

Computer-aided Design of Horizontally Curved Girders by the V-load Method

WILLIAM N. POELLOT, JR.

In May 1963, USX published a report entitled *USS Structural Report, Analysis and Design of Horizontally Curved Steel Bridge Girders*.¹ It presented a simplified analysis of a two-girder curved bridge, along with verification of the simplified method by a rigorous indeterminate analysis. The 1963 simplified method produced good results but was time consuming for use in actual design.

Further studies were conducted and led to a more practical formulation of the simplified method, which produced results of the same order of accuracy as the 1963 approach. The new formulation was likewise developed for a two-girder system but was shown to be equally applicable to multi-girder systems. It was presented in 1965 as a chapter in U.S. Steel's *Highway Structures Design Handbook*² and came to be known as the V-Load Method, for reasons seen later in this report.

The V-Load Method was computer programmed about 1966 and made available to many designers through the organization CEPA (Society for Computer Applications in Engineering, Planning and Architecture). The program was used for the analysis and design of many curved bridges in the 1970s.³ A limitation, however, was that the method and the program were applicable only to structures with radial support lines. During this period the FHWA-sponsored CURT (Consortium of University Research Teams) program was carried out and it shifted the thrust of curved girder research to more rigorous approaches. However, practicing engineers continued to produce designs by whatever means they could, with heavy reliance on the V-Load Method where it was applicable. Rigorous solutions were used when needed for irregular structures.

In the early 80s, U.S. Steel concluded the V-Load Method might continue to serve as a valuable state-of-the-art design tool if it could be applied to skewed bridges. This

improvement was achieved and validated through the combined effort of USS Research and Richardson, Gordon and Associates. It is available through USS Engineers and Consultants (UEC) in a computer program entitled "V-Load."⁴

This paper traces the derivation of the V-Load Method in detail through its stages of development. We will then look at its application in a design example. The design example will deal with the structural analysis, the sizing of the curved girder sections by the provisions of the *AASHTO Guide Specifications for Horizontally Curved Highway Bridges*⁵ and the computation of such items as lateral flange bending, centrifugal force and forces in crossframes.

The emphasis will be on methods and procedure rather than on specific computer programs, on the assumption that once known and understood, the theory is readily programmable to suit individual needs and equipment.

APPROXIMATE METHOD THEORY

Two Girder System

The theory of the approximate method is centered around the static behavior of a curved flange carrying an axial stress or force (Fig. 1).

If a short segment of such a flange is considered, it is seen the axial forces or stresses on the segment are not co-linear due to curvature. This results in a radial component which,

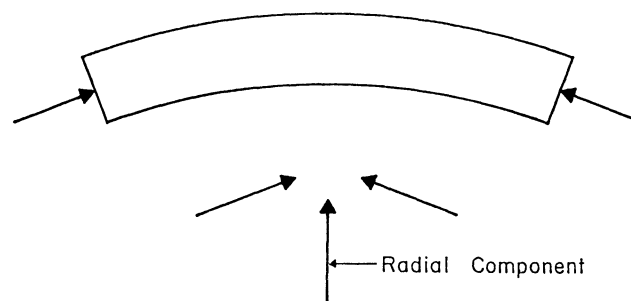


Fig. 1. Radial flange force component

William N. Poellot, Jr. is a Structure Engineer with the consulting firm of HDR-Richardson Gordon, Pittsburgh, Pennsylvania.

over some length of flange, manifests itself as a radial distributed force on the flange as shown in Fig. 2.

To determine magnitude of the radial flange force it is convenient to draw an analogy between the curved flange problem and the classical problem of hoop tension (Fig. 3).

In the hoop tension problem an internal pressure q produces a tensile force T in the hoop equal to qR . The curved girder problem is the same, except the situation is reversed. T is analogous to the flange force in a curved girder which can be represented as the girder moment M divided by the girder depth h . The curvature of this flange force produces q rather than q producing hoop tension. Solving for q gives $q = M/Rh$ as the radial component of flange force on a curved girder flange.

If such a radial force exists, it can be treated as a radial load on the flange. It then becomes logical to introduce diaphragms to resist this lateral load at intervals along the girder (Fig. 4).

A lateral reactive force H can be assumed to exist at each diaphragm. Since the axial forces in the top and bottom flanges are opposite in direction, the q forces at the top and bottom flanges will likewise be opposite in direction, as will the reactive H forces. The top and bottom H forces make up a couple, equivalent to a concentrated torque that the diaphragm is applying to the girder, and

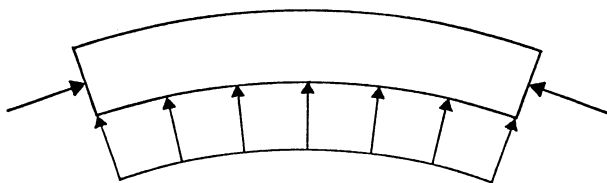
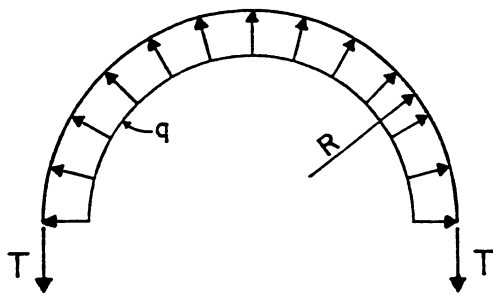


Fig. 2. Radial distributed flange force



$$T = qR$$

$$\text{but } T = F = M/h$$

$$qR = M/h$$

$$\text{and } q = M/Rh$$

Fig. 3. Hoop tension analogy

correspondingly, a concentrated end moment that the girder is applying to the diaphragm.

Figure 5 shows the interaction forces between girder and floorbeam consist of these H forces along with vertical shears V . If the diaphragms are spaced at a distance d , then the radial flange reaction at Girder 1 is $H_1 = q_1 d_1$ or $H_1 = M_1 d_1 / h R_1$ and at Girder 2 is $H_2 = M_2 d_2 / h R_2$. With concentric girders and radial diaphragms $d_1 / R_1 = d_2 / R_2$ and we can use simply a constant d/R from here on. The vertical shear force V between diaphragm and girder is obtained from the statics of the diaphragm as $V = (H_1 + H_2)h/D = (M_1 + M_2)/RD/d = M_1 + M_2/K$, where K is the constant RD/d . These V -loads, from whence comes the name "V-Load Method," produce moments in the girder outside of the bridge centerline which add to those that would exist in a straight girder of the same developed length, and moments in the girder inside of the bridge centerline which subtract from those that would exist in a straight girder of the same developed length.

The moment in the outside girder M_1 may thus be expressed as $M_1 = M'_1 + M_{V1}$, where M'_1 is the moment in the developed girder due to the external loads, and M_{V1} is the moment in the developed girder due to the V -loads, or secondary moment. Likewise, the moment in the inside girder M_2 may be expressed as $M_2 = M'_2 + M_{V2}$. However,

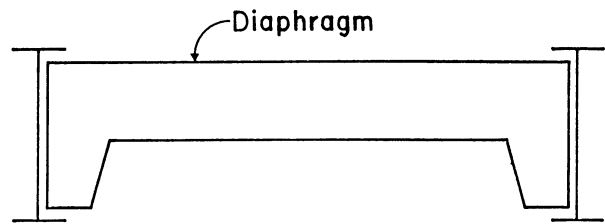


Fig. 4. Diaphragm

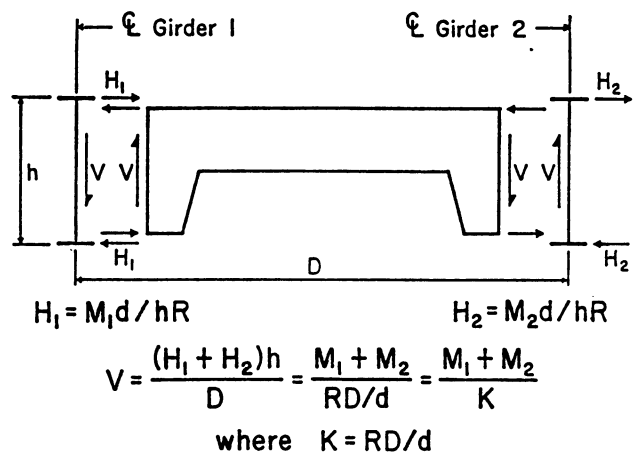


Fig. 5. Girder/diaphragm interaction forces

M_{V2} is equal to $-M_{V1}$ times the ratio of girder span lengths L_2/L_1 . The sum total girder moments $M_1 + M_2$ is then equal to the sum of the primary moments $M'_1 + M'_2$ plus $M_{V1}(1 - L_2/L_1)$. Since M_{V1} is small compared to M'_1 and M'_2 , and since $1 - L_2/L_1$ is also small, the whole quantity $M_{V1}(1 - L_2/L_1)$ is a second order term and may be neglected. Thus, $M_1 + M_2 = M'_1 + M'_2$ and $V = M'_1 + M'_2/K$. This is a key equation for the approximate or V-load method of curved girder analysis. The V-loads at any diaphragm or point along the structure are obtained simply by summing the primary moments in the girders at that point and dividing by K .

With the V-loads thus determined, everything is known that is needed to compute the normal bending moments and shears in the girder. Additionally, we have the lateral q loading on each flange which the flanges must carry acting as continuous beams supported at the diaphragms. This is illustrated in Fig. 6.

As shown, $q = M/Rh$. An approximation of the maximum lateral moment that can occur due to this loading is $M_{Lat} = qd^2/10$. The lateral flange bending stresses may be computed as M_{Lat}/S_{Lat} where S_{Lat} is the lateral section modulus of the flange. These stresses are equivalent to the so-called warping normal stresses that are discussed in rigorous treatment of torsion. They must be superimposed on the ordinary vertical bending stresses.

Finite element analysis has shown that $Md^2/12Rh$ is a more correct expression of M_{Lat} .⁶

APPROXIMATE METHOD THEORY

Multi-girder System

The V-load theory is extended relatively easily to include multiple girder systems. For multiple girder systems the behavior of the structure is broken down into two independent effects: the grid effect and the curvature effect. The grid effect is assumed to be present in any multi-girder bridge whether it is straight or curved and depends on the relative stiffnesses of the girders and connecting diaphragms. With the V-Load Method, this effect is taken care

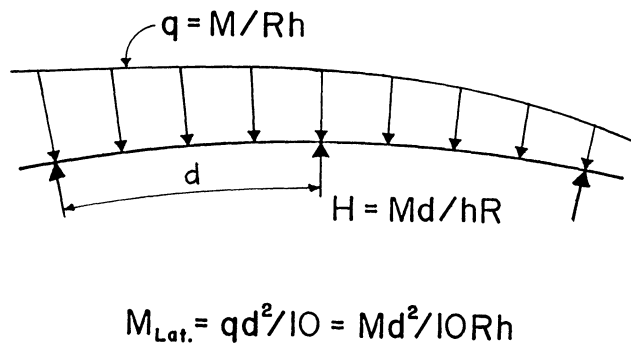


Fig. 6. Lateral flange bending

of in the same manner as for straight girders using the distribution factors given in the AASHTO Specifications.

The curvature effect is considered by the V-Load Method to be more a function of geometry than of relative stiffness. It is assumed that loading on girders outside the centerline of the system is increased and loading on girders inside the centerline is decreased. Evaluating the curvature effect is done using an approach similar to that used for a two-girder system. A reasonable assumption is that V-loads on any girder are proportional to the distance of the girder from the longitudinal centerline of the system as shown in Fig. 7 for a four girder system.

Free bodies in this figure are the girders and portions of the connecting diaphragms. Forces shown are those acting on these free bodies. The diaphragms are cut at assumed inflection points.

The shears on the diaphragms are V_1, V_2 and V_3 . The load on the outside girder is taken as V and V equals V_1 . By proportion of distance from centerline, the load on interior girders will be $(1/3)V$. Expressing the effect of curvature in the same way as in the approximate solution of a two-girder system results in these equations:

$$Hh = V_1a = M_1 d/R$$

$$V_1(D/3 - a) + V_2b = M_2 d/R$$

$$V_2(D/3 - b) + V_3c = M_3 d/R$$

$$V_3(D/3 - c) = M_4 d/R$$

where M_1, M_2, M_3 and M_4 are the total moments in the main girders at any diaphragm location.

Adding the equations together we have:

$$V_1 + V_2 + V_3 = (M_1 + M_2 + M_3 + M_4)/RD/3d \\ = M_{Total}/RD/3d$$

Substituting $V = V_1 = V_3$ and $V_2 = V_1 = (1/3)V$, and recognizing, as was shown for the two-girder system, that $\Sigma M_{Total} \approx \Sigma M_{Primary}$:

$$10/3V = \Sigma M_{Primary}/RD/3d \text{ or } V = \Sigma M_{Primary}/(10/9)RD/d$$

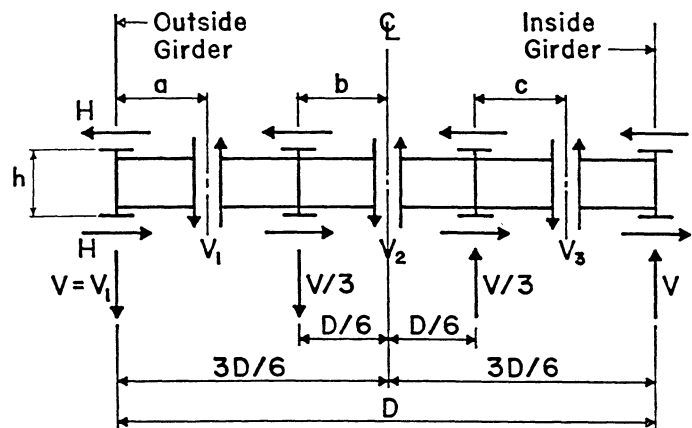


Fig. 7. Forces in multiple girder system

Table 1. Coefficients of K

System	Coefficient Fraction	Coefficient Decimal
2 Girders	1	1.00
3 Girders	1	1.00
4 Girders	10/9	1.11
5 Girders	5/4	1.25
6 Girders	7/5	1.40
7 Girders	14/9	1.56
8 Girders	12.7	1.72
9 Girders	15/8	1.88
10 Girders	165/81	2.04

Using $K = RD/d$ as for a two-girder system,
 $V = \Sigma M_{Primary} / (10/9)K$

It is seen that this equation is of the same form as that for the two girder system except that an additional coefficient 10/9 appears in the denominator. Derivation of the equation shows that this coefficient varies with the number of girders, according to Table 1.

Note that for analysis of ordinary moments and shears by the V-Load Method one need not yet have determined the diaphragm spacing. Any convenient number of diaphragms or diaphragm spacing may be initially assumed because the analysis results are independent of the spacing. Computer programs have typically assumed diaphragms at the tenth points of the spans.

In summary, the required procedure for analyzing a curved, stringer-type bridge by the V-Load Method is as follows:

1. Determine primary moments by the same procedures as for a system of straight girders using the developed lengths of the curved girders.
2. From primary moments, compute V-loads using the formula:

$$V = \Sigma M_{Primary} / \text{Coef.} \times K$$

3. Compute V-load moments.
4. Set actual diaphragm spacing.
5. Compute lateral bending moments using approximate formula:

$$M_L = Md^2 / 12Rh$$

M = primary moment plus secondary moment at any diaphragm location; all other symbols are as previously designated.

6. Determine cross sections required to provide for vertical and lateral forces computed under Items 1 to 5, inclusive.

The V-Load Method as derived in the foregoing pages is essentially the method developed and computer-programmed in the mid-60s for regular curved bridges with radial supports. Skew capability was added in the early 80s

Table 2. Geometry

Girder	Radius	Arc	Δ°
G1-1	300.0000	110.0000	21.00844
G1-2	300.0000	60.0000	11.45917
G1-3	600.0000	40.0000	3.81972
G2-1	291.1667	110.4751	21.73931
G2-2	291.1667	53.1651	10.46183
G2-3	591.1667	39.4111	3.81972
G3-1	282.3333	111.0056	22.52711
G3-2	282.3333	46.2739	9.39067
G3-3	582.3333	38.8222	3.81972
G4-1	273.5000	111.6008	23.37936
G4-2	273.5000	39.3167	8.23650
G4-3	573.5000	38.2333	3.81972

by assuming fictitious skewed diaphragms connecting the tenth points of the girders, in the computer program V-LOAD.

APPLICATION EXAMPLE

The four-girder bridge with the framing plan shown in Fig. 8 is an example of the type of curved structure that can be analyzed by the V-Load Method. Table 2 gives the geometry. Two of the supports are skewed and there is a point of compound curvature in the second span at which the radius of the outside girder G1 changes from 300 ft to 600 ft.

Figure 9 shows the cross section of the bridge. Dead loads and live load distribution factors are computed for Girders G1, G2, G3 and G4 exactly as they would be if the bridge were straight.

The current AASHTO Specifications require that centrifugal force be treated as a primary load for curved bridges. The Specifications are clear in defining the magni-

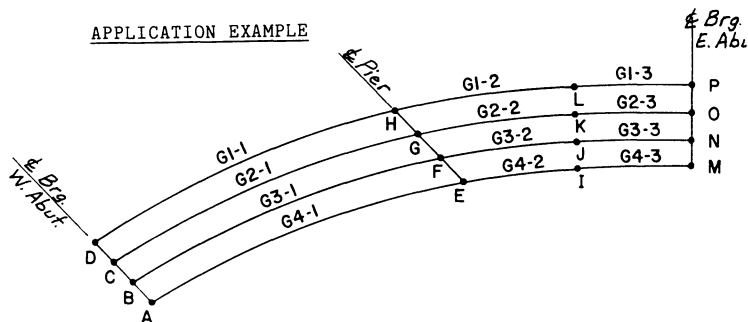


Fig. 8. Plan of curved bridge

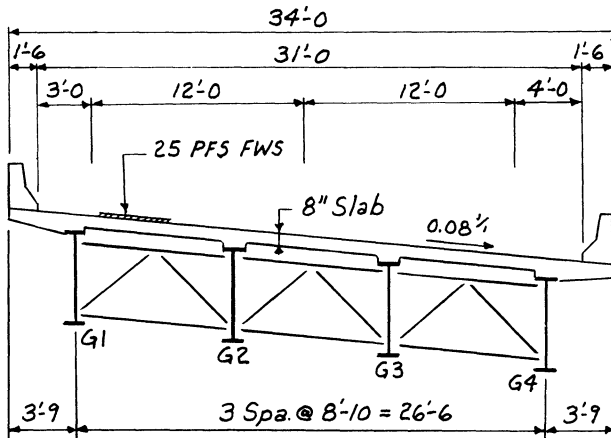
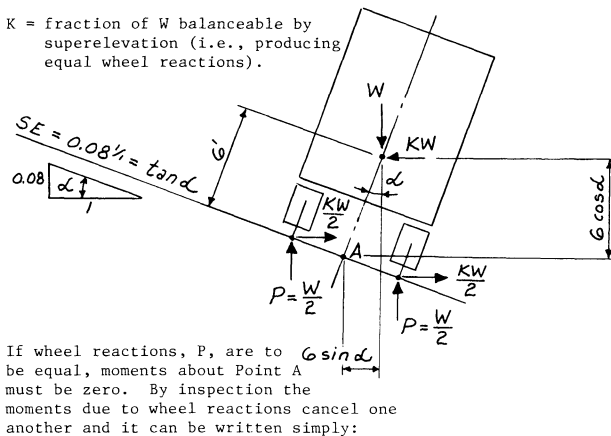


Fig. 9. Cross section



$$W(6 \sin \alpha) - KW(6 \cos \alpha) = 0$$

$$6 \sin \alpha W = 6 \cos \alpha KW$$

$$K = \frac{6 \sin \alpha W}{6 \cos \alpha W} = \tan \alpha = SE = 0.08$$

Fig. 10. Superelevation effect

tude of the centrifugal force as a percentage of the live load, applied through the center of gravity of the vehicle, 6 ft above the deck. Beyond this, designers must make their own assumptions as to how the centrifugal force is distributed through the structure. A possible procedure is outlined in the following paragraphs. The CF force may be considered to be transmitted to the bridge as a shear and an overturning moment at the deck level. The shear is delivered directly to the bridge bearings by way of the deck slab and the crossframes at the supports, with no measurable effect on the bridge other than a contribution to the member forces in the diagonals of the support crossframes.

The overturning moment may be reasonably assumed to increase the vertical loading on the outside girders and decrease the vertical loading on the inside girders. Not all of the CF is doing this, however. A given superelevation of the deck is capable of balancing a certain amount of centrifugal

force by equalizing the outside and inside wheel load reactions of the vehicle. When these wheel reactions are equal for a portion of the CF loading there should be no overturning moment from that portion. We can assume that only the CF over and above the amount balanceable by superelevation is producing overturning moment on the structure.

The amount of centrifugal force balanceable by superelevation is easily derived. As shown in Fig. 10, if it is expressed as a fraction of live load, this fraction is exactly equal to the superelevation rate.

Proceeding on this basis to the structure at hand, the CF will be a function of the design speed, taken as 30 miles per hour for this two lane ramp on a centerline radius of slightly less than 287 ft. The superelevation rate is 0.08 ft per ft, or 8.0%. The formula from AASHTO Article 1.2.21 gives a centrifugal force value of 21.0%, but 8.0% is balanceable by the superelevation, leaving 13.0% as producing an overturning moment about the mid-depth of the slab. With two lanes of live load on the bridge, the CF force in terms of lanes is 0.260 lanes.

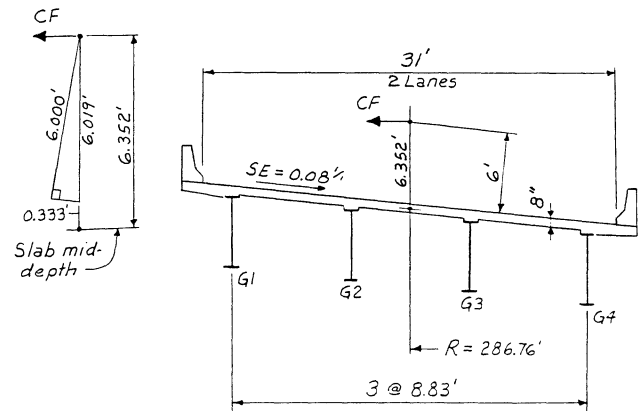


Fig. 11. Centrifugal force application

$$S = 30 \text{ mph}$$

$$R = 286.76 \text{ ft}$$

$$C = 6.68S^2/R = (6.68)(30)^2/286.76 = 21.0\%$$

$$\text{Balanceable by superelevation} = (100)(0.08) = \frac{8.0\%}{13.0\%}$$

$$CF = (0.130)(2) = 0.260 \text{ lanes}$$

Pile group distribution is used to determine the vertical CF loads on the girders due to the overturning moment. The moment of inertia of the four girders treated as piles is 389.8 ft.² Loads of 0.056 and 0.019 lanes are computed for G1 and G2. The CF vertical loads on G3 and G4 are assumed to be zero, since such loads would act upward and be subtractive from the other vertical loads. This is a valid loading case corresponding to the situation in which live load is on the structure but not moving.

$$I_{\text{Pile Group}} = (2) \left[\left(\frac{8.83}{2} \right)^2 + (1.5 \times 8.83)^2 \right] = 389.8 \text{ ft}^2$$

$$\text{CF load on G1} = \frac{(0.260)(6.352)(1.5)(8.83)}{389.8}$$

$$= 0.056 \text{ lanes}$$

$$\text{CF load on G2} = \frac{(0.260)(6.352)(0.5)(8.83)}{389.8}$$

$$= 0.019 \text{ lanes}$$

CF load on G3 and G4 = 0 lanes by inspection, assuming $S = 0$ mph

It is recalled that the V-loads are a function of the summation of primary moments transversely across the structure at each crossframe line. Since the CF loads act downward on G1 and G2, and upward on G3 and G4, the summation of moments is essentially zero and therefore the V-loads are zero. Thus, for centrifugal force loading all that need be analyzed is the primary load effect, which can be done on an isolated straight girder basis.

An analysis of DL_1 and DL_2 and $L + I$ by Program V-LOAD, and of CF on an individual girder basis, yields moments and shears at the tenth points of the spans from which the moment curves in Fig. 12 can be plotted along with a similar set of shear curves.

Design of the girder sections then consists of sizing for these forces using the provisions of the AASHTO *Guide*

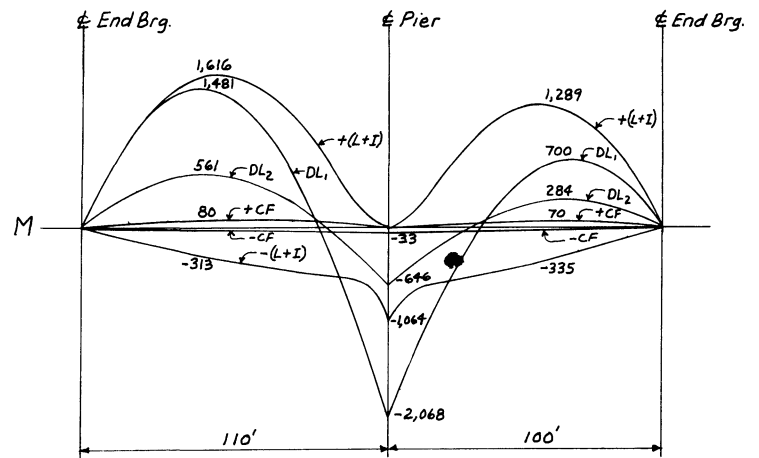


Fig. 12. Moment curves for girder G1

Specifications for Horizontally Curved Highway Bridges.

At this point, the actual diaphragm layout can be set. The chosen arrangement is shown in Fig. 13. Having the diaphragm spacing will enable the computation of allowable stresses and lateral bending stresses.

Figure 14 shows a section that will be investigated as the maximum positive moment section in Span 1 of Girder G1. The section is composite with an effective slab width of 90 in. Section properties are computed in the normal fashion for the steel section and long and short term composite sections. Using these properties and the moments at the section, the stresses are calculated as follows:

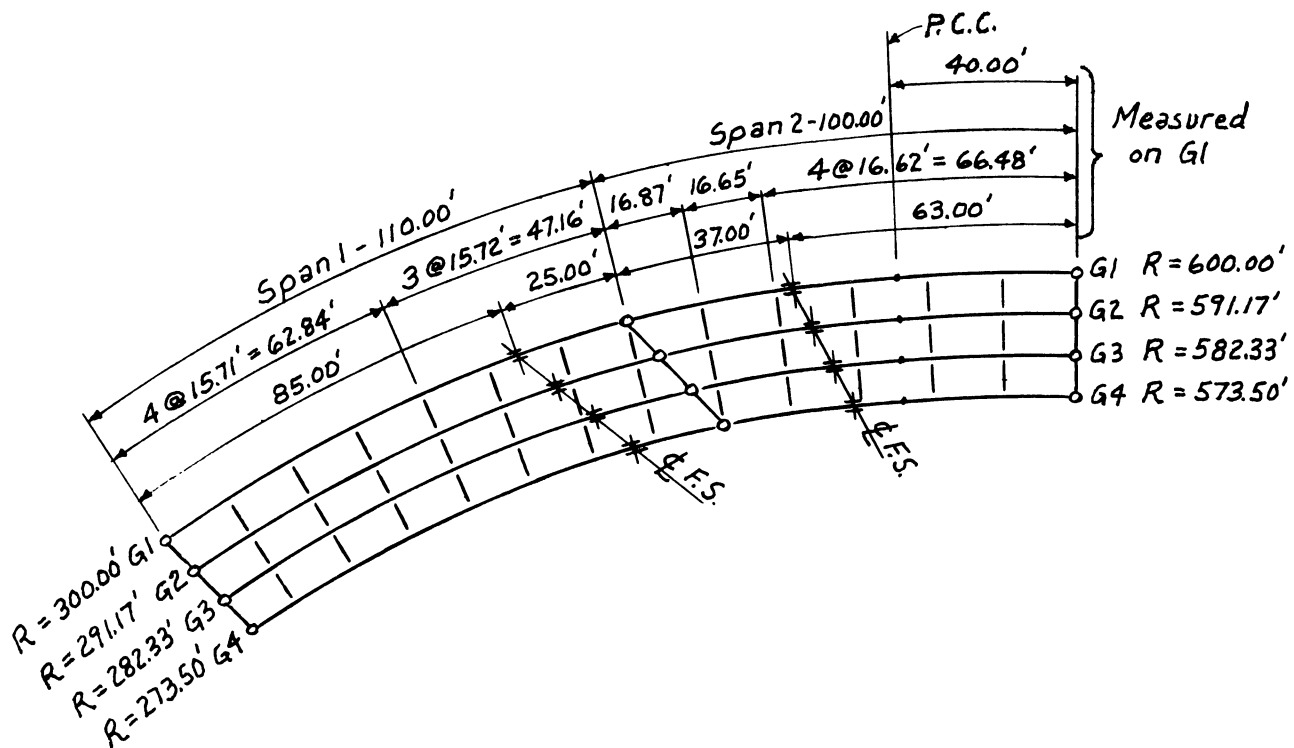


Fig. 13. Framing plan

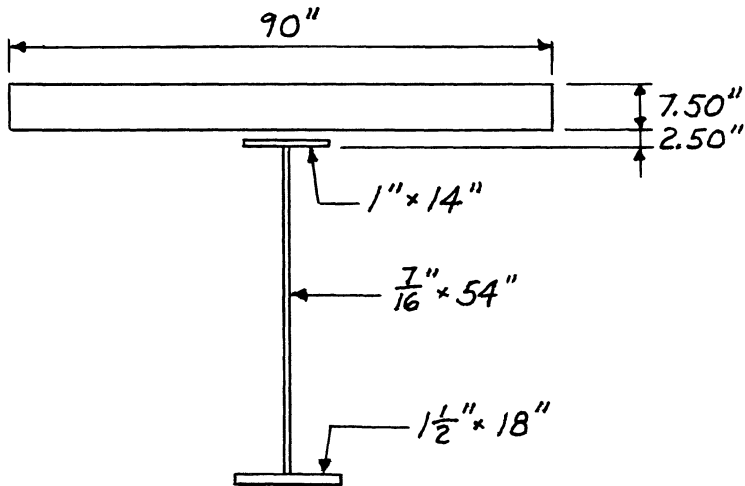


Fig. 14. Maximum positive moment section

Bending Moments 44 ft from End Support

	DL_1	DL_2	$L + I$	CF
M , kip/ft	1,481	561	1,616	80
$M_{\text{Top Flg. Lat } DL_1}$	$= \frac{M_{DL_1} d^2}{12Rh} = \frac{(1,481)(15.71)^2}{(12)(300)\left(\frac{55.25}{12}\right)} = 22.1 \text{ k/ft}$			
$M_{\text{Bot Flg. Lat } DL_1}$	$= 22.1 \text{ k/ft}$			
$M_{\text{Bot Flg. Lat } DL_2}$	$= \frac{(561)(15.71)^2}{(12)(300)\left(\frac{55.25}{12}\right)} = 8.4 \text{ k/ft}$			
$M_{\text{Bot Flg. Lat } L+I}$	$= \frac{(1,616)(15.71)^2}{(12)(300)\left(\frac{55.25}{12}\right)} = 24.1 \text{ k/ft}$			
$M_{\text{Bot Flg. Lat } CF}$	$= \frac{(80)(15.71)^2}{(12)(300)\left(\frac{55.25}{12}\right)} = 1.2 \text{ k/ft}$			

Steel Stresses Due to Maximum Design Loads

Top of steel (compression)

Vertical bending:

$$\text{For } DL_1: f_b = \frac{1,481 \times 12}{1,043} \times 1.30 = 22.2$$

$$\text{For } DL_2: f_b = \frac{561 \times 12}{2,969} \times 1.30 = 2.9$$

$$\text{For } L + I: f_b = \frac{1,616 \times 12}{7,819} \times 1.30 \times \frac{5}{3} = 5.5$$

$$\text{For CF: } f_b = \frac{80 \times 12}{7,816} \times 1.30 = \frac{0.2}{30.8 \text{ ksi}}$$

Bottom of steel (tension)

$$\text{For } DL_1: f_b = \frac{1,481 \times 12}{1,543} \times 1.30 = 15.1$$

$$\text{For } DL_2: f_b = \frac{561 \times 12}{1,871} \times 1.30 = 4.7$$

$$\text{For } L + I: f_b = \frac{1,616 \times 12}{2,022} \times 1.30 \times \frac{5}{3} = 21.4$$

$$\text{For CF: } f_b = \frac{80 \times 12}{2,022} \times 1.30 = \frac{0.6}{41.8 \text{ ksi}}$$

Lateral bending:

Top of steel (compression)

$$\text{For } DL_1: f_w = \frac{22.1 \times 12}{32.7} \times 1.30 = 10.5$$

$$\text{For } DL_2: f_w = 0.0$$

$$\text{For } L + I: f_w = 0.0$$

$$\text{For CF: } f_w = \frac{0.0}{10.5 \text{ ksi}}$$

Bottom of steel (tension)

$$\text{For } DL_1: f_w = \frac{22.1 \times 12}{81.0} \times 1.30 = 4.3$$

$$\text{For } DL_2: f_w = \frac{8.4 \times 12}{81.0} \times 1.30 = 1.6$$

$$\text{For } L + I: f_w = \frac{24.1 \times 12}{81.0} \times 1.30 \times \frac{5}{3} = 7.7$$

$$\text{For CF: } f_w = \frac{1.2 \times 12}{81.0} \times 1.30 = \frac{0.2}{13.8 \text{ ksi}}$$

Note there are no lateral bending stresses from DL_2 and $L + I$ on the top flange because that flange is supported by the slab under those loads. The compression flange geometry is checked and found to satisfy the requirements for compactness by a narrow margin. The allowable stress is given by Article 2.12A of the Guide Specifications:

Allowable Steel Stresses

Top flange (compression):

$$\frac{b}{t} = \frac{14}{1} = 14.0$$

$$\frac{b}{t_{\text{Max. for compact flg.}}} = \frac{3,200}{\sqrt{F_y}} = \frac{3,200}{\sqrt{50,000}} = 14.31 > 14.0$$

∴ Flange is compact

$$\bar{\rho}_B = \frac{1}{1 + \frac{\ell}{b} \left(1 + \frac{\ell}{6b}\right) \left(\frac{\ell}{R} - 0.01\right)^2}$$

$$= \frac{1}{1 + \frac{15.71}{14/12} \left[1 + \frac{15.71}{(6)(14/12)}\right] \left(\frac{15.71}{300} - 0.01\right)^2}$$

$$= 0.92729$$

$$\lambda = \frac{1}{\pi} \frac{\ell}{b} \sqrt{\frac{F_y}{E}} = \frac{1}{\pi} \left(\frac{15.71}{14/12}\right) \sqrt{\frac{50}{29,000}} = 0.17798$$

$$F_{bs} = F_y(1 - 3\lambda^2) = 50[1 - (3)(0.17798)^2] = 45.248$$

$$\bar{\rho}_w = 0.95 + 18 \left[0.1 - \frac{\ell}{R}\right]^2 + \frac{f_w \left[0.3 - 0.1 \frac{\ell}{R} \frac{\ell}{b}\right]}{\bar{\rho}_B F_y / F_{bs}}$$

$$= 0.95 + 18 \left[0.1 - \frac{15.71}{300}\right]^2$$

$$+ \frac{10.5 \left[0.3 - 0.1 \left(\frac{15.71}{300}\right) \left(\frac{15.71}{14/12}\right)\right]}{(0.92729)(50)/45.248}$$

$$= 1.06719$$

$$\bar{\rho}_B \bar{\rho}_w = (0.92729)(1.06719) = 0.98959 < 1.0$$

$$F_{bu} = F_{bs} \bar{\rho}_B \bar{\rho}_w = (45.248)(0.98959) = 44.8 \text{ ksi} > 30.8$$

The bottom flange is checked in a similar manner, using the criteria for tension flanges:

$$\bar{\rho}_B = \frac{1}{1 + \frac{15.71}{18/12} \left[1 + \frac{15.71}{(6)(18/12)}\right] \left(\frac{15.71}{300} - 0.01\right)^2}$$

$$= 0.95092$$

$$F_{bs} = F_y = 50$$

$$\bar{\rho}_w = 0.95 + 18 \left[0.1 - \frac{15.71}{300}\right]^2$$

$$+ \frac{13.8 \left[0.3 - 0.1 \left(\frac{15.71}{300}\right) \left(\frac{15.71}{18/12}\right)\right]}{(0.95092)(50)/50}$$

$$= 0.90573$$

$$\bar{\rho}_B \bar{\rho}_w = (0.95092)(0.90573) = 0.86128 < 1.0$$

$$F_{bu} = (50)(0.86128) = 43.1 \text{ ksi} > 41.8$$

The seemingly inefficient top flange (30.8 ksi actual stress vs. 44.8 ksi allowable stress) is justified if one considers, alternatively, a design with a thinner and therefore non-compact top flange. In this case, the following material and stresses would result:

	Material	Bot. Flg. Stress	Top Flg. Stress
Top flg.	$\frac{15}{16} \times 16$	Actual 24.6 ksi	Actual 28.2 ksi
Web	$\frac{7}{16} \times 54$	Allowable 24.9 ksi	Allowable 28.2 ksi
Bot. flg.	$2\frac{7}{8} \times 18$		

This section actually requires 40% more steel because the allowable stresses are substantially lower than for a section with compact compression flange. It is obvious such computations can and should be done by computer. Going beyond the V-Load analysis, there is virtually no limit to the extent to which these ancillary operations can be programmed.

The one other aspect of curved bridge analysis and design that needs to be mentioned is that of diaphragm or cross-frame analysis. The cross-frame forces due to curvature are easily computed from V-Load Theory. Looking at any cross-frame line in the positive moment region of a four-girder system (Fig. 15), the assumed distribution of V-loads and torque loads to the girders is as shown below. Girder G1 is the outside girder.

Corresponding distribution of end shears on the cross-frames follows in Fig. 16:

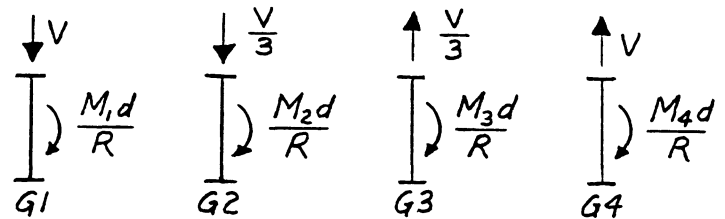


Fig. 15. Girder loads from diaphragms

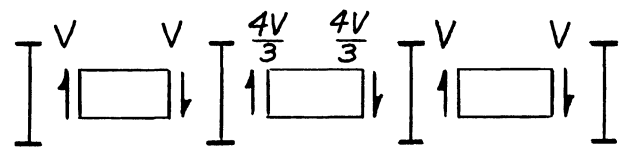


Fig. 16. Diaphragm end shears

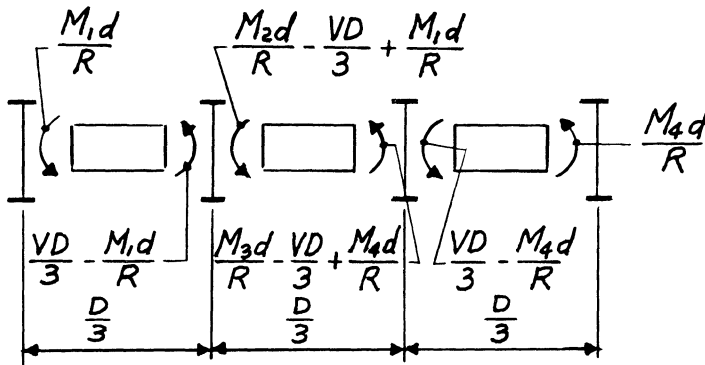


Fig. 17. Diaphragm end moments

It is then just a matter of statics to derive the expressions shown in Fig. 17 for end moments on the diaphragms:

In the Figs. 16 and 17:

$$V = (M_{1P} + M_{2P} + M_{3P} + M_{4P})/K(RD/d)$$

$M_{1P}, M_{2P}, M_{3P}, M_{4P}$ = primary moments in G1, G2, G3, G4 at cross-frame line under investigation

M_1, M_2, M_3, M_4 = total moments in G1, G2, G3, G4 at cross-frame line under investigation

D = width, G1 to G4

d = cross-frame spacing on G1

R = radius of G1

K = a coefficient (10/9 for a four girder system)

SUMMARY

We have seen a design approach for curved bridges that is extremely simple and easily understood. The V-Load Method is not only a valuable tool from the practical standpoint, but familiarity with the method should contribute to the engineer's overall "feel" of the curved girder problem. Although curved girder analysis and design by the V-Load Method is straightforward, it is not a procedure that can be done readily by longhand. But it can be done by a variety of manageable, fast-running computer programs, starting with Program V-LOAD.

REFERENCES

1. Richardson, Gordon and Associates Analysis and Design of Horizontally Curved Steel Bridge Girders *United States Steel Structural Report, ADUSS 88-6003-01, 1963.*
2. United States Steel Corporation Highway Structures Design Handbook *ADUSS 88-1895-01, Vol. 1, Chapter 12, 1965.*
3. Subcommittee on Curved Girders of the Joint ASCE-AASHTO Committee on Flexural Members Survey of Curved-Girder Bridges *Civil Engineering, February 1973.*
4. USS Engineers and Consultants V-Load—A Computer Program for the Analysis of Horizontally Curved Steel Bridge Girders *Pittsburgh, Pa.*
5. American Association of State Highway and Transportation Officials Guide Specifications for Horizontally Curved Highway Bridges *1980.*
6. United States Steel Corporation V-Load Analysis *USS Highway Structures Design Handbook, ADUSS 88-8535-01, Vol. 1, Chapter 12, July 1984.*
7. United States Steel Corporation Highway Structures Design Handbook *ADUSS 88-1895-01, Vol. 2, Chapter 6, 1986.*