.55 \\ 1116.55 & -4269.85 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} d_x - d_{y1} \\ d_z - d_{y2} \end{bmatrix} \text{ or}$$

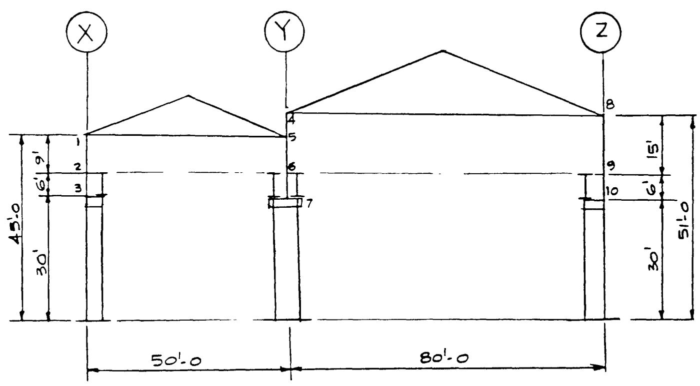


Figure 3

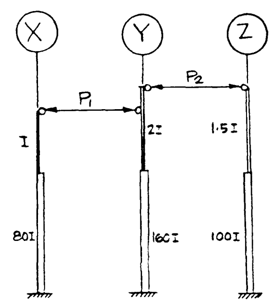


Figure 4

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \frac{1}{7433.86} \begin{bmatrix} 3.8241 & -1.0 \\ 1.0 & -2.00251 \end{bmatrix} \begin{bmatrix} (d_x - d_{y1}) \\ (d_z - d_{y2}) \end{bmatrix} \quad (3)$$

Once the values and the directions of the prop forces are found shear forces and bending moment values can be determined in each column along its length. The following load cases are considered as examples.

1. Crane surge loads in bay Y-Z, 15 kip load acting 36 ft above column bases.

   $d_x = 0$
   $d_{y1} = -15 \times 249.2 = -3738.0$
   $d_{y2} = -15 \times 326.8 = -4902.0$
   $d_z = -15 \times 477.3 = -7159.5$
   $(d_x - d_{y1}) = (0 - (-3738.0)) = 3738.0$
   $(d_z - d_{y2}) = (-7159.5 - (-4902.0)) = -2257.2$

   Substituting the above in Eq. 3 we get $P_1 = 2.227$ kips and $P_2 = 1.111$ kips.

   The loading and the resulting moments on each column are shown in Fig. 5.

2. Crane surge loads in bays X-Y and Y-Z, 10 kips and 15 kips respectively acting 36 ft above column bases.

   $d_x = 10 \times 498.4 = 4984.0$
   $d_{y1} = (15 + 10) \times 249.2 = 6230.0$
   $d_{y2} = (15 + 10) \times 326.8 = 8170.0$
   $d_z = 15 \times 477.3 = 7159.5$
   $(d_x - d_{y1}) = (4984.0 - 6230.0) = -1246.0$
   $(d_z - d_{y2}) = (7159.5 - 8170.0) = -1010.5$

   Substituting the above in Eq. 3 we get $P_1 = -0.505$ kips and $P_2 = +0.105$ kips.

   The loading and resulting moments on each column are shown in Fig. 6.

3. Moments in bay Y-Z of 450 ft-kips and 500 ft-kips on columns Y and Z respectively due to crane vertical loads.

   $d_x = 0$
   $d_{y1} = +450 \times 5.62 = 2529.0$
   $d_{y2} = +450 \times 6.75 = 3037.5$
   $d_z = -500 \times 10.8 = -5400.0$
   $(d_x - d_{y1}) = (0.0 - 2529.0) = -2529.0$
   $(d_z - d_{y2}) = (-5400.0 - (3037.5)) = -8437.5$

   Substituting the above in Eq. 3 we get $P_1 = -0.166$ kips and $P_2 = +1.9327$ kips.

   The loading and resulting moments on each column are shown in Fig. 7.

4. Wind loads on the frame as follows: on the windward side, a load of 0.72 kip/ft on column X and on exposed area of column Y, on the leeward side a load of 0.45 kip/ft on column Z, a lateral wind load of 9.0 kips and 14.4 kips along the low and high truss bottom chords respectively assumed distributed equally to each end of the truss. The deflections at top of the columns due to a uniform load of 1.0 kip/ft for each column for the loaded length are as follows (see Appendix A for derivation):

   $d_1 = 12656.25/EI$ ft
   $d_4 = 8724.38/EI$ ft
   $d_5 = 5585.6\ /EI$ ft
   $d_8 = 24420.15/EI$ ft

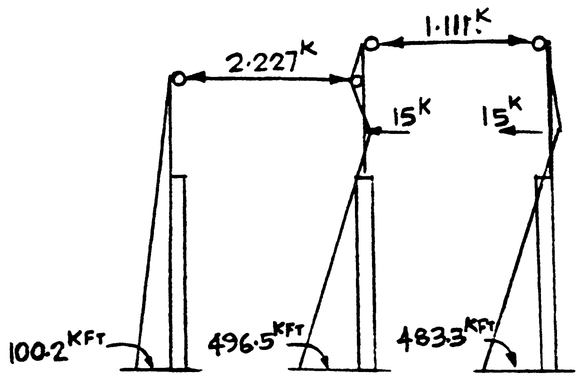


Figure 5

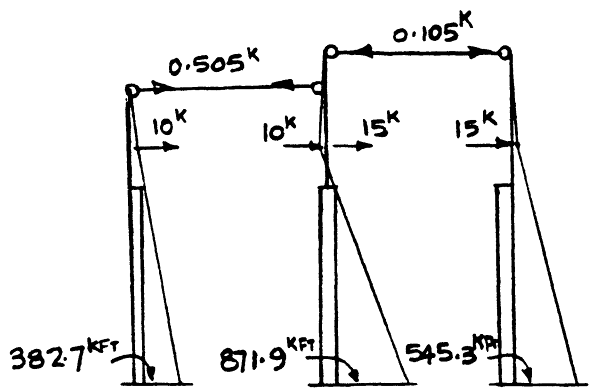


Figure 6

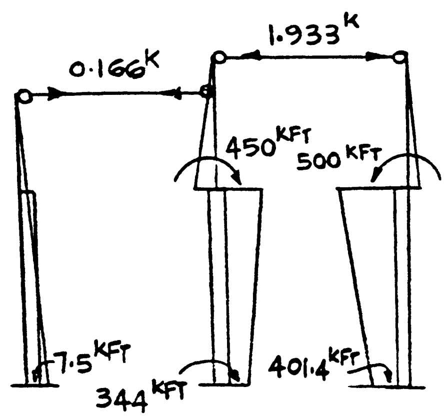


Figure 7

The imposed deflections at the tops of columns are given below.

$d_x = 0.72 \times 12656.26 + 4.5 \times 1490.6 = 15820.21$
$d_{y1} = 0.72 \times 5585.6 + 4.5 \times 745.3 + 7.2 \times 1116.55 = 15414.64$
$d_{y2} = 0.72 \times 8724.38 + 4.5 \times 1116.55 + 7.2 \times 1800.55 = 24269.99$
$d_z = 0.45 \times 24420.15 + 7.2 \times 2469.3 = 28768.03$
$(d_x - d_{y1}) = (15820.21 - 15414.64) = 405.57$
$(d_z - d_{y2}) = (28768.03 - 24269.99) = 4498.04$

Substituting the above in Eq. 3, we get $P_1 = -0.3964$ kips and $P_2 = -1.1571$ kips.

The loading and resulting moments on each column are shown in Fig. 8.

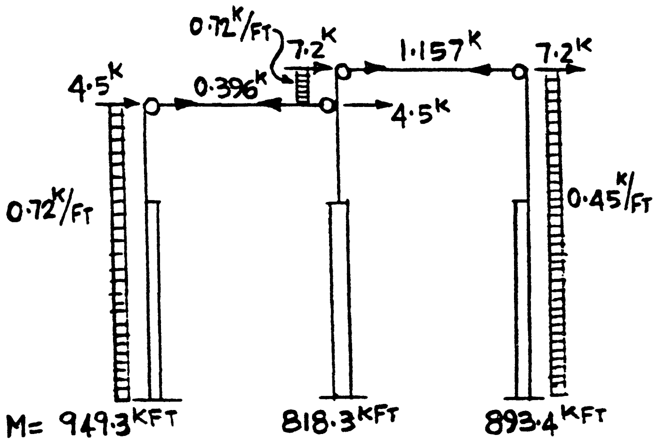


Figure 8

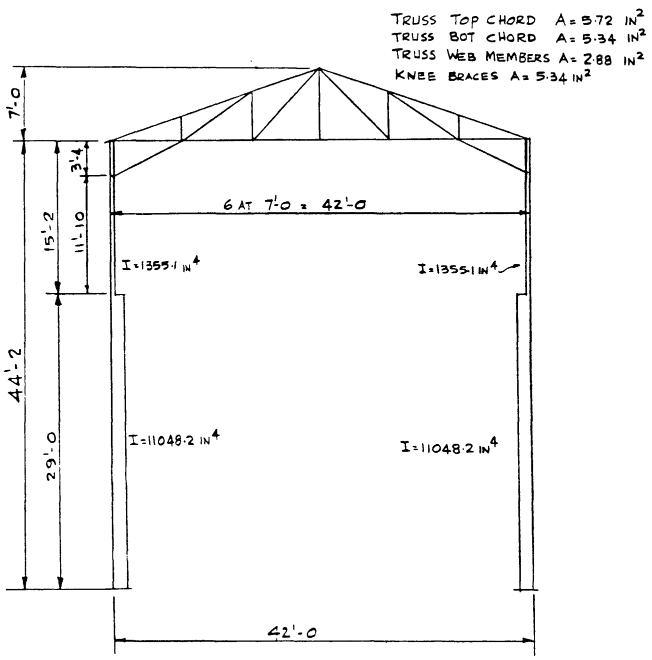


Figure 9

TRUSS SYSTEMS WITH KNEE BRACES

In Ref. 2, the authors report a test performed on an actual mill column which was loaded by an eccentric vertical load (due to crane loads). A structural model was proposed to fit the measurements made on that column. The same model was extended for the crane side thrust case. In addition to the column bases being considered fixed the model assumes a pin in the column at the mid height of knee brace with sway considered prevented. For the case of wind loads a different model was proposed with fixity assumed at the bottom of the knee brace. Reference 2 details the steps involved.

To compare the results given in the above reference with the analyses used here in this paper, the column sizes, lengths and loadings considered are the same as in that reference (see Fig. 9 for details). The roof span, rise and its members are assumed. The analysis is done in two steps. First, a temporary lateral restraint is introduced at truss eaves level preventing the frame from swaying. For a given loading the knee brace loads and lateral restraint reactions are calculated (initial distribution). Then the frame is released to sway. The knee-brace loads for this sway correction is determined. The addition of the two sets of values gives the final results.

Initial Distribution

Referring to the propped cantilever example in Fig. 2, consider location 3, on the column some distance from the prop. The deflection at that location is given by the deflection due to P minus the deflection due to R, both calculated on the basis of a cantilever, i.e., $d_3 = P \times d_{32} - R \times d_{31}$. Substituting R with $P \times d_{12}/d_{11}$, $d_3 = P \times d_{32} - P \times d_{12} \times d_{31}/d_{11} = P(d_{11} \times d_{32} - d_{12} \times d_{31})/d_{11}$. The imposed deflections d_{g1} and d_{h1} in the column at the knee brace location are calculated in a manner similar to d_3 above. If the only lateral load on the column is a unit load at the location 3, then the deflection at 3, d_{33p}, is given by $d_{33p} = 1 \times d_{33} - R \times d_{31} = d_{33} - (d_{13} \times d_{31}/d_{11}) = d_{33} - (d_{31}^2/d_{11}) = (d_{11} \times d_{33} - d_{31}^2)/d_{11}$, which is the flexibility coefficient based on a propped cantilever. The flexibility coefficients d_{hhp} and d_{ggp} are calculated in a similar manner as d_{33p} above. If there is a second support at the location 3 the reaction at that support is such that $R_3 \times d_{33p} = d_3$, or $R_3 = d_3/d_{33p}$. Substituting from above, $R_3 = P \times (d_{11} \times d_{32} - d_{12} \times d_{31})/(d_{11} \times d_{33} - d_{31}^2)$. If it is a settling support at location 3, such as an elastic prop or a knee brace or attachment to a flexible system, then the value of the reaction R_3 is influenced by the response of the settling support. All the above deliberations are on the assumption that the prop at the end of the column does not deflect (sway prevented case). Now, the equations for the initial distribution will be developed (Fig. 10).

Consider the imposed deflections due to loads on each of the columns at the knee brace locations acting to the right. For this condition the brace forces in members GC and HD will be compressive and tensile respectively. At point G deflection d_{g1} due to external loads is to the right and restoring deflection due to $P_1 \cos\theta_1$ is equal to $P_1 \cos\theta_1 d_{ggp}$. The net deflection d_g acting to the right is $d_{g1} - P_1 \cos\theta_1 d_{ggp}$. Similarly net deflection d_h in column EF at H acting to the right is $d_{h1} - P_2 \cos\theta_2 d_{hhp}$. Corresponding net deflection d_c in truss bottom chord at C is $P_1 \sin\theta_1 d_{cc} - P_2 \sin\theta_2 d_{cd}$ and d_d at D is $P_2 \sin\theta_2 d_{dd} - P_1 \sin\theta_1 d_{cd}$. All these are shown in Fig. 10. Now relating the net deflections in columns and beams through the knee brace axial elongation or shortening we have at knee brace GC

$$d_g \cos\theta_1 - d_c \sin\theta_1 = P_1 d_{brc1} \quad \text{or}$$
$$P_1(\sin^2\theta_1 d_{cc} + \cos^2\theta_1 d_{ggp} + d_{brc1}) - (P_2 \sin\theta_1 \sin\theta_2 d_{cd}) = d_{g1} \cos\theta_1 \qquad (4)$$

Similarly at knee brace HD

$$-P_1(\sin\theta_1 \sin\theta_2 d_{cd}) + P_2(\sin^2\theta_2 d_{dd} + \cos^2\theta_2 d_{hhp} + d_{brc2}) = d_{h1} \cos\theta_2 \qquad (5)$$

Grouping Eqs. 4 and 5 in matrix form and solving,

$$\begin{bmatrix} -P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} (\sin^2\theta_1 d_{cc} + \cos^2\theta_1 d_{ggp} + d_{brc1}) & (-\sin\theta_1 \sin\theta_2 d_{cd}) \\ (-\sin\theta_1 \sin\theta_2 d_{cd}) & (\sin^2\theta_2 d_{dd} + \cos^2\theta_2 d_{hhp} + d_{brc2}) \end{bmatrix}^{-1} \begin{bmatrix} d_{g1}\cos\theta_1 \\ d_{h1}\cos\theta_2 \end{bmatrix} \qquad (6)$$

A negative sign is assigned to the value of P_1 so that all negative values of P are compressive and positive values are tensile. The values of d_{g1} and d_{h1} are positive when they produce deflections towards the right of each column. Having obtained the knee brace loads the next step is to calculate sway shear and sway correction.

For the example that is chosen the matrix coefficients are determined as follows (Fig. 9):

$$\theta_1 = \theta_2 = \tan^{-1}(40/84) = 25.463°.$$
$$\cos\theta_1 = \cos\theta_2 = 0.9028 \qquad \sin\theta_1 = \sin\theta_2 = 0.4299$$

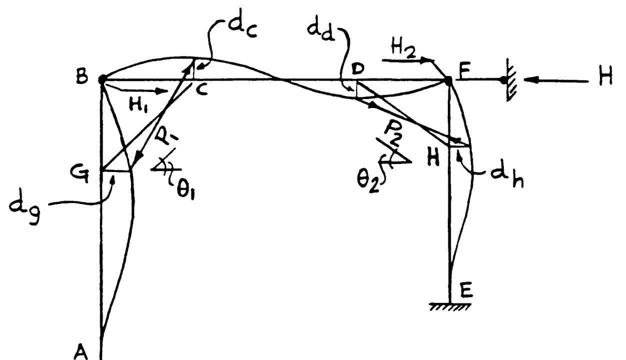
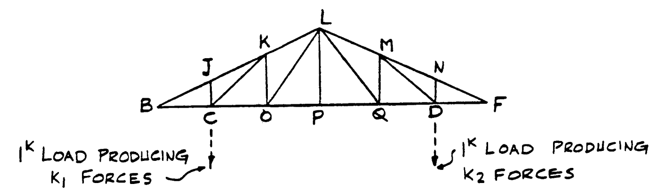


Figure 10

Table 1



MEMBER	LENGTH L IN	AREA A IN²	L/A IN⁻¹	K₁ KIPS	K₂ KIPS	K₁²·L/A	K₁K₂ L/A
BJ	88.54	5.72	15.48	-2.635	-0.527	107.48	21.50
JK	↓	↓	↓	-2.635	-0.527	107.48	21.50
KL	↓	↓	↓	-1.054	-0.527	17.20	8.60
LM	↓	↓	↓	-0.527	-1.054	4.30	8.60
MN	↓	↓	↓	-0.527	-2.635	4.30	21.50
NF	↓	↓	↓	-0.527	-2.635	4.30	21.50
BC	84.0	5.34	15.73	+2.5	+0.5	98.31	19.66
CO	↓	↓	↓	+1.0	+0.5	15.73	7.87
OP	↓	↓	↓	+0.5	+0.5	3.93	3.93
PQ	↓	↓	↓	+0.5	+0.5	3.93	3.93
QD	↓	↓	↓	+0.5	+1.0	3.93	7.87
DF	↓	↓	↓	+0.5	+2.5	3.93	19.66
KO	56.0	2.88	19.44	-0.5	—	4.86	—
MQ	56.0	↓	19.44	—	-0.5	—	—
CK	100.96	↓	35.06	+1.803	—	113.97	—
OL	118.79	↓	41.25	+0.707	—	20.63	—
LQ	118.79	↓	41.25	—	+0.707	—	—
MD	100.96	↓	35.06	—	+1.803	—	—
					Σ =	514.28	166.12

$d_{cc} = \Sigma K_1^2 L/AE = 514.28/E$ IN/KIP

$d_{cd} = \Sigma K_1 K_2 L/AE = 166.12/E$ IN/KIP

BY SYMMETRY $d_{cc} = d_{dd} = 514.28/E$ IN/KIP

Flexibility coefficients for the truss bottom chord at knee brace locations in vertical direction from Table 1 are:

$$d_{cc} = d_{dd} = 514.28/E \text{ in./kip} \qquad d_{cd} = 166.12/E \text{ in./kip}$$

Flexibility coefficients for each column, at locations indicated, in the horizontal directions are:
(see Appendix B for derivations)

$$d_{bb} = d_{ff} = 5792.78/E \text{ in./kip}$$
$$d_{gg} = d_{hh} = 4167.51/E \text{ in./kip}$$
$$d_{bg} = d_{fh} = 4863.26/E \text{ in./kip}$$
$$d_{ggp} = d_{hhp} = 84.6175/E \text{ in./kip}$$
$$d_{brc1} = d_{brc2} = L/AE = 93.04/(5.34E) = 17.42/E \text{ in./kip}$$

Substituting the above values in Eq. 6

$$\begin{bmatrix} -P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 181.431 & -30.699 \\ -30.699 & 181.431 \end{bmatrix}^{-1} \begin{bmatrix} d_{g1}\cos\theta_1 \\ d_{h1}\cos\theta_2 \end{bmatrix}$$

$$= \frac{1}{1041.558} \begin{bmatrix} 5.91 & 1.0 \\ 1.0 & 5.91 \end{bmatrix} \begin{bmatrix} d_{g1}\cos\theta_1 \\ d_{h1}\cos\theta_2 \end{bmatrix}$$

Having solved for the knee-brace loads, the next step is to calculate the lateral restraint reaction at the truss bottom chord level which is then released to obtain the sway correction.

SWAY CORRECTION

In Fig. 11, let the sway of the frame be to the right. Two equations are needed to solve for the knee brace loads, one equation for the frame base shears added to equal the external sway load H, and another for the deflection at the top of each column to be equal. For the direction of the sway the knee brace load is tensile in GC and compressive in HD. Consider the deflected shape of the frame as shown in Fig. 11. For column AB the net horizontal offset between points G and B, d_g, is given by the quantity deflection at B minus the deflection at G.

$$d_g = (P_1\text{Cos}\theta_1 d_{gb} - H_1 d_{bb}) - (P_1\text{Cos}\theta_1 d_{gg} - H_1 d_{gb}) \quad (7)$$

d_g can also be written in terms of the deflection that the brace suffers at its ends, i.e.,

$$d_g\text{Cos}\theta_1 - d_c\text{Sin}\theta_1 = P_1 d_{brc1}$$

where $d_c = P_1\text{Sin}\theta_1 d_{cc} - P_2\text{Sin}\theta_2 d_{cd}$.

Substituting the values of d_c and d_g in Eq. 7 and rearranging:

$$H_1 = \frac{P_1\{(d_{gb} - d_{gg})\text{Cos}^2\theta_1 - d_{cc}\text{Sin}^2\theta_1 - d_{brc1}\} + P_2\text{Sin}\theta_1\text{Sin}\theta_2 d_{cd}}{(d_{bb} - d_{gb})\text{Cos}\theta_1} \quad (8)$$

Base shear for column AB then is $P_1\text{Cos}\theta_1 - H_1$ (9)

Deflection at the top of column is $P_1\text{Cos}\theta_1 d_{gb} - H_1 d_{bb}$ (10)

Similarly for column EF

$$H_2 = \frac{P_2\{(d_{hf} - d_{hh})\text{Cos}^2\theta_2 - d_{dd}\text{Sin}^2\theta_2 - d_{brc2}\} + P_1\text{Sin}\theta_1\text{Sin}\theta_2 d_{cd}}{(d_{ff} - d_{hf})\text{Cos}\theta_2} \quad (11)$$

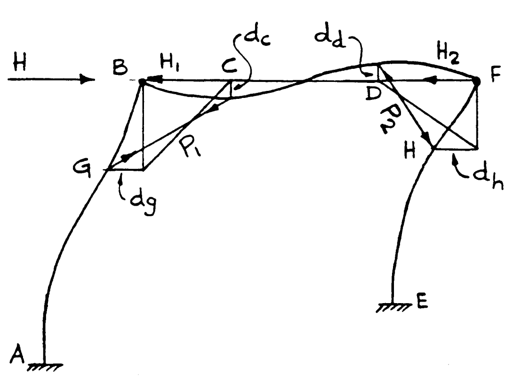


Figure 11

Base shear for column EF then is $P_2\text{Cos}\theta_2 - H_2$ (12)

Deflection at the top of column is $P_2\text{Cos}\theta_2 d_{hf} - H_2 d_{ff}$ (13)

For the mill building frame in the example,

$d_{bb} = 5792.78/E$ $d_{gb} = 4863.26/E$ $d_{gg} = 4167.51/E$
$d_{cc} = 514.28/E$ $d_{cd} = 166.12/E$ $d_{bb} - d_{gb} = 929.52/E$

Substituting these values in the equations, we get for column AB:

$$H_1 = \frac{454.647P_1 + 30.699P_2}{839.208} = 0.54176P_1 + 0.03658P_2$$

Base shear $= 0.36108P_1 - 0.03658P_2$
Deflection $= 1252.449P_1 - 211.8999P_2$

For column EF (the values are the same as those for column AB):

$$H_2 = 0.03658P_1 + 0.54176P_2,$$

Base shear $= -0.03658P_1 + 0.36108P_2$ and
Deflection $= -211.8999P_1 + 1252.449P_2$

Adding the base shears for both columns $H = 0.3245P_1 + 0.3245P_2$. The columns are of the same stiffness $P_1 = P_2$. Therefore, $H = 0.649P_1 = 0.649P_2$ or $P_1 = P_2 = 1.5408H$. If the columns were not of the same stiffness then two equations have to be solved for P_1 and P_2.

Examples

a. Eccentric moment of 2882 in.-kips on each column as shown in Fig. 12.

Sway prevented case:

If top of crane leg is identified as J on the column AB then due to unit moment at J,

$d_{bj} = 11.21/E$ in./in.-kip moment
$d_{gj} = 9.95/E$ in./in.-kip moment

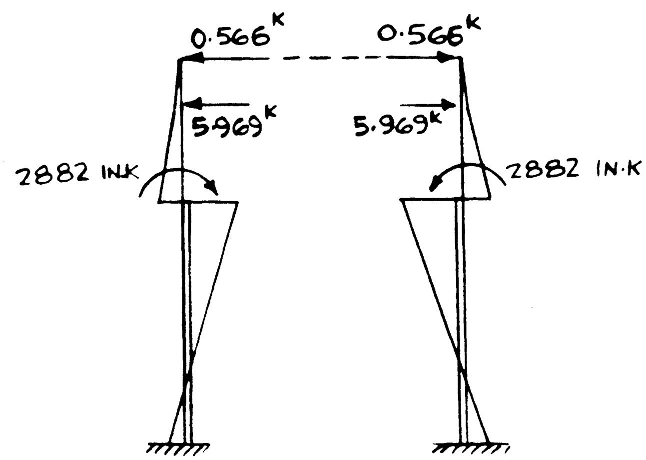


Figure 12

Net deflection d_{g1} at G due to a moment of 2882 in.-kips at J is

$$Ed_{g1} = 2882(d_{bb} \times d_{gj} - d_{gb} \times d_{bj})/d_{bb}$$
$$= 2882(5792.78 \times 9.95 - 4863.26 \times 11.21)/5792.78$$
$$d_{g1} = 1552.8/E \text{ in.}$$

Similarly $d_{h1} = -1552.8/E$ in.

$$\begin{bmatrix} -P_1 \\ P_2 \end{bmatrix} = \frac{1}{1041.558} \begin{bmatrix} 5.91 & 1.00 \\ 1.00 & 5.91 \end{bmatrix} \begin{bmatrix} +1552.8 \times 0.9028 \\ -1552.8 \times 0.9028 \end{bmatrix}$$

$P_1 = -6.61$ kips (compression), $P_2 = -6.61$ kips (compression). Sway shear is zero because equal eccentric moment in each column of the same size induces loads in the truss bottom chord which are equal in value and opposing to each other.

The values of knee-brace loads shown are the final ones. The prop reaction at roof level is equal to (2882 × 11.21 − 5.9675 × 4863.26)/5792.78 = 0.5672 kips.

$H_1 = 0.5672$ kips and $H_2 = 0.5672$ kips

The moments along both columns are as given below:

at the knee brace point 22.6 in.-kips;
at the surge girder level 715.34 in.-kips;
at the top of crane leg 950.59/−1931.41 in.-kips;
at the base of column 342.77 in.-kips.

The above results will be different if vertical load due to the crane is not the same on each of the columns, which is very possible.

b. Crane side thrust load of 16.9 kips on each column as shown in Fig. 13 (point K on column AB).

Sway prevented case:

Due to unit load at K the flexibility coefficients are (see Appendix B):

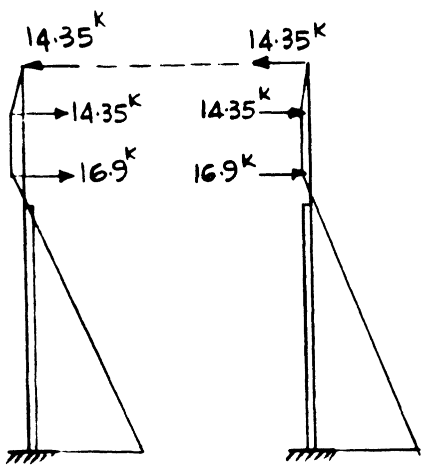


Figure 13

$d_{bk} = 2753.98/E$ in./kip
$d_{gk} = 2470.3\ /E$ in./kip

Net deflection at knee brace points is

$$Ed_{g1} = 16.9(d_{bb} \times d_{gk} - d_{bk} \times d_{gb})/d_{bb}$$
$$= 16.9(5792.78 \times 2470.3 - 2753.98 \times 4863.26)/5792.78$$
$$d_{g1} = 2674.208/E \text{ in.}$$

Similarly, $d_{h1} = 2674.208$ in.

$$\begin{bmatrix} -P_1 \\ P_2 \end{bmatrix} = \frac{1}{1041.558} \begin{bmatrix} 5.91 & 1.00 \\ 1.00 & 5.91 \end{bmatrix} \begin{bmatrix} 2674.208 \times 0.90284 \\ 2674.208 \times 0.90284 \end{bmatrix}$$

$P_1 = -16.018$ kips (compressive), $P_2 = +16.018$ kips (tensile). The prop reaction at roof level for both columns is:

$H_1 = H_2 = (16.9 \times 2753.98 - 16.018 \times 0.90284 \times 4863.26)/5792.78 = -4.105$ kips.

Therefore, sway shear = 2(16.018 × 0.90284 − 4.105) = 20.71 kips acting to the right.

Sway correction case:

The sway shear is 20.71 kips.

$P_1 = 1.5408 \times 20.71 = 31.91$ kips (tensile),
$P_2 = 31.91$ kips (compressive).

$H_1 = 0.57834P_1 = 18.455$ kips and $H_2 = 18.455$ kips

Final results:

The net knee-brace loads are,

$P_1 = 31.91 - 16.02 = 15.89$ kips (tensile),
$P_2 = -31.92 + 16.02 = -15.89$ kips (compressive)
$H_1 = 18.455 - 4.105 = 14.35$ kips
$H_2 = 14.35$ kips

The moments along both columns are as given below:
at the knee-brace point 574.0 in.-kips;
at the surge girder level 574.0 in.-kips;
at the top of crane leg −34.4 in.-kips;
at the base of column −5915.6 in.-kips.

c. Roof wind of 10.0 kips at truss eaves level as shown in Fig. 14.
This is a direct sway shear case for 10.0 kips.

$P_1 = 1.5408 \times 10.0 = 15.408$ kips (tensile),
$P_2 = 15.408$ kips (compressive).
$H_1 = 0.57834\ P_1 = 8.911$ kips and $H_2 = 8.911$ kips.

The final moments along both columns are,

at the knee-brace point 356.44 in.-kips;
at the surge girder level −173.56 in.-kips;
at the top of crane column −353.55 in.-kips;
at the base of column −2093.54 in.-kips.

d. Side wind load of 0.5 kip/ft up to the eaves level only on the windward column as shown in Fig. 15.

Sway prevented case:

In this case instead of using flexibility coefficients actual deflections are calculated. Due to 0.5 kip per ft of load on column AB, the deflection at the top of column as a cantilever is

$d_1 = 40897.1/E$ in. at B
$d_3 = 36073.047/E$ in. at G

Prop reaction at truss $= d_1/d_{bb}$
$= 40897.1/5792.78$
$= 7.06$ kips

The net deflection in the windward column, on the basis of propped cantilever, at the knee-brace point is $d_{g1} = d_3 - R \times d_{gb} = 36073.047 - 7.06 \times 4863.26 = 1738.431/E$ in. The deflection $d_{h1} = 0.0$

$$\begin{bmatrix} -P_1 \\ P_2 \end{bmatrix} = \frac{1}{1041.558} \begin{bmatrix} 5.91 & 1.00 \\ 1.00 & 5.91 \end{bmatrix} \begin{bmatrix} 1738.431 \times 0.90284 \\ 0.0 \end{bmatrix}$$

$P_1 = -8.906$ kips (compressive),
$P_2 = 1.507$ kips (tensile).

The reaction at the roof prop level is

$H_1 = (40897.1 - 8.9058 \times 0.9028 \times 4863.26)/5792.78$
$= 0.3097$ kips
$H_2 = 1.5069 \times 0.9028 \times 4863.26/5792.78$
$= 1.1422$ kips.

The total load on the windward column is 22.0833 kips.

The base shear for the windward column is $= 22.0833 - 0.3097 - 0.9028 \times 8.9058 = 13.733$ kips.

The base shear for the leeward column is $= 1.1422 - 0.9028 \times 1.507 = -0.2183$ kips.

Sway shear $= 22.0833 - 13.7333 + 0.2183 = 8.5683$ kips.

Sway correction case:

$P_1 = 1.5408 \times 8.5683 = 13.202$ kips (tensile), and
$P_2 = -13.202$ kips (compressive).

The reaction at the roof prop level is

$H_1 = 0.57834P_1 = 7.6354$ kips and $H_2 = 7.6354$ kips.

Final results:

The net knee-brace loads are:

$P_1 = -8.906 + 13.202 = 4.296$ kips (tensile), and
$P_2 = 1.507 - 13.202 = -11.695$ kips (compressive).
$H_1 = 0.3097 + 7.6354 = 7.9451$ kips, and
$H_2 = 7.6354 - 1.1422 = 6.4932$ kips.

The final moments along the windward column are:

at the knee-brace point 284.47 in.-kips;
at the surge girder level 304.72 in.-kips;
at the top of crane leg 205.09 in.-kips;
at the base of the column −3541.94 in.-kips.

The final moments along the leeward column are:

at the knee-brace point 259.73 in.-kips;
at the surge girder level −171.25 in.-kips;
at the top of crane column −317.63 in.-kips;
at the base of the column −1732.56 in.-kips.

Discussion

The results in this paper were checked by a computer program. These values, together with those obtained with a Murray-Graham model, are shown in Table 2 for comparison. It can be seen the governing design moments calculated here are within 1 to 7% of the computer analyzed values.

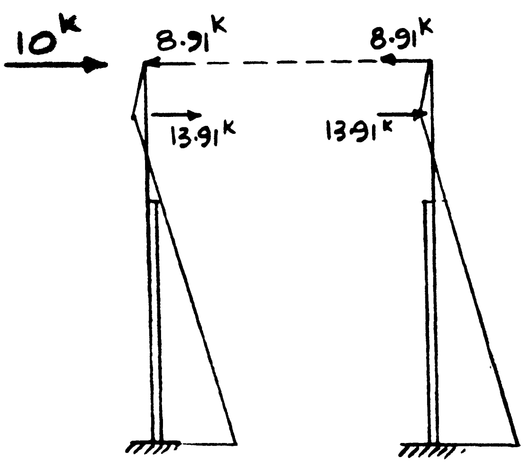


Figure 14

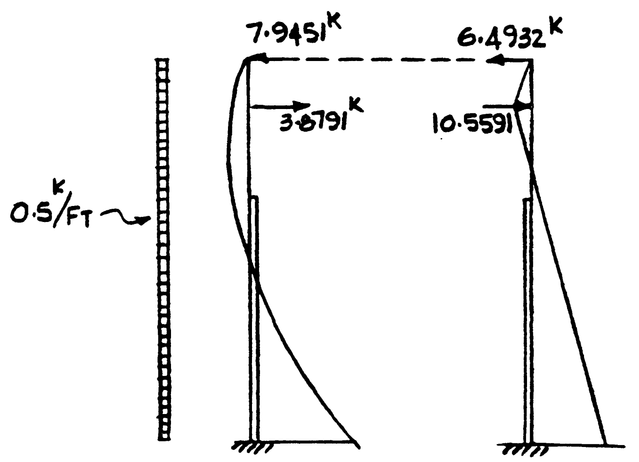


Figure 15

Table 2

COMPARISON OF RESULTS.

(KNEE BRACE LOADS IN KIPS, MOMENT IN IN·KIPS)

OAD CASE / TS		2882 IN·KIP MOMENT			16·9 KIPS SIDE THRUST			10·0 KIPS ROOF SHEAR		
		MURRAY-GRAHAM MODEL	THIS PAPER	COMPUTER ANALYSIS	MURRAY-GRAHAM MODEL	THIS PAPER	COMPUTER ANALYSIS	MURRAY-GRAHAM MODEL	THIS PAPER	COMPUTER ANALYSIS
KNEE BRACE		+124	+23	+28	+180	+574	+599	+638	+356	+380
SURGE GIRDER		+781	+715	+709	+1131	+574	+599	+108	-174	-151
TOP OF CRANE LEG		+1005 -1877	+951 -1931	+940 -1942	+848	-34	8·8	-72	-354	-331
BASE		+280	+343	+291	-1910	-5916	-5890	-1812	-2094	-2071
E .E DS	P_1	—	+6·6	+6·3	—	+15·9	+16·6	—	+15·4	+16·0
	P_2	—	-6·6	-6·3	—	-15·9	-16·6	—	-15·4	-16·0

RESULTS / LOAD CASE		SIDE WIND LOAD					
		WINDWARD COLUMN			LEEWARD COLUMN		
		MURRAY-GRAHAM MODEL	THIS PAPER	COMPUTER ANALYSIS	MURRAY-GRAHAM MODEL	THIS PAPER	COMPUTER ANALYSIS
MOMENT IN COL. AT	KNEE BRACE	+65	+284	+302	+573	+260	+268
	SURGE GIRDER	+307	+305	+314	+97	-171	-155
	TOP OF CRANE LEG	+283	+205	+212	-65	-318	-299
	BASE	-2735	-3542	-3563	-1627	-1733	-1686
KNEE BRACE LOADS	P_1	—	+4·3	+4·9			
	P_2				—	-11·7	-11·8

Table 3

COMPARISON OF RESULTS FOR TOTAL WIND LOAD

8.334^K ROOF SHEAR + SIDE WIND FOR FULL HEIGHT.

(KNEE BRACE LOADS IN KIPS, MOMENT IN IN-KIPS)

RESULTS		WINDWARD COLUMN			LEEWARD COLUMN		
		MURRAY-GRAHAM MODEL	THIS PAPER	COMPUTER ANALYSIS	MURRAY-GRAHAM MODEL	THIS PAPER	COMPUTER ANALYSIS
MOMENT IN COL. AT	KNEE BRACE	+703	+581	+619	+1211	+557	+585
	SURGE GIRDER	+415	+160	+188	+205	-316	-281
	TOP OF CRANE LEG	+211	-90	-64	-137	-613	-575
	BASE	-4547	-5287	-5289	-3438	-3478	-3412
KNEE BRACE LOADS	P_1	—	+17·1	+18·2			
	P_2				—	-24·5	-25·1

In the case of moment on the column at the top of its crane leg, Murray-Graham model considers translation of the column prevented at the mid height of knee brace. In the example here both columns are subjected to moment of equal value. Due to equal stiffness columns, the sway shear and therefore the translation at the top of columns is zero. To that extent both methods should give the same results. From Table 2 it can be seen this paper predicts a larger moment in the bottom portion while the Murray-Graham model predicts a larger moment in the upper portion of the column. The difference between the two sets of values can be attributed to the following: The methodology in this paper considers both rotational restraint offered by the truss and actual column geometry in the analysis, while the Murray-Graham model considers only a lateral restraint closer to the load point.

In the case of crane surge loads, the Murray-Graham model considers the truss bottom chord bracing provides lateral restraint to the column at mid height of knee brace which cannot be fully realized. Total restraint load in that example is 2 × 8.99 or 17.98 kips. The sharing of this load by adjacent rows of columns would reduce the translation but not prevent it totally. In the example studied in this paper the restraint reaction, before sway correction, is 20.71 kips. This initial distribution then would give one bounding value if indeed the roof bracing restrains the columns laterally. The final results after sway correction would then give another bounding value if roof bracing is considered ineffective in providing lateral restraint (Table 2 shows only values after sway correction). Therefore, even with an effective roof bracing, the resulting values would be somewhere between the two bounding values. The above reasoning will also apply to the case of unequal moments applied at top of crane legs in a given row of columns (or equal moments on columns of different stiffnesses) to produce translation.

In the case of wind loads, each row is designed to carry the full value of loads tributary to that row. Transferring of loads to adjacent rows is done in specific instances where the column in question is either stopped at an intermediate level or is not sufficiently strong to carry them. The Murray-Graham model considers the column fixed at the bottom of knee brace on the basis of a truss with infinite stiffness, at the same time allowing translation. In actuality it is flexible to the extent the truss and knee braces allow it to rotate. In this paper truss member sizes are generously assumed to reflect a stiff truss system that is reasonable. Both roof shear and side wind cases may be grouped together to obtain a realistic comparison of values. From Ref. 2, the total wind load on the frame is 10 kips of roof shear plus side wind load up to the bottom of knee brace of 20.417 kips, i.e., 30.417 kips. To avoid any duplication of side load over top 40 in., equivalent roof shear is 30.417 minus side wind load of 22.083 for the full height, i.e., 8.334 kips. Table 3 shows the comparison for the total wind on the frame. By referring to Tables 2 and 3 it is evident the Murray-Graham model underestimates column moments in the lower lengths and on foundations by overestimating the stiffness of the truss system.

In conclusion, the truss members and their connections should be designed for additional loads from the prop action and/or knee-brace loads as applicable. For any changes in the area or the moment of inertia of the members the procedure is repeated, iteratively, until acceptable solutions are obtained. Directions of reactions H_1 and H_2 should be properly taken to obtain correct results.

For multiple bay buildings with knee braces, equations can be developed relating column flexibilities, truss and knee-brace stiffness in a manner similar to that described here.

ACKNOWLEDGMENT

The author wishes to acknowledge contributions from S. Shankar who provided results of computer analysis for examples of mill buildings with knee braces.

NOMENCLATURE

Truss systems without knee braces:

P_1, P_2 = Truss bottom chord prop loads, kips.

d_{ij} = Deflection at i due to a unit load (moment) at j

Truss systems with knee braces:

P_1, P_2 = Brace loads for knee braces GC and HD, kips

θ_1, θ_2 = Angles of knee braces 1 and 2 to the horizontal

d_{cc} = Deflection of roof truss at c due to a unit load at c, truss spanning from column to column

d_{dd} = Deflection of roof truss at d due to a unit load at d, truss spanning from column to column

d_{cd} = Deflection of roof truss at c due to a unit load at d, truss spanning from column to column

d_{ggp} = Deflection of column AB at g due to a unit load at g on the basis of a propped cantilever

d_{gg} = Deflection of column AB at g due to a unit load at g on the basis of a cantilever

d_{gb} = Deflection of column AB at g due to a unit load at b on the basis of a cantilever

d_{bb} = Deflection of column AB at b due to a unit load at b on the basis of a cantilever

d_{hhp} = Deflection of column EF at h due to a unit load at h on the basis of a propped cantilever

d_{hh} = Deflection of column EF at h due to a unit load at h on the basis of a cantilever

d_{hf} = Deflection of column EF at h due to a unit load at f on the basis of a cantilever

d_{ff} = Deflection of column EF at f due to a unit load at f on the basis of a cantilever

d_{g1} = Deflection of column AB at g due to external loading on it on the basis of a propped cantilever

d_{h1} = Deflection of column EF at h due to external loading on it on the basis of a propped cantilever

d_{brc1} = Shortening of brace GC due to a unit load along GC

d_{brc2} = Shortening of brace HD due to a unit load along HD

H_1 = Reaction on column AB at truss bottom chord level

H_2 = Reaction on column EF at truss bottom chord level

H = Sway shear on the frame

REFERENCES

1. *Moore, W. E., Jr.* A Programmable Solution for Stepped Crane Columns *AISC Engineering Journal, 2nd Qtr., 1986, Chicago, Ill. (pp. 55–58).*
2. *Murray, J. J. and T. C. Graham* The Design of Mill Buildings *AISC National Engineering Conference Proceedings, 1959.*

APPENDIX A

Flexibility coefficients and deflections for the example under "Truss systems without knee braces," calculated as a cantilever (Figs. 16 and 17).

Flexibility coefficients:

For column X:

EId_{11} due to unit load at 1 = $(15 \times 15 \times 10/2) + (15 \times 30 \times 30/80) + (30 \times 30 \times 35/(2 \times 80))$

$d_{11} = 1490.6/EI$ ft/kip

EId_{12} due to unit load at 2 = $(6 \times 6 \times 13/2) + (6 \times 30 \times 30/80) + (30 \times 30 \times 35/(2 \times 80))$

$d_{12} = 498.4/EI$ ft/kip

For column Y:

EId_{44} due to unit load at 4 = $(21 \times 21 \times 14/(2 \times 2)) + (21 \times 30 \times 36/160) + (30 \times 30 \times 41/(2 \times 160))$

$d_{44} = 1800.55/EI$ ft/kip

EId_{45} due to unit load at 5 = $(15 \times 15 \times 16/(2 \times 2)) + (15 \times 30 \times 36/160) + (30 \times 30 \times 41/(2 \times 160))$

$d_{45} = 1116.5/EI$ ft/kip

EId_{55} due to unit load at 5 = $(15 \times 15 \times 10/(2 \times 2)) + (15 \times 30 \times 30/160) + (30 \times 30 \times 35/(2 \times 160))$

$d_{55} = 745.3/EI$ ft/kip

EId_{56} due to unit load at 6 = $(6 \times 6 \times 13/(2 \times 2)) + (6 \times 30 \times 30/160) + (30 \times 30 \times 35/(2 \times 160))$

$d_{56} = 249.2/EI$ ft/kip

EId_{46} due to unit load at 6 = $(6 \times 6 \times 19/(2 \times 2)) + (6 \times 30 \times 36/160) + (30 \times 30 \times 41/(2 \times 160))$

$d_{46} = 326.8/EI$ ft/kip

EId_{57} due to unit moment at 7 = $(30 \times 30/160)$

$d_{57} = 5.62/EI$ ft/kip-ft of moment

EId_{47} due to unit moment at 7 = $(30 \times 30/160)$

$d_{47} = 6.75/EI$ ft/kip-ft of moment

For column Z:

EId_{88} due to unit load at 8 = $(21 \times 21 \times 14/(2 \times 1.5)) + (21 \times 30 \times 36/100) + (30 \times 30 \times 41/(2 \times 100))$

$d_{88} = 2469.3/EI$ ft/kip

EId_{89} due to unit load at 9 = $(6 \times 6 \times 19/(2 \times 1.5)) + (6 \times 30 \times 36/100) + (30 \times 30 \times 41/(2 \times 100))$

$d_{89} = 447.3/EI$ ft/kip

EId_{810} due to unit moment at 10 = $(30 \times 36/100)$

$d_{810} = 10.8/EI$ ft/kip-ft of moment

Deflection in columns due to side wind load of 1 kip/ft:

At 1 $EId_1 = (112.5 \times 15 \times 3 \times 15/(3 \times 4)) + (112.5 \times 30 \times 30/80) + (450 \times 30 \times 35/(2 \times 80)) + (450 \times 30 \times 37.5/(3 \times 80))$

$d_1 = 12656.25/EI$ ft/(1 kip/ft) of loaded length

At 4 $EId_4 = (18 \times 6 \times 3 \times 6/(3 \times 4 \times 2)) + (18 \times 15 \times 13.5/2) + (90 \times 15 \times 16/(2 \times 2)) + (108 \times 30 \times 36/160) + (180 \times 30 \times 41/(2 \times 160))$

$d_4 = 8724.38/EI$ ft/(1 kip/ft) of loaded length

At 5 $EId_5 = (18 \times 15 \times 7.5/2) + (90 \times 15 \times 10/(2 \times 2)) + (108 \times 30 \times 30/160) + (180 \times 30 \times 35/(2 \times 160))$

$d_5 = 5585.6/EI$ ft/(1 kip/ft) of loaded length

At 8 $EId_8 = (220.5 \times 21 \times 3 \times 21/(3 \times 4 \times 1.5)) + (220.5 \times 30 \times 36/100) + (630 \times 30 \times 41/(2 \times 100)) + (450 \times 30 \times 43.5/(3 \times 100))$

$d_8 = 24420.15/EI$ ft/(1 kip/ft) of loaded length

APPENDIX B

Flexibility coefficients and deflections for the example under "Truss systems with knee braces," calculated as a cantilever (Figs. 16 and 18);
Flexibility coefficients:

$Ed_{bb} = Ed_{ff} = (182 \times 182 \times 121.33/(2 \times 1355.1)) + (182 \times 348 \times 356/11048.2) + (348 \times 348 \times 414/(2 \times 11048.2))$

$d_{bb} = d_{ff} = 5792.78/E$ in./kip

$Ed_{gg} = Ed_{hh} = (142 \times 142 \times 94.67/(2 \times 1355.1)) + (142 \times 348 \times 316/11048.2) + (348 \times 348 \times 374/(2 \times 11048.2))$

$d_{gg} = d_{hh} = 4167.51/E$ in./kip

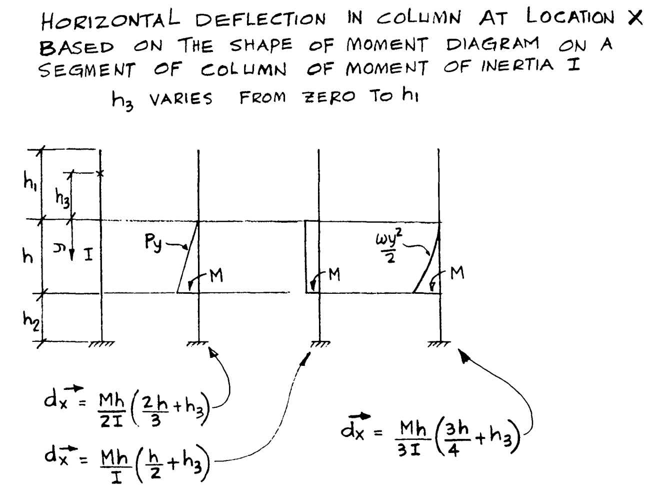


Figure 16

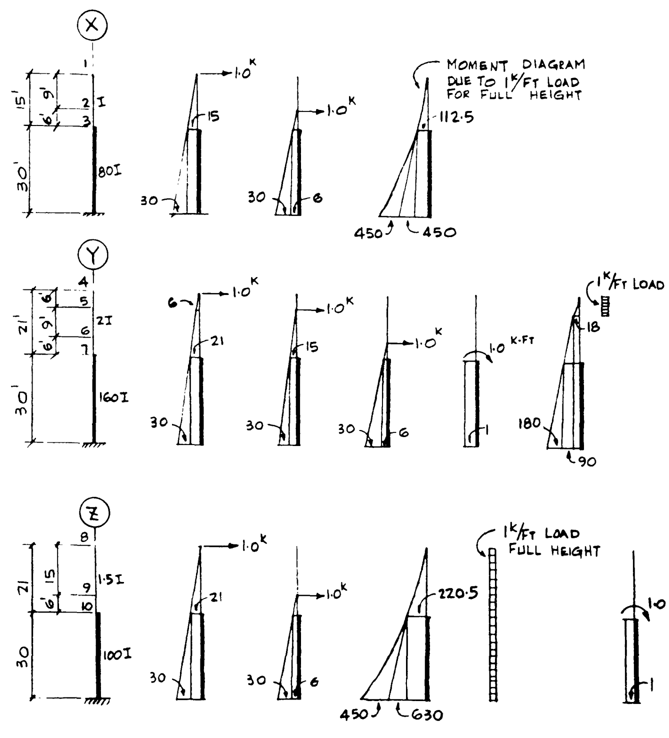


Figure 17

$$Ed_{bg} = Ed_{fh} = (142 \times 142 \times 134.67/(2 \times 1355.1)) + (142 \times 348 \times 356/11048.2) + (348 \times 348 \times 414/(2 \times 11048.2))$$

$$d_{bg} = \ d_{fh} = 4863.26/E \text{ in./kip}$$

$$d_{ggp} = (d_{bb} \times d_{gg} - d_{bg}^2)/d_{bb} = (5792.78 \times 4167.51 - 4863.26^2)/5792.78E = 84.6175/E \text{ in./kip}$$

Similarly $d_{hhp} = 84.6175/E$ in./kip

If the top of crane leg is identified as J on column AB, then horizontal cantilever deflections due to unit moment at J are,

$$d_{bj} = (1 \times 348 \times 356/11048.2E) = 11.21/E \text{ in./kip-in. of moment}$$

$$d_{gj} = (1 \times 348 \times 316/11048.2E) = 9.95/E \text{ in./kip-in. of moment}$$

If the location of crane side thrust is identified as K on column AB, then horizontal cantilever deflections due to unit load at K are,

$$Ed_{bk} = (36 \times 36 \times 170/(2 \times 1355.1)) + (36 \times 348 \times 356/11048.2) + (348 \times 348 \times 414/(2 \times 11048.2))$$

$$d_{bk} = 2753.98/E \text{ in./kip}$$

$$Ed_{gk} = (36 \times 36 \times 130/(2 \times 1355.1)) + (36 \times 348 \times 316/11048.2) + (348 \times 348 \times 374/(2 \times 11048.2))$$

$$d_{gk} = 2470.3/E \text{ in./kip}$$

Horizontal cantilever deflections of column AB due to side wind load are:

At B $$Ed_1 = (690.08 \times 182 \times 136.5/(3 \times 1355.1)) + (690.08 \times 348 \times 356/11048.2) + (2639 \times 348 \times 414/(2 \times 11048.2)) + (2523 \times 348 \times 443/(3 \times 11048.2))$$

$$d_1 = 40897.1/E \text{ in.}$$

At G $$Ed_3 = (33.33 \times 142 \times 71/1355.1) + (236.67 \times 142 \times 94.67/(2 \times 1355.1)) + (420.08 \times 142 \times 3 \times 142/(3 \times 4 \times 1355.1)) + (690.08 \times 348 \times 316/11048.2) + (2639 \times 348 \times 374/(2 \times 11048.2)) + (2523 \times 348 \times 403/(3 \times 11048.2))$$

$$d_3 = 36073.047/E \text{ in.}$$

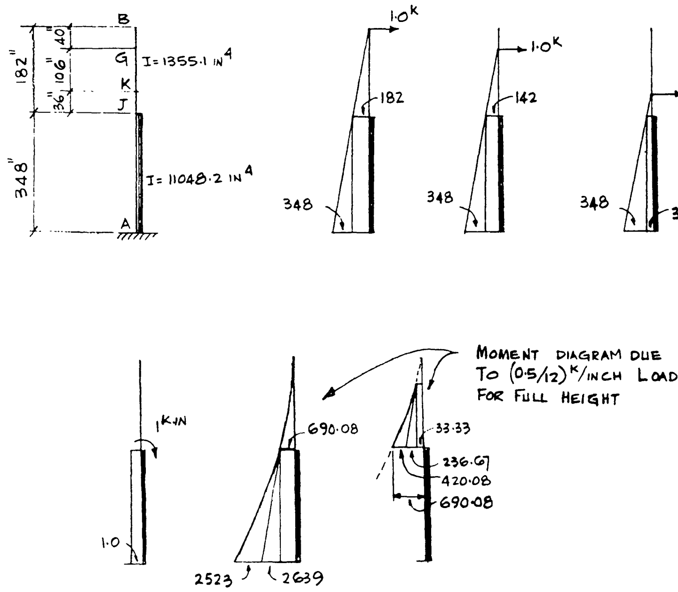


Figure 18