Lateral Load Distribution in Multi-girder Bridges

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This study of the distribution of internal forces in bridge structures is motivated in part by the sense that simple formulations may be perhaps inaccurate, restrictive and uneconomical in the design process. In the age of computerized structural analysis, the differences between the results of so-called exact, and now less costly, analyses and simplified load distribution rules are apparent.

The availability of a variety of structural analysis systems programs greatly expands the options available to the bridge engineer. For slab and longitudinal girder bridges, a structural solver which will handle a three-dimensional frame combined with plate and shell finite elements can produce an "exact" analysis. SAP, NASTRAN, FINITE, ANSYS and SAP80 are examples of this type of program. These systems are implemented on machines ranging from supercomputers to desktop microcomputers. Programs such as SAP80 are accessible to the individual engineer on a personal, not corporate basis. However, the size of the problem and the range of analytical refinement available to the PC is restricted at present. Some level of simplification is needed, but even so considerable refinement over the present distribution rules can be achieved.

Application of the analysis system does not involve the development of new computer programs—rather an application of an input "language" to the task of describing the loads and structure for analysis. Important also is a clear understanding of the idealization and specific structural theory underlying the system to be used. Once a paradigm input is written, the solution for a family of bridge types is usually simple.

This paper provides an overview of insights gained on idealization and simplification for analysis and selected illustrative results developed on AISI Project 332, "Lateral Load Distribution for Multigirder Bridges." Detailed information will become available in the project final report.[1] Discussed are structural idealizations of the bridge and deck systems accomplished by use of (1) the "exact" idealization of a shell bending element (RFSHELL) with axial membrane forces placed eccentric to the girders and diaphragm elements, (2) the grid with plate bending elements (RPB12) and (3) the simple grid for which both the transverse effect of the deck and composite action are taken into account (GRID). Selected results will be used representing simple and continuous bridges with a variety of span lengths, widths, number of girders and slab thicknesses. A rich background in the literature is available; selected items have been listed as Refs. 2–11.

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Current Specification Philosophy

Practice in highway bridge design has emphasized approaches which idealize the deck and multiple girder structure as a beam; hence it is most convenient to deal with wheel loads and apportion the total moment produced to the interior or exterior girders by a design factor. When multiple lane loads are used for bridges sufficiently long that a single vehicle does not control, then the specification is silent on the details of lateral distribution. However an equal distribution to the girders is often used and is satisfactory.

These remarks are directed at the problems associated with design of slab and longitudinal girder (with more than two longitudinal girders) bridges. In the case of only two girders or the deck system of a truss bridge, cable-stayed design with two girders, or suspension bridges with main cables in two planes, the task of live-load distribution to the main longitudinal structural system is accomplished by static distribution.

Scope of Parameters Studied

Using the three analytical models outlined, bridge parameters were chosen to represent a range of steel girder bridge designs. Span lengths range from 50 to 400 ft to obtain a wide variation of structural parameters for right, i.e. non-skewed, straight bridges. Curved bridges clearly require a different approach for simplification, although in principle, they can be treated using FEM approach in an identical manner as used here. Trussed diaphragms are considered at 25-ft spacing where appropriate; duplicate solutions are made with diaphragms omitted for comparison. An initial goal was to assess the significance of such bracing in increasing the efficiency of load distribution.

Bridge Structural Properties

Composite action between longitudinal girders and deck is assumed throughout. Representative values of girder moment of inertia and section changes along the span are estimated or taken from published typical designs. No attempt is made to make all parameter combinations strictly representative of a given class of designs. The parameter selection seeks to bracket possible practice for slab and girder construction. Only I-type girders are considered, that is, sections with significant torsional resistance are not included. A typical variation in combinations of girder spacing and slab thickness is tabulated below for one of the several span lengths considered. The corresponding stiffness parameter H (as used by Newmark and others sometimes in a scaled form) is also tabulated. The parameter H is a measure of the relative stiffness of the slab to that of the girders.[4,12]

Typical Combinations Used for H Variation

Slab Thickness in.	Span	Number of Girders	Girder Spacing ft	b/a	H Stiffness Parameter
6	200	5	9	0.045	33.3
7	200	3	18	0.09	29
7	200	5	9	0.045	20.9
8	200	3	18	0.09	20
9	200	3	18	0.09	14
8	200	5	9	0.045	14
10	200	3	18	0.09	10.2
9	200	5	9	0.045	9.85
10	200	5	9	0.045	7.18
12	200	3	18	0.09	5.92

These combinations are tabulated in descending order of H, that is increasing relative slab stiffness. A similar spread in H is developed for all span lengths as illustrated in Fig. 1

COMMENTS ON ANALYTICAL MODELS

The three analytical models discussed above have been used. The so-called exact (RFSHELL) finite element model works well but is more costly in computer time; it is well represented by the grid and plate element model (RPB12). A detailed comparison and assessment of computational times will be a part of the AISI Project 332 final report.

PRESENTATION OF SELECTED RESULTS

A sampling of results is presented here to show the general nature of the non-uniform distribution of moments in slab and girder bridges and to illustrate the effect of diaphragms. The three analytical models discussed above are represented in the available results, but for extended parameter studies the RPB12 model has been used; special attention will be given to the GRID model. Far more useful output describing the behavior of the bridge is available than can be presented herein. For example, deflection vectors (six components: three displacements and three rotations) are available for all nodes in the model; the exact distribution of all internal forces and displacements are part of the output.

Influence surfaces may be readily generated, and the data is simple to store and plot when the grid model is used. Also, it is possible to draw conclusions on the prediction of both deck forces (slab moments and axial forces usually available indirectly as part of the assumption of composite action in longitudinal bending) and truss diaphragm member forces.

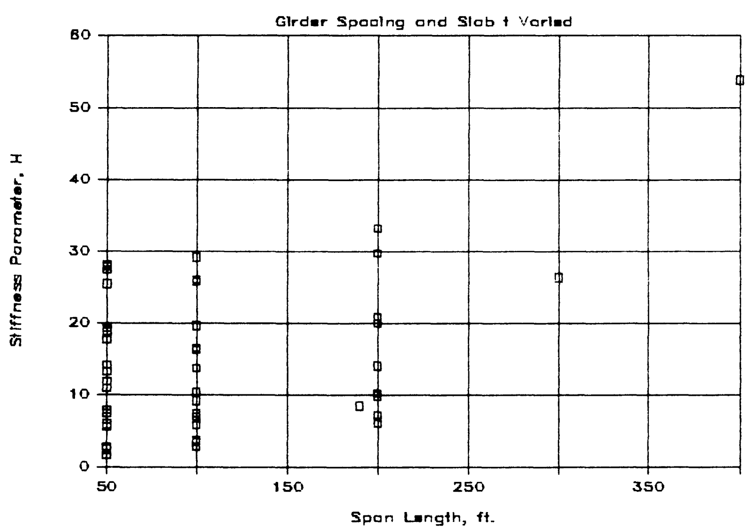

Fig. 1. Variation in stiffness parameter

Results for Two-span Continuous

Selected results are presented to illustrate the effect of transverse trussed diaphragms on the lateral distribution of bending moments at both midspan and over the interior pier. These results also serve to indicate the general non uniform distribution of bridge response when the load is applied in a relatively concentrated mode. In all studies, two point loads are applied spaced 6 ft apart to simulate the transverse arrangement of the AASHTO Code. The structure for which results are presented below has two continuous 200-ft spans with five girders. The total bridge width is 44 ft; the five girders are spaced at 9 ft.

In Figs. 2 and 3 the midspan and pier moments are plotted in units of in.-kips vs. position of the girder corresponding to the moment value shown, for edge loading and center loading respectively. In the legend, *ND* denotes no diaphragm and *D* with diaphragm; *M* and *I* denote midspan positive and interior pier negative moment regions, respectively. For eccentrically placed loads the influence of the inclusion of transverse trussed diaphragms is small, nearly negligible. For the centrally placed loading the effect is

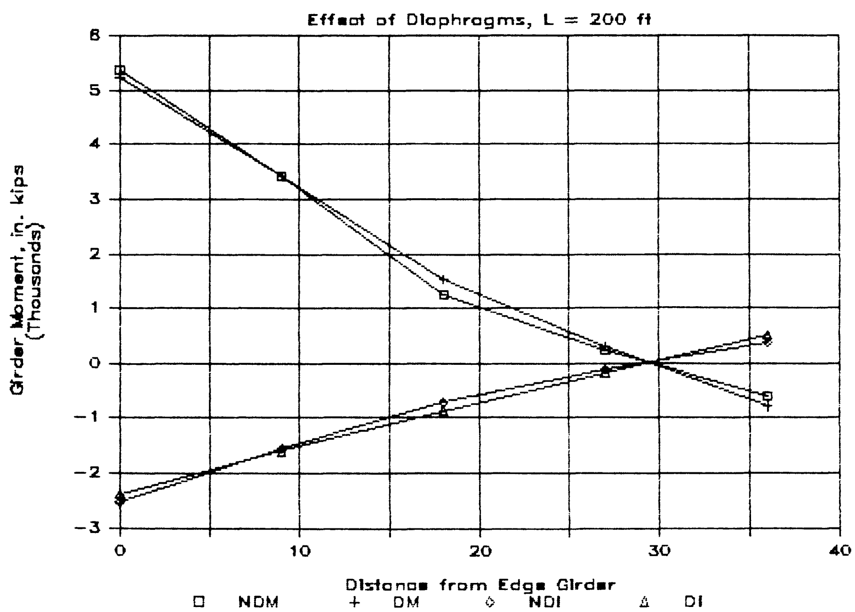

Fig. 2. Two-span continuous bridge

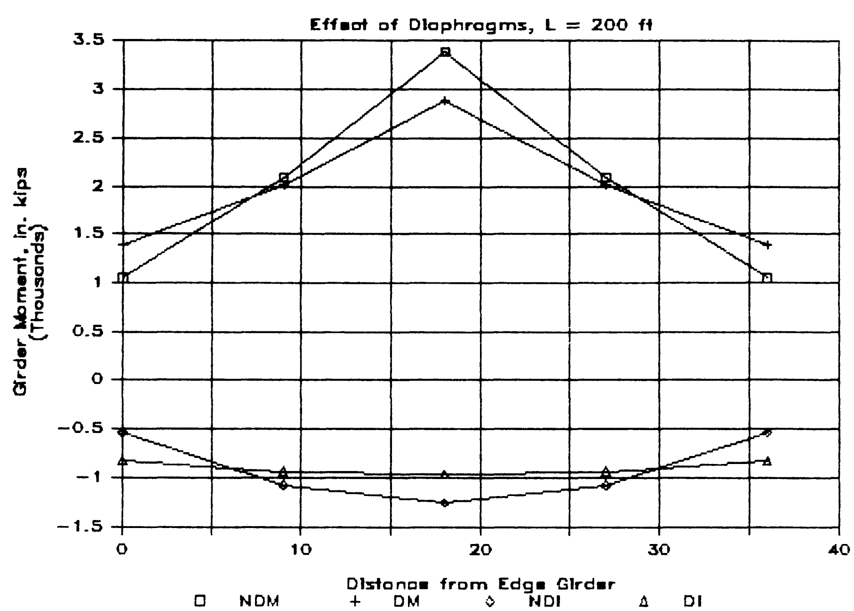

Fig. 3. Two-span continuous bridge

more pronounced and serves to make the distribution of moments somewhat more uniform.

These same results are presented as fractions of the total moment in the span at the cross section of concern in Figs. 4 and 5. Here it is clear that for edge loading, (1) the effects of diaphragms is very small and (2) the relative distribution of moments is essentially the same for midspan moments and for those over the interior pier. The data for these four figures is also summarized in Table 1.

While no numerical results are presented herein, it should be emphasized that when more loads are present across the section (more lanes loaded), the distribution of moments becomes much more uniform. When the structures analyzed for this discussion are loaded with three full lane loads the five girders share the total moment also equally.

Results for Three- and Four-span Continuous Bridges

Results for the behavior of three and four span continuous bridges will be available in the final report of the project but none are included herein. The picture of the lateral distribution of effects is much the same on a qualitative basis as for the two-span or indeed the simple span bridges. A case could be argued for the development of an equivalent simple span bridge to distribute moments in the positive moment regions.

ASSESSMENT OF THE AASHTO *s/D* RULE

In most solutions run in the course of the study the moments across the span were compared with the appropriate AASHTO *s/D* rule, for the interior or exterior as appropriate. Some trends have emerged and selected cases are

Table 1

Data Worksheet for RPB 12 Model — 5-Girder Bridge
Two-span Continuous, L = 200 ft — S = 9 ft
File: work2

Girder	Location ft	Moment in-kips w/o Diaph	w/Dia	Fraction of Total Beam Moment w/o Diaph	w/Dia
(a) Two 10 kip loads, 6-ft spacing One load over Girder A					
A	0.0	5395.0	5242.0	0.55538	0.53974
B	9.0	3432.0	3427.0	0.35330	0.35286
C	18.0	1261.0	1530.0	0.12981	0.15754
D	27.0	232.0	304.0	0.02388	0.03130
E	36.0	−606.0	−791.0	−0.06238	−0.08145
Totals =		9714.0	9712.0		
(b) Load as in (a), Negative Moment over Support					
A	0.0	−2513.0	−2377.0	0.55919	0.52940
B	9.0	−1535.0	−1596.0	0.34157	0.35546
C	18.0	−721.0	−862.0	0.16044	0.19198
D	27.0	−98.0	−167.0	0.02181	0.03719
E	36.0	373.0	512.0	−0.08300	−0.11403
Totals =		−4494.0	−4490.0		
(c) Two kip loads centered					
A	0.0	1051.0	1392.0	0.10866	0.14374
B	9.0	2087.0	2009.0	0.21578	0.20746
C	18.0	3396.0	2882.0	0.35112	0.29760
D	27.0	2087.0	2009.0	0.21578	0.20746
E	36.0	1051.0	1392.0	0.10866	0.14374
Totals =		9672.0	9684.0		
(d) Load as in (c), Negative Moment over Support					
A	0.0	−534.0	−823.0	0.11965	0.18346
B	9.0	−1069.0	−935.0	0.23952	0.20843
C	18.0	−1257.0	−970.0	0.28165	0.21623
D	27.0	−1069.0	−935.0	0.23952	0.20843
E	36.0	−534.0	−823.0	0.11965	0.18346
Totals =		−4463.0	−4486.0		

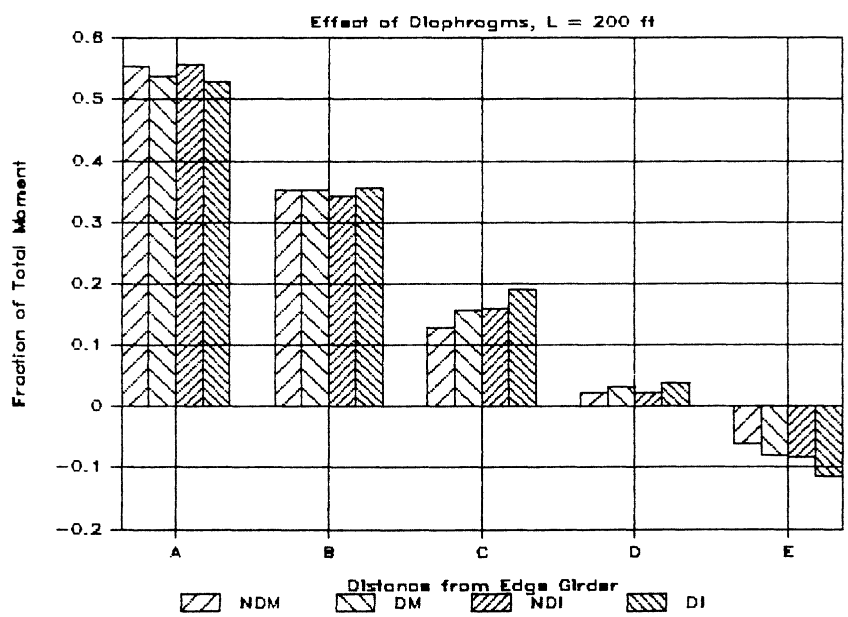

Fig. 4. Two-span continuous bridge

discussed in the following as representative of the more complete results planned. It is important to qualify these comments with the note that neither skew nor curvature of the bridges have been taken into account.

Parameter Studies for Comparison with AASHTO Distribution Rules

So as to have a wide range of relative slab to girder stiffness values, over 60 solutions were run with a wide and sometimes impractical slab thickness variation (5 to 12 in.).

The basic comparisons which were made for a wide variation H as noted above were accomplished with the RPB12 model. A sampling of results is given in Figs. 6 through 8 in which the proportion of moment carried by the girder is expressed in an equivalent D from the s/D format and plotted versus H. The distribution of moment is given for edge beam (Girder A) or an interior beam—either adjacent to the edge beam (B) or the center interior girder (C). In the legend the girders are denoted A, B, or C and the prefix M or I signifies midspan positive or interior negative moment region, respectively. It is seen that the $s/5.5$ rule from AASHTO overestimates the interior beam moment. The same is true in a number of cases for the edge girder, but to a lesser extent. In other cases the AASHTO may slightly underestimate moment—but these are relatively less common parameter combinations which need not represent reasonable design practice (i.e. unworkable slab thickness and girder spacing combinations).

Values of D at Midspan:

H	Exterior Girder	Center Girder
2.48	5.99	7.96
5.87	5.8	7.14
13.2	5.75	6.58
18.8	5.75	6.34
28.1	5.75	6.1

For a two-span structure with 50-ft spans supported on seven girders (an aspect ratio for the slab of 0.12) the following values of equivalent D for positive moment at midspan are seen:

Similar results would be seen for the corresponding girders over the support. Lower values for the exterior girder can be obtained only by doing to a girder spacing which is below usual practice, 4.5 ft for a 50-ft span of 44 ft total width.

Perhaps more representative are results for a 100-ft two-span continuous bridge, five girders spaced at 9 ft:

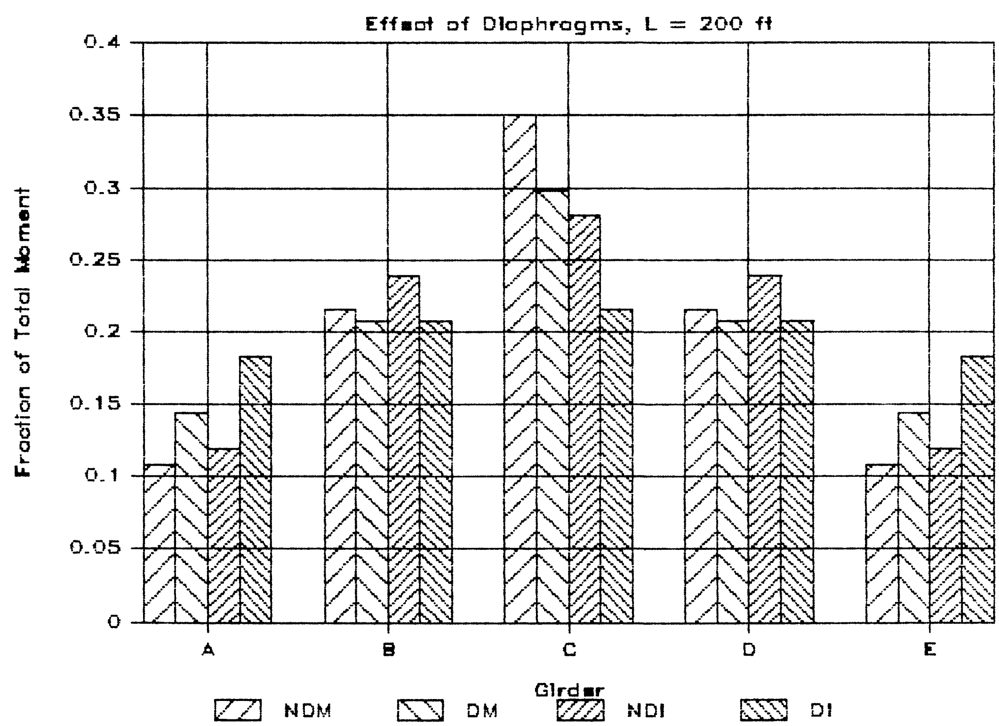

Fig. 5. Two-span continuous bridge

Values of D at Midspan:

H	Exterior Girder	1st Int. Girder	Center Girder
3.54	7.8	10.4	10.8
6.91	7.51	9.66	9.76
10.3	7.39	9.2	9.22
16.4	7.27	8.69	8.67
26	7.21	8.38	8.4

The largest D seen was about 12 for interior beams. Note trend with increasing H (stiffness parameter).

CONCERNING THE AASHTO CODE

Should simple formulas be replaced? Certainly the most direct method for obtaining better load distribution values is to refine the analysis—provided the extra effort is warranted by the need for a more competitive design, and computational resources are available. In simple cases and for preliminary *early* design iterations the *s*/*D* scheme is best. But to do more, and better, with the grid analysis takes a computer resource no more elaborate than a desktop personal computer. This same machine may in a few years also house the electronic implementation of the specification.

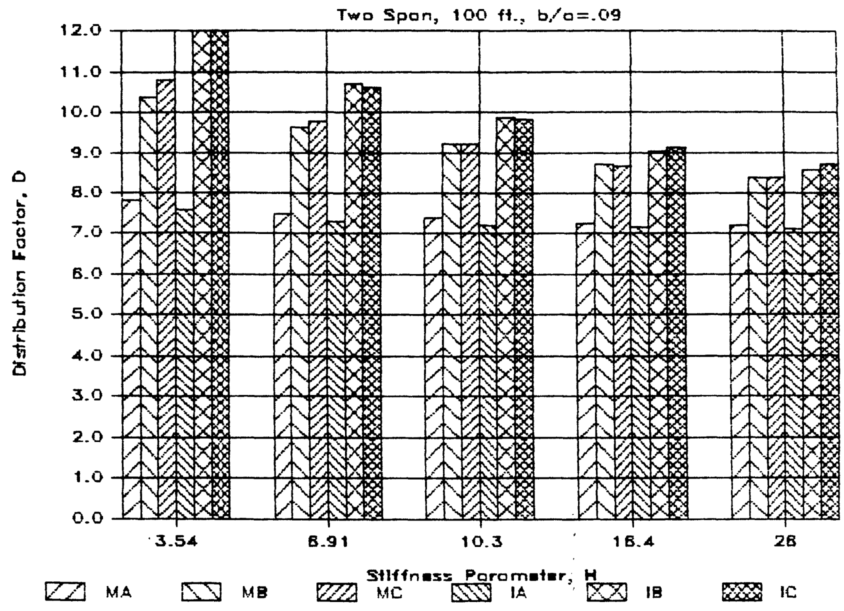

Fig. 6. Distribution factor D

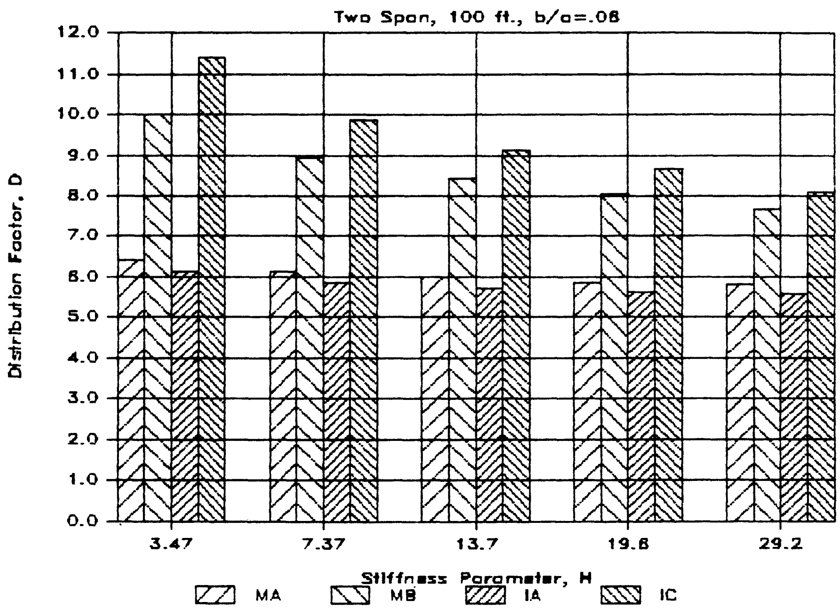

Fig. 7. Distribution factor D

Of course even as AISI Project 332 rounds to a close, the power of the desktop machine has increased dramatically so most of the studies at the exact level could be accomplished on a desktop super microcomputer. But the exact analysis is formidable to input for analysis unless a well written preprocessor for describing the structure is implemented, and equally important, a post processor reduces the flood of results to the needed design factors for load distribution. Hence the simple grid model yielding useful and improved results, capable of implementation on simple personal computers, and requiring no auxiliary charts and look-ups has appeal for addition to the AASHTO code.

The use of a simple grid analysis (or better) instead of formula (*s*/*D*) or refined formula with charts has advantages. It can be incorporated into a computer based specification format and implementation directly. A more accurate load distribution procedure is attractive for both rating and assessment of stress range for fatigue analysis. The grid analysis can be performed on a personal computer using either available software or special purpose software design to output only the desired distribution coefficients. The latter could be based on any of several textbook implementations of a grid analysis program. Based on the project experience a 15-node grid problem would require about 30-sec. solution time; the size of the grid run would more likely be limited by the memory or memory access limit inherent in the hardware and software.

Hence an additional goal of this presentation is to introduce the grid model and indicate its range of usefulness and comment on the issues related to its implementation in a design office. Questions such as: (1) Does the grid serve as a useful simplification of RFSHELL or RPB12 models; (2) In formulating an equivalent grid, what are the refinements needed. Example: can simplifications be made in modelling changes in EI of the girder along the span; and (3) What limitations should be imposed?

Comparison of RFSHELL PB12 and GRID Models

Shown in Fig. 9 are the results of selected comparisons. The corresponding data are given in Table 2. The terms GRID2, GRID4, 8 or 20 denote, respectively, grid models with 2, 4, 8 or 20 longitudinal subdivisions of the span. The grid models are constructed on the basis of simple rules; transverse beams represent the equivalent slab and diaphragms if present and longitudinal girders model the composite moment of inertia for longitudinal bending. The fact that a crude grid by chance gives a better representation of a certain moment value is not to be generalized. The grid model has limitations in the way it can represent torsion in the deck. In all cases it gives a better representation of bridge behavior than does a simple two-value *s*/*D* rule.

Load Representation

If a simple grid model is permitted and becomes a primary scheme for AASHTO lateral distribution calculations, then explicit guidance for placement of loads is appropriate. In keeping with present practice the longitudinal distribution to multiple axles will be neglected and the presence of a single vehicle will be represented by a pair of loads placed 6 ft apart (the specification value). These loads must then be placed transversely so as to produce the maximum effect in the beam of concern. For this purpose influence surfaces could be generated but would seem to be an unneeded refinement for most situations.

In the present study placing the outer load of the pair of loads over the edge girder give a good value for the distribution of moment to the edge girder. This would be modified only if there exists an extreme overhanging deck extending, and loaded, say a lane width beyond the exterior girder. Of course for the assessment of cantilever moments in the slab or floor beams in the region outboard of the exterior girder, the grid model may be of questionable value or should be used with care. Multiple loaded lanes can be treated with

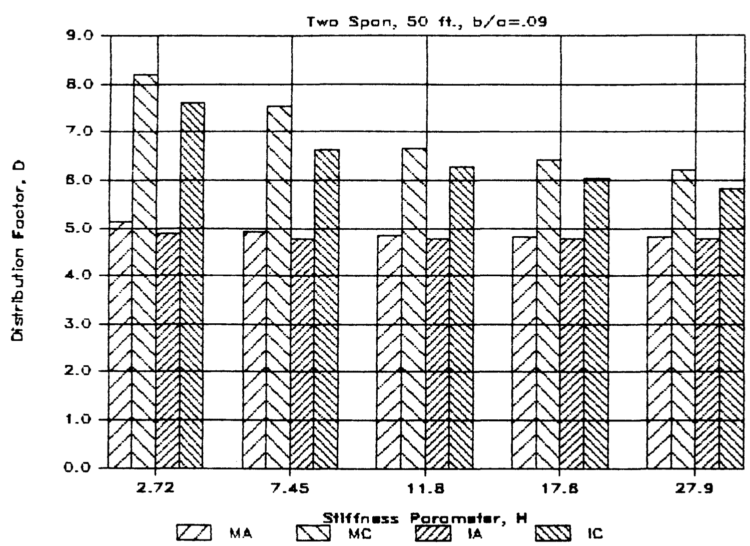

Fig. 8. Distribution factor D

Table 2. Grid Model Comparisons
Load Fraction in Girder A at Mid-span

H	RFSHELL	RPB12	Grid 2	Grid 4	Grid 8	Grid 20
7.180	0.520	0.546	0.555	0.569	0.571	0.572
9.850	0.533	0.555	0.560	0.574	0.575	0.576
14.000	0.547	0.566	0.567	0.579	0.581	0.581
20.900	0.561	0.577	0.576	0.587	0.587	0.588
33.300	0.575	0.589	0.588	0.596	0.596	0.596
7.180	1.000	1.050	1.067	1.094	1.098	1.100
9.850	1.000	1.041	1.051	1.077	1.079	1.081
14.000	1.000	1.035	1.037	1.059	1.062	1.062
20.900	1.000	1.029	1.027	1.046	1.046	1.048
33.300	1.000	1.024	1.023	1.037	1.037	1.037

appropriate numbers of paired loads placed at 6-ft centers and separated so as to give the correct lane width. (Where RFSHELL and RPB12 models are used and where sufficient elements are present then actual distributed load values can be used directly.)

A Further Simplification

It should be emphasized the truss diaphragm can be included as an element in the grid model. It is treated as an equivalent beam with suitably adjusted values of flexural and shear stiffness. It has been seen for the two span continuous beam that the moments over the interior pier were transversely distributed in much the same fashion as the midspan positive moments. Although not documented here, it is also seen that transverse distribution of moments at the point of maximum is very nearly the same as for a convenient adjacent location—say at midspan. The distribution of positive moments in the continuous structure is also well correlated with that of a geometrically and structurally similar simple span segment.

An equivalent simple span may thus be considered a further simplification of the grid model approach. Its chief virtue would be ease of data input, since only the relative stiffness parameter H, a set of truss diaphragm parameters, number of girders and their spacing and the slab overhang would be needed to describe the structure.

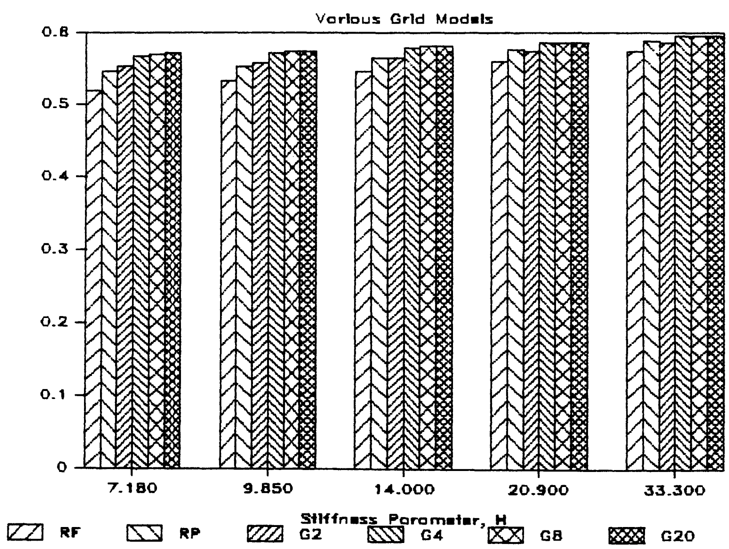

Fig. 9. Comparison of models

SUMMARY AND CONCLUSIONS

This presentation is based on studies in progress under the AISI Project 332 investigation entitled "Lateral Load Distribution for Multi-Girder Bridges." Three analytical models have been implemented in the study and the applicability and limitations of the AASHTO *s/D* rule have been explored. The use of a grid model to represent the essential predictions of the more exact analytical models has been explored. Grid model is seen as a useful option for a lateral load distribution provision in the specification. A simple microcomputer implementation of a grid model based provision are seen to be workable possibilities.

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