# Design of Diagonal Cross Bracings Part 1: Theoretical Study

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Diagonal bracing is commonly used in steel structures to resist horizontal loads. In current practice, the design of this type of bracing system is based on the assumption the compression diagonal has negligible capacity and the tension diagonal resists the total load (Fig. 1).

If the diagonals are connected at their intersection point (usual practice), this design procedure is conservative because the effect of this connection on the out-of-plane buckling capacity of the compression diagonal is ignored.[1,2,3,4] The restraint provided to the compression diagonal by the loaded tension diagonal is generally sufficient to consider that the effective length of the compression diagonal is 0.50 times the diagonal length ($KL = 0.5L$) for out-of-plane buckling as for in-plane buckling. Analytical and experimental results[1,2] have also shown the ultimate horizontal load on the bracing system is much higher than the horizontal component of the yielding strength of the tension member, because of load sharing between the diagonals. The assumption that the compression diagonal has negligible capacity usually results in overdesign.

This paper presents the results of a theoretical study aimed at the determination of the transverse stiffness offered by the tension diagonal in cross-bracing systems and at the evaluation of the effect of this stiffness on the out-of-plane buckling resistance of the compression diagonal. The theory is supported by seven transverse stiffness tests and 15 buckling tests. The test results are reported in the second part of the paper.

## BUCKLING OF THE COMPRESSION DIAGONAL

In double diagonal cross bracing, the tension diagonal acts as an elastic spring at the point of intersection of the compression diagonal as shown in Fig. 2a, where $\alpha$ is the spring stiffness (kips/in. or kN/mm). If $\alpha = 0$, then $K = 1.0$ (Fig. 2b) and the elastic critical load $C_{ce}$ is equal to:

$$C_{ce} = \frac{\pi^2 EI_c}{L^2} = C_e \qquad (1)$$

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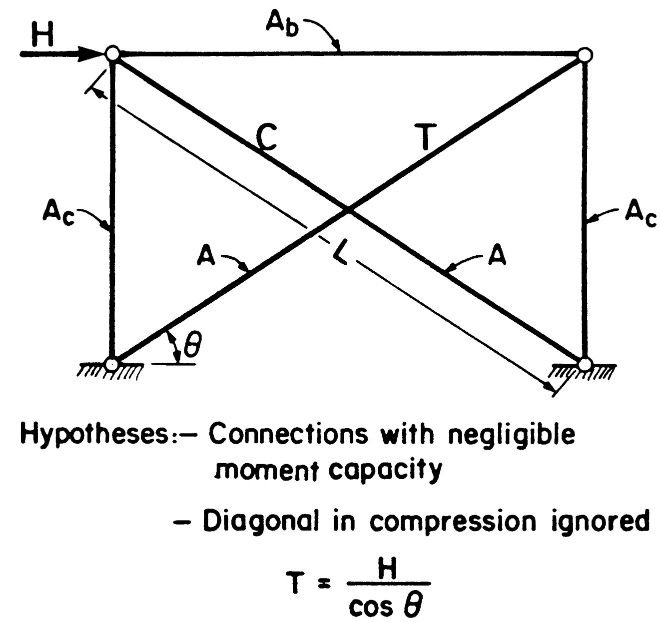


*Fig. 1. Usual design hypotheses*

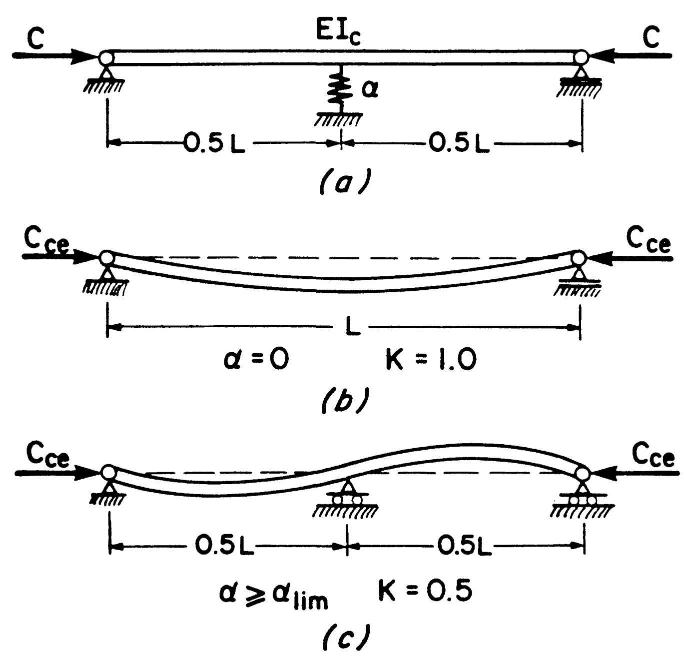


*Fig. 2. Buckling modes for compression diagonal*

In this equation, $I_c$ is the moment of inertia of the compression diagonal considering out-of-plane buckling.

If $\alpha = \infty$, then $K = 0.5$ and the elastic critical load is equal to:

$$C_{ce} = \frac{4\pi^2 EI_c}{L^2} = 4\ C_e \tag{2}$$

Therefore, the effective length factor is given by:

$$K = \sqrt{\frac{C_e}{C_{ce}}} \tag{3}$$

Timoshenko and Gere[5] demonstrated there is a limit spring stiffness above which the elastic critical load is $4C_e$ (Fig. 2c). In other words, the spring does not have to be infinitely stiff to obtain $K = 0.5$. The limit spring stiffness is given by:[5]

$$\alpha_{\lim} = \frac{16\pi^2 EI_c}{L^3} \tag{4}$$

Let us define the non-dimensional spring stiffness as:

$$\gamma = \frac{\alpha L}{C_e} \tag{5}$$

From Eqs. 1, 4 and 5, the limit value of the non-dimensional spring stiffness is equal to:

$$\gamma_{\lim} = \alpha_{\lim} \frac{L}{C_e} = 16 \tag{6}$$

If we assume a linear relationship between $C_{ce}$ and $\gamma$, we obtain:

$$C_{ce} = \left(1 + \frac{3\gamma}{16}\right) C_e \leq 4C_e \tag{7}$$

The variation of the dimensionless elastic buckling load is shown in Fig. 3 in terms of the dimensionless spring stiffness. The exact solution obtained by Timoshenko and Gere is also shown in the figure. It can be seen that Eq. 7 is slightly conservative.

Using Eqs. 3 and 7, the effective length factor becomes:

$$K = \frac{4}{\sqrt{16 + 3\gamma}} \geq 0.5 \tag{8}$$

The variation of $K$ is plotted in Fig. 4 against the dimensionless spring stiffness. The exact solution, given in Refs. 5 and 6, is also shown in the figure.

## TRANSVERSE STIFFNESS OF THE TENSION DIAGONAL

To determine the parameters $\alpha$ and $\gamma$, we will consider a prismatic member subjected to a transversely applied point load $Q$ and a concentrically applied tensile force $T$, as shown in Fig. 5. For the problem studied here, $Q$ represents the transverse force transmitted to the tension diagonal by the compression diagonal at buckling.

The general solution of the differential equation obtained from equilibrium of the tension member shown in Fig. 5 is classical, assuming elastic behavior. The elastic behavior assumption is easily satisfied for X-bracing, since both diagonals are identical. When the compression diagonal buckles, the tension diagonal is still behaving elastically.[1,2]

The transverse stiffness or spring stiffness provided by the tension diagonal is obtained from the solution of the differential equation and is given by:[1,2]

$$\alpha = \left(\frac{16EI_t}{L^3}\right) \frac{v^3}{(v - \tanh v)} \tag{9}$$

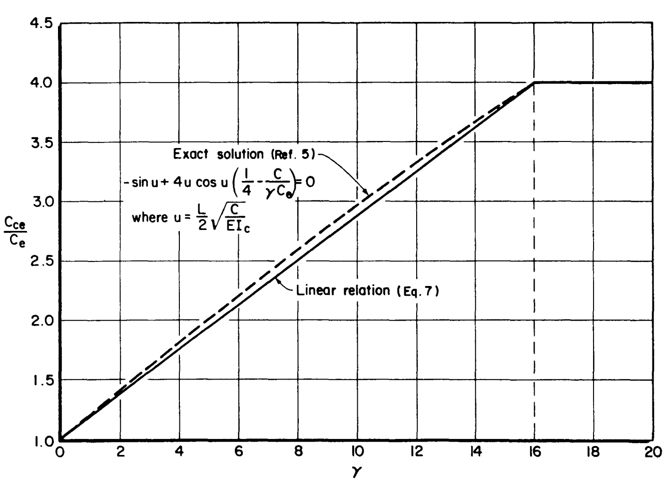


*Fig. 3. Relationship between elastic buckling load and non-dimensional spring stiffness*

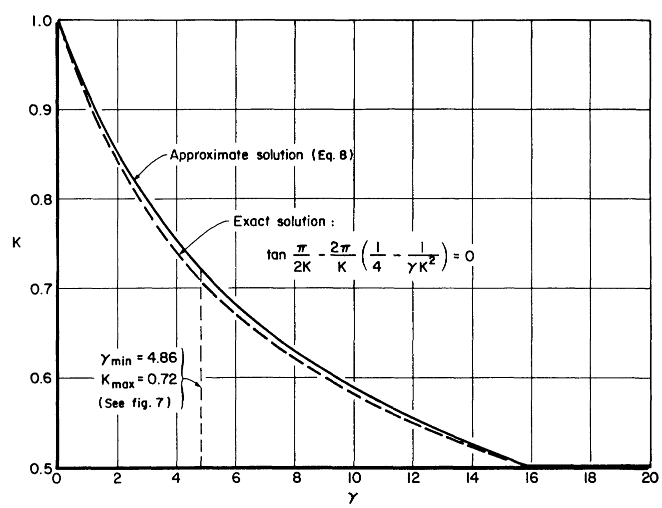


*Fig. 4. Relationship between effective length factor and non-dimensional spring stiffness*

where $v$ is defined as:

$$v^2 = \left(\frac{L}{2}\right)^2 \frac{T}{EI_t} \tag{10}$$

In Eqs. 9 and 10, $E$ is the elastic modulus of the material and $I_t$ is the moment of inertia of the tension diagonal, considering out-of-plane bending of the X-bracing.

Combining 1, 5 and 9, we obtain the following equation for the evaluation of the dimensionless spring stiffness:

$$\gamma = \left(\frac{16}{\pi^2}\right)\left(\frac{I_t}{I_c}\right)\frac{v^3}{(v - tanh\ v)} \tag{11}$$

In X-bracing systems, both diagonals are identical and $I_t = I_c = I$. However, in X-trusses supporting loads which always act in the same direction, the compression diagonal could be stiffer. Since this paper deals with double diagonal bracings, Eq. 11 can be rewritten as:

$$\gamma = \left(\frac{16}{\pi^2}\right)\frac{v^3}{(v - tanh\ v)} \tag{12}$$

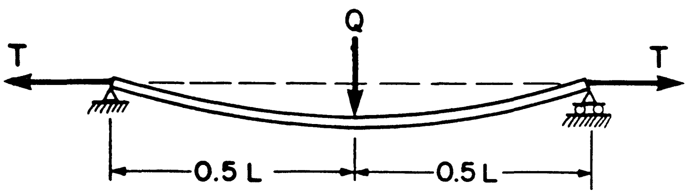


*Fig. 5. Tension member subjected to transverse force at mid-span*

The following approximation for Eq. 12 is proposed:

$$\gamma = \frac{16}{\pi^2}(3 + 1.09\ v^2) \tag{13}$$

Eqs. 12 and 13 are compared in Fig. 6. It can be seen Eq. 13 very closely approximates Eq. 12 when $0 \le v^2 \le 80$. This practical range for parameter $v^2$ was evaluated as follows.

Let us first assume the out-of-plane slenderness ratio of the compression diagonal, i.e., both diagonals, should not exceed 200. Since the diagonals are connected at their intersection, we get:

$$\frac{L/2}{r} \le 200$$

$$\left(\frac{L}{2}\right)^2 \le 40\,000\frac{I}{A} \tag{14}$$

The maximum force in the tension diagonal being $AF_y$, Eq. 10 together with Eq. 14 and $E = 29\,000$ ksi leads to:

$$v^2 \le \left(40\,000\frac{I}{A}\right)\frac{AF_y}{EI} = 1.38\ F_y$$

Considering a maximum value for $F_y$ of 58 ksi, then $0 \le v^2 \le 80$.

In the previous equation, the constant 1.38 is in $(\text{ksi})^{-1}$. If SI units are used $E = 200\,000\ MPa$ and the constant is equal to $0.20\ (MPa)^{-1}$. Then $F_y$ is in $MPa$.

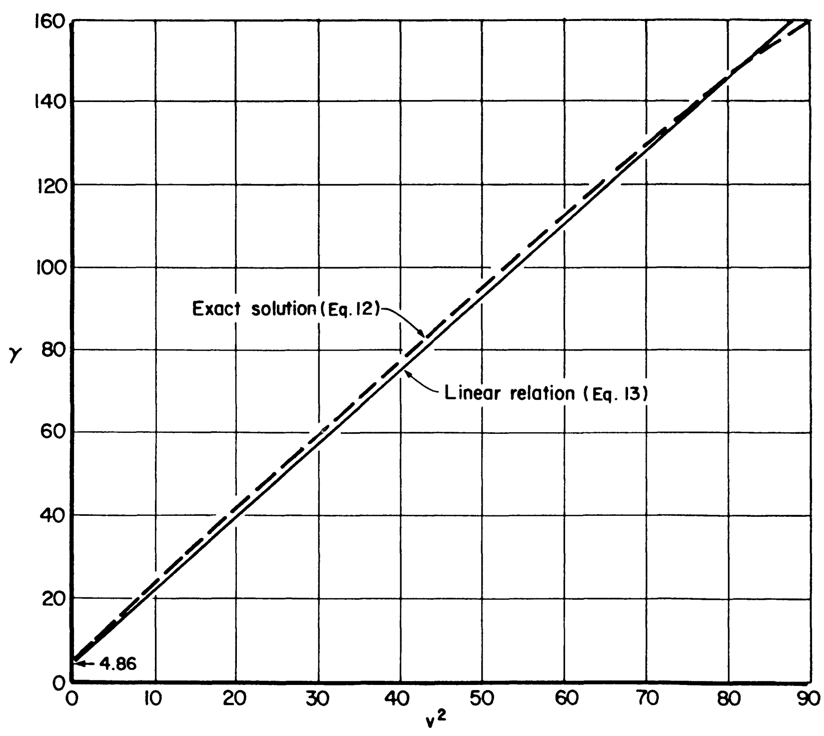


*Fig. 6. Comparison of Eqs. 12 and 13*

Eq. 13, being an acceptable approximation (Fig. 6), will be used in the theoretical analysis. Since $I_c = I_t = I$, combining Eqs. 1, 5, 10 and 13 gives:

$$\alpha = \frac{C_e\gamma}{L} = \frac{48EI}{L^3} + 4.36\frac{T}{L} \qquad (15)$$

It is interesting to note the first term in Eq. 15 represents the flexural stiffness of the tension diagonal. Then $T = 0$, the spring stiffness is equal to the flexural stiffness. Therefore $\alpha$ is never equal to zero and $K$ is always smaller than 1.0 ($\alpha_{min} = 48\,EI/L^3$; from Eq. 5 or Eq. 18, $\gamma_{min} = 4.86$; from Eq. 8, $K_{max} = 0.72$ (see Fig. 4).

The results of the transverse stiffness tests reported in the second part of the paper demonstrate the validity of Eqs. 13 and 15.

Eq. 10 can be rewritten as:

$$v^2 = \left(\frac{\pi L}{2}\right)^2 \frac{T}{\pi^2 EI} = 2.47\frac{T}{C_e} \qquad (16)$$

Introducing Eq. 16 into Eq. 13 gives:

$$\gamma = 4.86 + 4.36\frac{T}{C_e} \qquad (17)$$

From Eq. 3, $C_e = K^2C_{ce}$. Therefore, Eq. 17 can be rewritten as:

$$\gamma = 4.86 + 4.36\left(\frac{T}{K^2C_{ce}}\right) \qquad (18)$$

Combining 8 and 18 gives:

$$K = \sqrt{0.523 - \frac{0.428}{C_{ce}/T}} \geq 0.50 \qquad (19)$$

During the buckling tests reported in the second part of the paper, the force in the tension diagonal was kept constant. Therefore, the value of $T$ is known. If the compression diagonal buckles in the elastic range, Eq. 19 can be used to determine the effective length factor ($C_{ce} = C_{cr}$ where $C_{cr}$ is the measured buckling load). For inelastic buckling, Eq. 19 is not valid. However, the measured inelastic buckling load can be introduced into AISC equations[7] (safety factor removed) or into CSA equations[8] (resistance factor removed) to determine the effective length factor. With this value of $K$, two values of $C_{ce}$ can be computed. One is obtained from Eq. 19 and the other from the following equation: $C_{ce} = \pi^2\, EI/(KL)^2$. These two values are compared to evaluate the accuracy of Eq. 19.

Two final comments should be made concerning the previous equations. These equations are valid for continuous diagonals which are connected at their intersection point. If one diagonal is interrupted at the intersection point, constant stiffness is not maintained and the connection at this point may become a weak link when the interrupted diagonal is compressed.[3]

In the derivations the rotational restraint of the connections was ignored. In Ref. 4, the rotational restraint was relatively important, and test results suggest use of an effective length factor of 0.85 times the half diagonal length, i.e., $K = 0.425$ if the total length of the diagonal is considered.

## COMPRESSION-TENSION RATIO

Since the behavior of the braced frame shown in Fig. 1 is elastic up to buckling of the compression diagonal, the ratio $C_{ce}/T$ is equal to the ratio of the force in the compression diagonal $C$ to that in the tension diagonal $T$ obtained from an elastic analysis of the frame. Eq. 19 can thus be rewritten as:

$$K = \sqrt{0.523 - \frac{0.428}{C/T}} \geq 0.50 \qquad (20)$$

Eq. 20 is plotted in Fig. 7. As seen, when the $C/T$ ratio is larger than 1.6, the effective length factor increases. It reaches its maximum value when the $C/T$ ratio is equal to

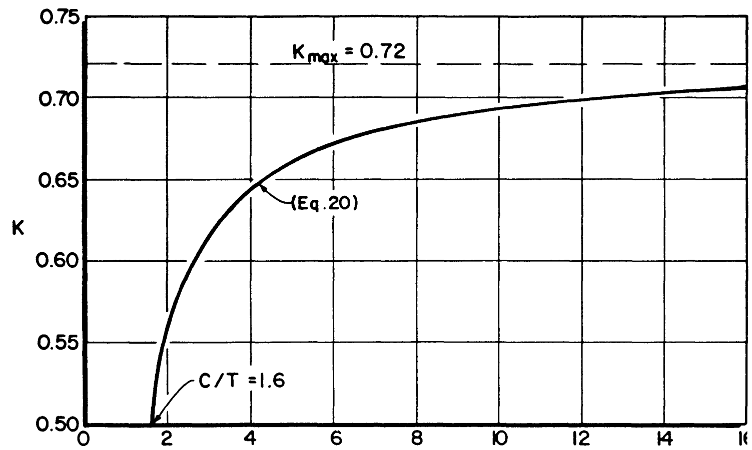


*Fig. 7. Effective length factor vs. C/T ratio*

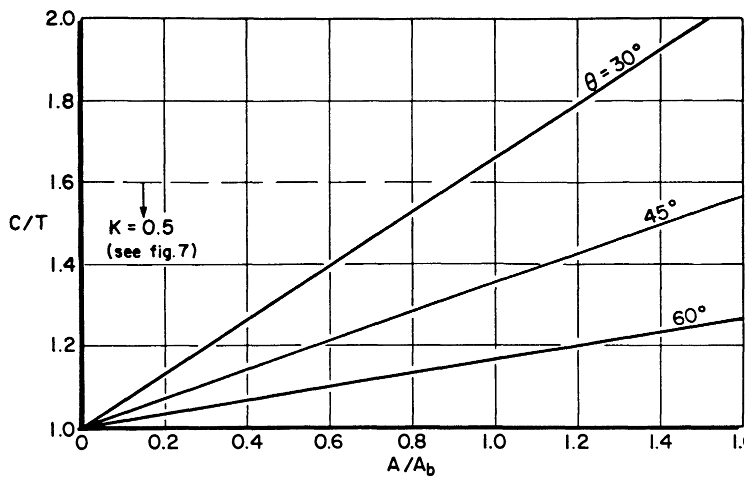


*Fig. 8. Variation in C/T ratio with $A/A_b$ ratio (Eq. 22)*

infinity, which corresponds to the minimum spring stiffness, equal to the flexural stiffness of the tension diagonal ($T = 0$ in Eq. 15 or Eq. 17).

By considering the elastic deformations of the braced frame shown in Fig. 1, Vickers[3] derived the following equation to compute the $C/T$ ratio:

$$\frac{C}{T} = 1 + \frac{\cos^3\theta}{\frac{A_b}{A}\left(1 + \frac{A}{A_c}\sin^3\theta\right)} \tag{21}$$

The parameters of this equation are defined in Fig. 1. Eq. 21 was derived for a load pushing against one side of the frame as in Fig. 1. This is the most critical loading condition for the design of the compression diagonal because the $C/T$ ratio exceeds one.[3]

In a typical braced frame, the area of the cross section of the columns $A_c$ is usually much greater than the area of the cross section of the diagonals $A$. Moreover, $\sin^3\theta$ is always smaller than one. Consequently, the term $A \sin^3\theta/A_c$ in Eq. 21 is small compared to 1 and can be neglected with only a minor loss in accuracy resulting in errors which are on the conservative side. Eq. 21 becomes:

$$\frac{C}{T} = 1 + \left(\frac{A}{A_b}\right)\cos^3\theta \tag{22}$$

Eq. 22 is plotted in Fig. 8. It can be seen that large values of $A_b$, i.e. small values of the $A/A_b$ ratio, tend to equalize the forces in the diagonals. In practical situations, the $A/A_b$ ratio is much smaller than one. Therefore, as shown in Fig. 8, the $C/T$ ratio is usually smaller than 1.6 and the $K$ value is equal to 0.5 (Fig. 7).

## CONCLUSION

The theoretical study reported in this paper shows that in double diagonal bracing systems the effective length of the compression diagonal is 0.5 times the diagonal length when the diagonals are continuous and attached at the intersection point.

Tests were done to demonstrate the validity of the equations used to determine the transverse stiffness or spring stiffness provided by the tension diagonal and the validity of the equations used to determine the effective length factor. The results of these tests are reported in the second part of the paper.

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