Earthquake-resistant Design of Double-angle Bracings

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Behavior of steel-braced frames during an earthquake strongly depends on the behavior of bracing members and their connections. In recent years, considerable research has been done on behavior of bracing members under cyclic loading. However, studies on the behavior of their connections are almost nonexistent.

A number of observers who have studied structures damaged by earthquakes have reported total or partial failures in the connections of bracing members. These observed failures clearly indicate a need for evaluation of current philosophies and procedures employed in design practice.

Simplified design procedures are used in practice even though it is known connections have complex stress distributions. Most of these design procedures are based on the studies of monotonically loaded connections and were intended to be used in design of connections subjected to nonreversible loads. Thus, the cyclic forces and deformations induced in the bracing members and their connections during a strong ground motion earthquake are not considered in such procedures.

To investigate the behavior of double angle bracing members and their connections, 17 full-size, double-angle test specimens were used in this study. The bracing members were placed in a diagonal position inside a loading frame and subjected to reversed cyclic deformations similar to those expected during a severe earthquake. Details of test specimens, test program and analysis of the experimental results can be found in Refs. 1, 2 and 3. Emphasis of the research program was on the evaluation of current design methods and developing modified procedures in order to ensure adequate seismic performance of bracing members.

This paper summarizes the most significant findings of the research program from a design perspective. Three design examples are included to illustrate step-by-step use of the recommended procedures for earthquake resistance design.

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TEST SPECIMENS

All test specimens were fabricated using hot rolled A36 steel unequal leg double angles. The angles were stitched together and were connected to the end gusset plates by bolts or fillet welds. Figure 1 shows a typical test specimen inside the loading frame.

Eight specimens had short legs of angles placed back-to-back. These specimens during compression buckled in the plane of the gusset plates which was in the same plane as the loading frame. Nine specimens had long legs of angles placed back to back. These specimens buckled out of the plane of the gusset plate. The test results indicated the most important parameter affecting behavior of double angle bracing with end gusset plates is the direction of buckling (in-plane or out-of-plane). Depending on direction of buckling, the cyclic behavior and failure modes change significantly.

Cyclic deformation history and details of major observations for some in-plane buckling specimens are shown in Figs. 2 and 3 and for out-of-plane buckling specimens in Figs. 4 and 5. Specimens designated with odd numbers

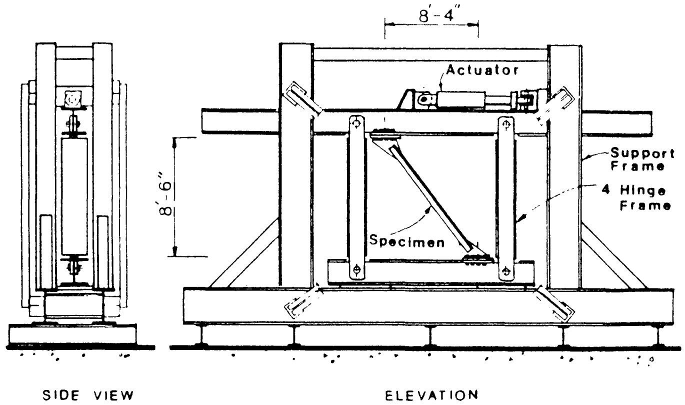

Fig. 1. Test specimen and loading frame

buckled in-plane and those with even numbers buckled out-of-plane. W and B in the designation of test specimens indicates welded and bolted connection details, respectively.

MODE OF BUCKLING AND EFFECTIVE LENGTH FACTOR

The deformed shape of all in-plane buckling specimens was close to a full cycle cosine curve with points of inflection at the two quarter points similar to the deformed shape of an axially loaded column with fixed end connections. The deformed shape of the out-of-plane buckling bracing members was close to a half sine curve similar to an axially loaded column with pin-ended connections. Thus, the effective length factor K for in-plane buckling double angle bracings can be approximated by 0.5. For out-of-plane buckling double angle bracing members the value of effec-

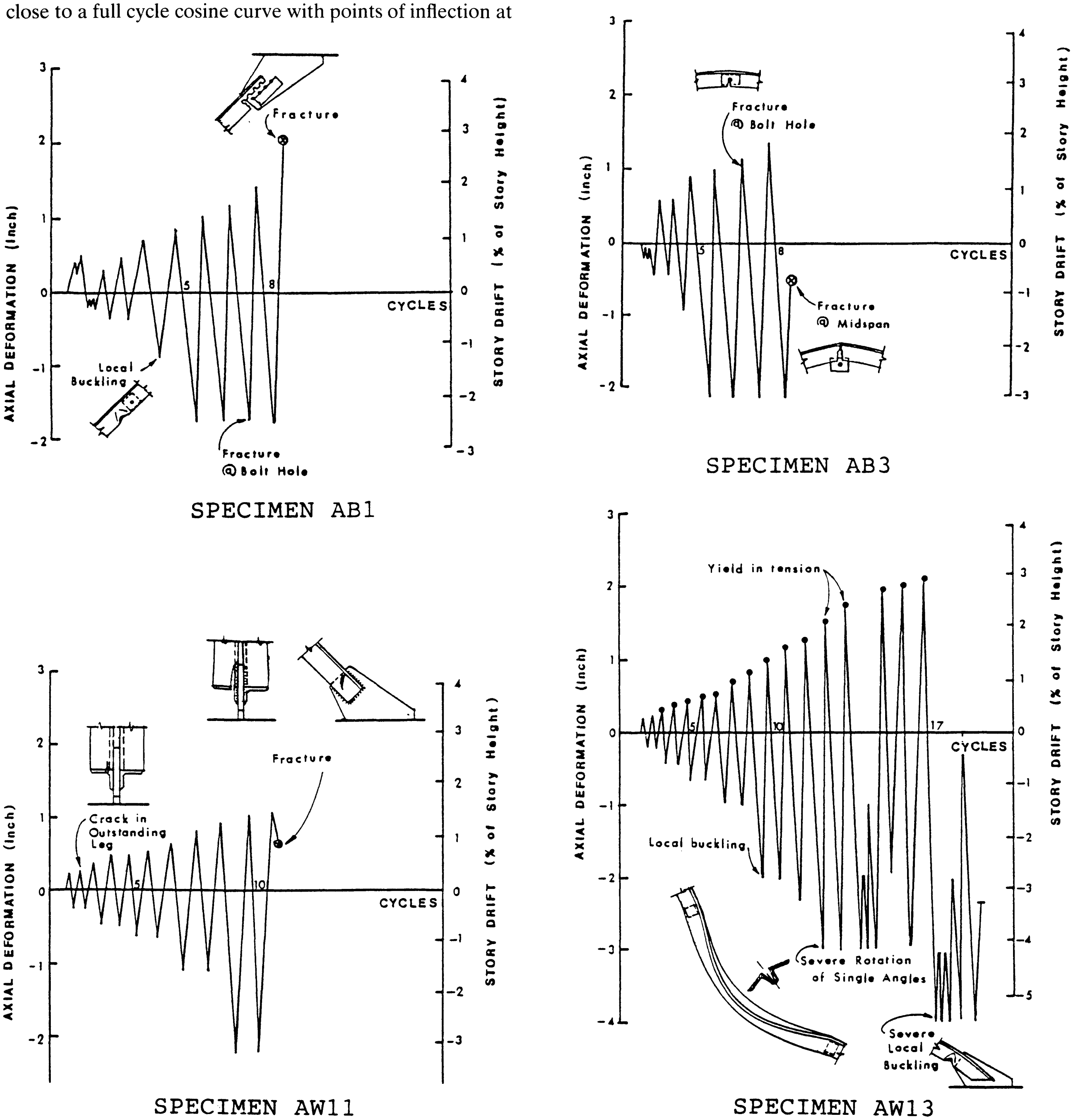

Fig. 2. Behavior of in-plane buckling specimens designed by AISC (1978) Specification

tive length factor is close to 1.0. Use of these effective length factors determines the critical effective slenderness ratio and mode of buckling for the double angle bracing members.

PLASTIC HINGES

Three plastic hinges generally form in a bracing member. In the in-plane buckling members one hinge forms at the midspan and the other two at the ends in the angles just before the connections. In out-of-plane buckling members one hinge forms at the midspan but the other two form in the end gusset plates.

BUCKLING LOAD

The first buckling load for members with welded connections was close to the value given by the AISC formulas

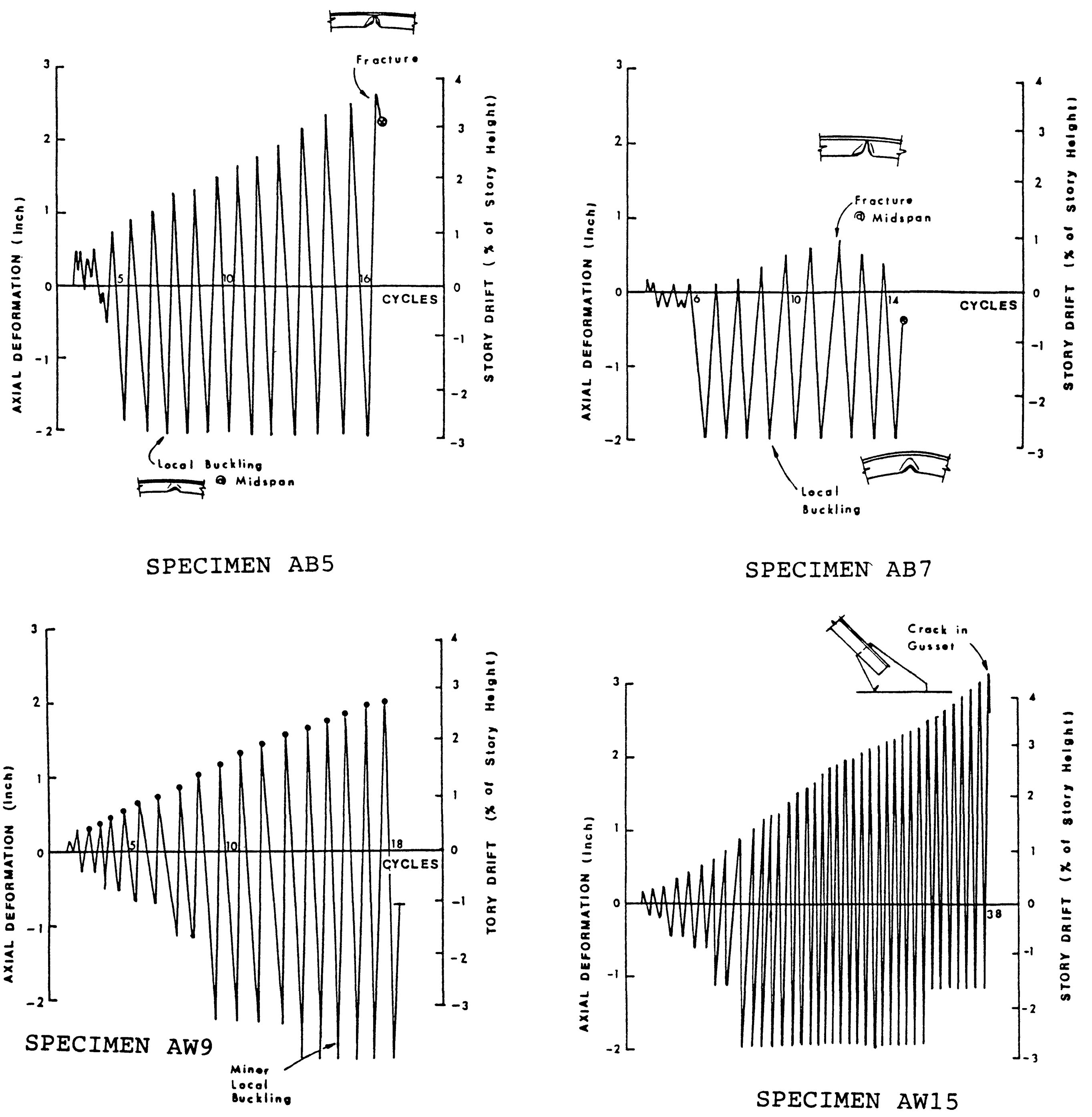

Fig. 3. Behavior of in-plane buckling specimens designed by modified procedure

increased by the factor of safety. Buckling capacity of bracing members decreases during cyclic loading. The decrease is most significant from first to second cycle and continues in subsequent cycles but at reduced rate.

LOCAL BUCKLING

Local buckling occurred in the back-to-back legs of in-plane buckling bracing specimens AB1, AB5, AB7, AW9 and AW13, as shown in Figs. 2 and 3. On the other hand, the outstanding legs experienced local buckling for the out-of-plane buckling specimens AB2 and AW8 (Figs. 4 and 5). The *b*/*t* ratio of these legs exceeded the limits permitted in Part 2 of the AISC Specification,[4] which for A36 steel is 8.5.

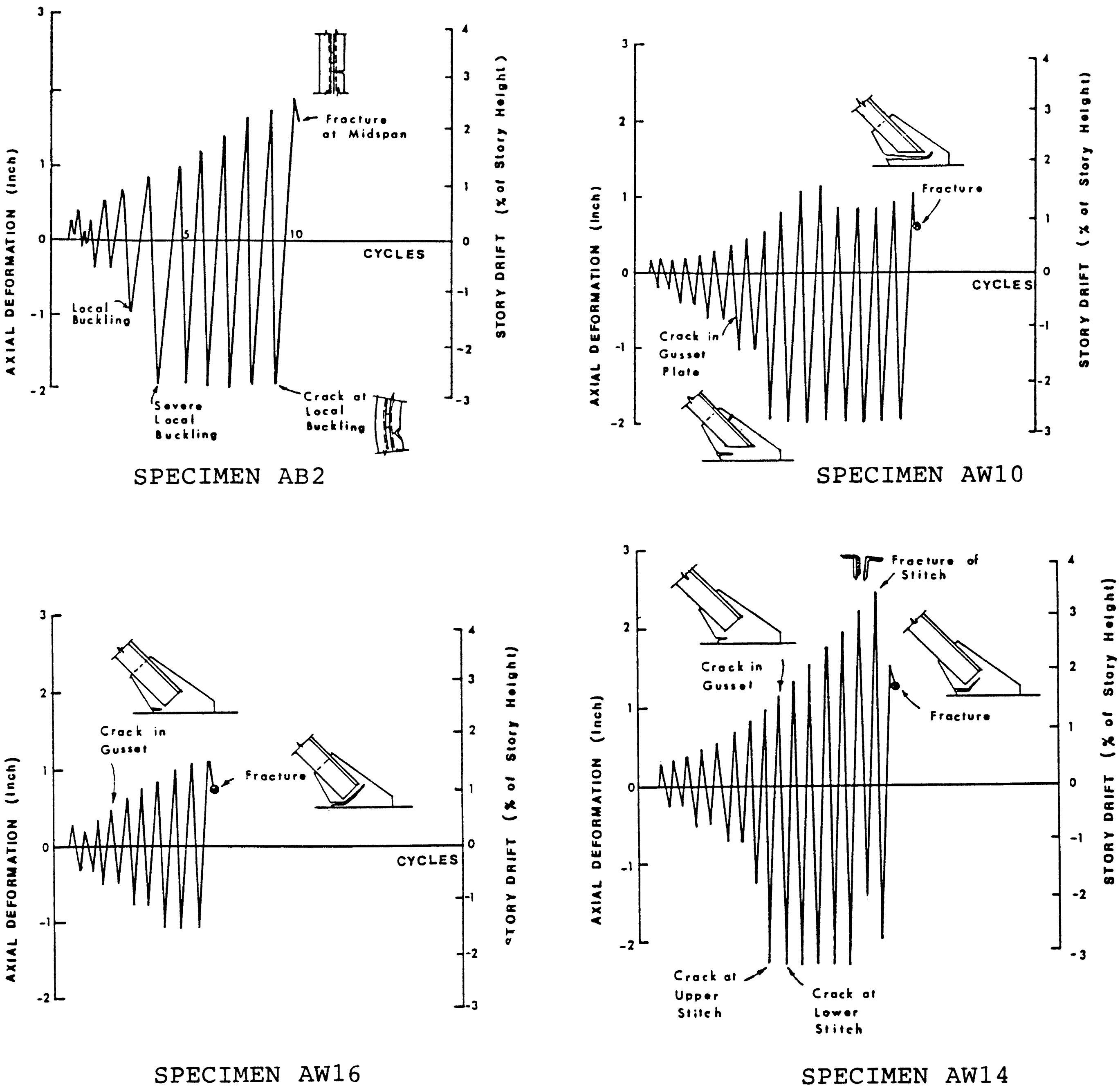

Fig. 4. Behavior of out-of-plane buckling specimens designed by AISC (1978) Specification

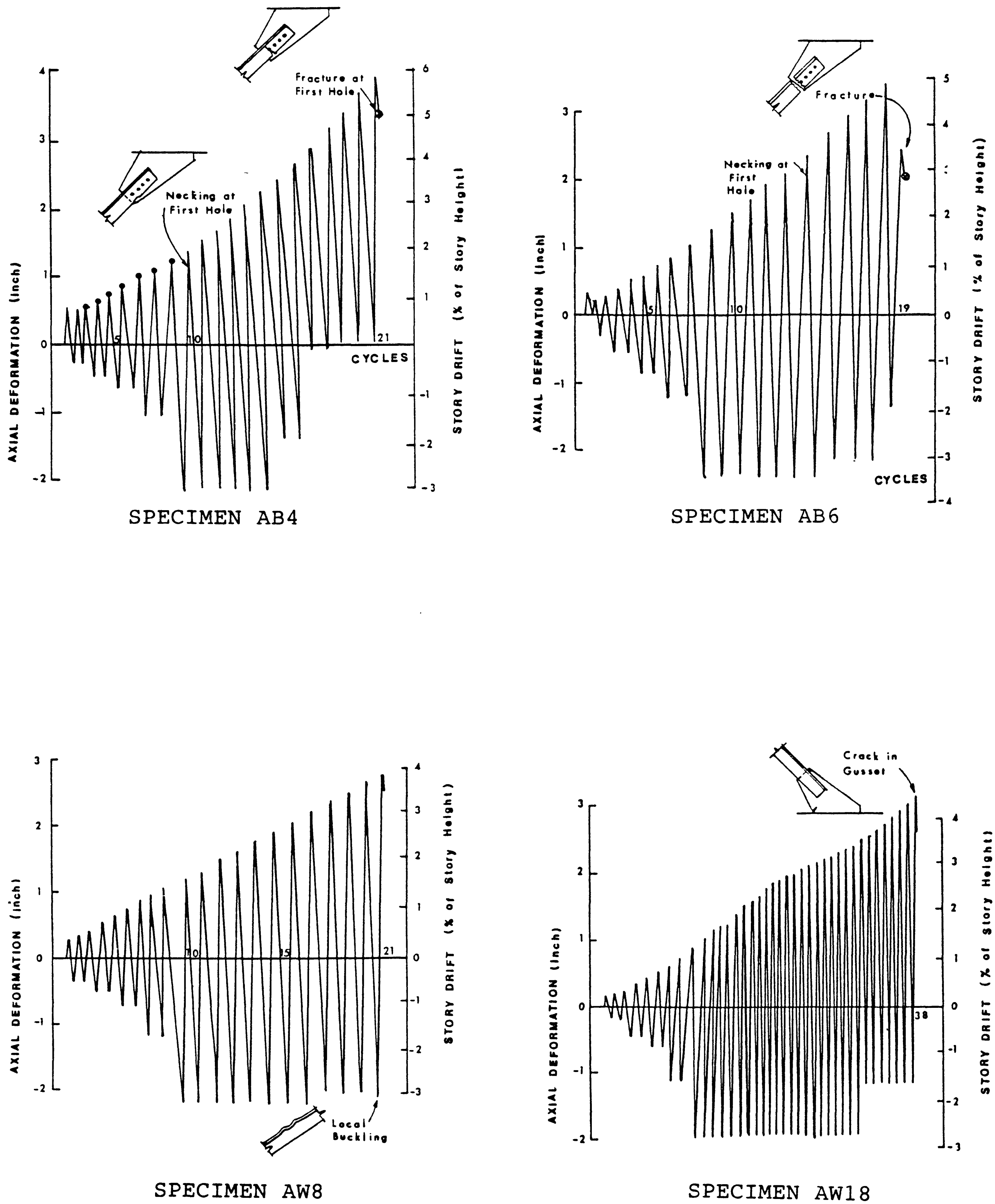

Fig. 5. Behavior of out-of-plane buckling specimens designed by modified procedure

EVALUATION OF CURRENT DESIGN PROCEDURES

Test specimens AB1, AB3, AW11, AW13, AB2, AW10, AW12, AW14 and AW16 were designed according to the current design practice and provisions of AISC Specification.[4] Local failures occurred at end connections and stitches of these specimens during early cycles of loading as shown in Figs. 2 and 4. Based on test observations and analysis of the results, it was concluded that some double angle bracing members designed by current practice may not have sufficient ductility to survive severe cyclic loadings. Modified design procedures were formulated and are presented in the following sections of this paper. Those modified procedures were used in the design of specimens AB5, AB7, AW9, AW15, AB4, AB6, AW8 and AW18. These specimens showed significant improvement in the ductility and performance under cyclic loading particularly with respect to the behavior of connections and stitches as illustrated in Figs. 3 and 5.

STITCHES

The stitch spacing according to AISC Specifications[4] should be adequate to prevent single angle buckling between the stitches before the overall buckling of bracing member. The forces in the stitches of in-plane buckling members are minimal. Therefore, the nominal stitches may be used in in-plane buckling double angle bracing members.

Based on measurements of actual forces in the out-of-plane buckling test specimens the forces transferred by the stitches are large. Nominal stitches are not adequate to withstand severe cyclic deformations. The stitches of out-of-plane buckling bracing members should be designed to transfer a force at least equal to ¼ of the total tension yield capacity P_y of the member. This force should be considered acting along the centroid of one angle (Fig. 6).

Bolted stitches, especially for in-plane buckling bracing members, should be avoided at midspan where the plastic hinge forms. Premature failure of net sections of the type shown in Fig. 2a is very likely at mid span of in-plane buckling bracing members if a stitch hole is located at this point.

DESIGN FORCES FOR END CONNECTIONS

The current design methods for connections of bracing members consider a tension force acting through the centroid of the member. The elements of connection are designed accordingly to resist such force. The tests of in-plane buckling specimens indicated that during cyclic loading a bending moment accompanies the axial force in the post-buckling stage. Based on test results it seems more appropriate to design the end connections for combined effects of bending moment and axial force as shown in Fig. 7a.

Moment-axial force interaction curves (*M-P* curves) of the member cross section can be used to establish design

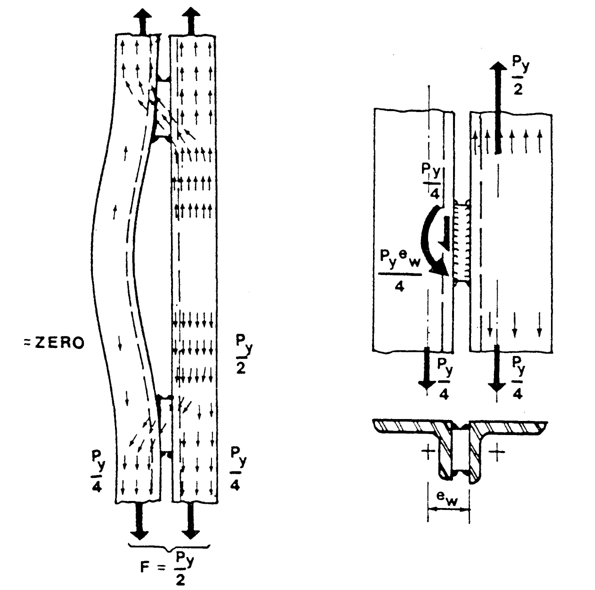

Fig. 6. Design forces for stitches in out-of-plane buckling bracings

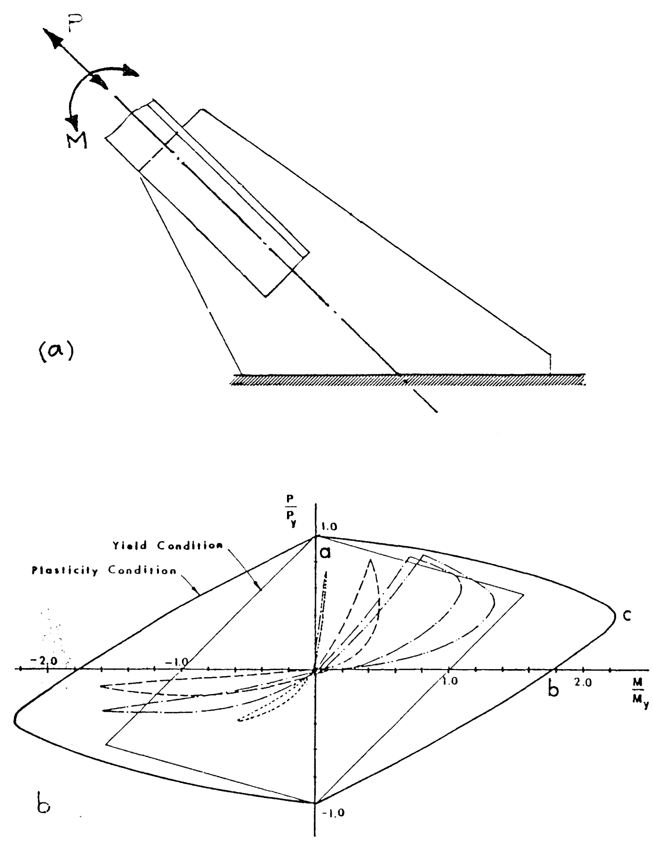

Fig. 7(a). Design forces for end connections of in-plane buckling bracings
(b). M-P curves for specimens AB1

forces for the connections of in-plane buckling member. Assuming elastic-perfectly plastic material, the M-P curve representing total yielding of cross section can be obtained. A typical curve is shown in Fig. 7b. The plasticity condition shown by solid line is the locus of M-P values causing total yielding of the cross section. The dashed lines are actual M-P curves recorded during the test. Points "a" and "b" correspond to pure axial loading and pure bending cases, respectively. It is clear that point "c" in Fig. 7b represents a critical combination of moment and axial force. Moment and axial force corresponding to this point were calculated for all double angle sections listed in the AISC Manual[5] with short legs back-to-back. It was found that for practical design purposes a value of $M = 2.5M_y$ together with $P = 0.5P_y$ can be used as ultimate design forces for the connections. In addition, adequacy of the connection should be checked separately for a tension force equal to P_y.

CONNECTION OF ANGLES TO GUSSET PLATE

Behavior of test specimens AB1 and AW11 indicated weakness of the current design practice in terms of ensuring adequate ductility. Both specimens failed at the angle to gusset plate connection during early cycles of loading (Fig. 2). These failures are related to eccentricity of the centroid of a single angle connection during early cycles of loading (Fig. 2). These failures were related to eccentricity of the centroid of a single angle in the direction normal to the gusset plate which caused out-of-plane bending of the angles. Such out-of-plane bending when combined with direct shear at the end connections can cause premature failure.

The problem of fracture within the bolt spacing caused by cyclic loading is far more complicated and the mechanics of failure cannot be generalized by studying only the fracture of specimen AB1. However, in the absence of more comprehensive studies, a conservative solution to prevent these failures would be to avoid severe yielding in bolt spacing and adjacent areas.

For this purpose simple behavior models are considered (Fig. 8). These models are based on principles of equilibrium of forces and observation of actual behavior of the test specimens.

For welded specimens maximum out-of-plane bending

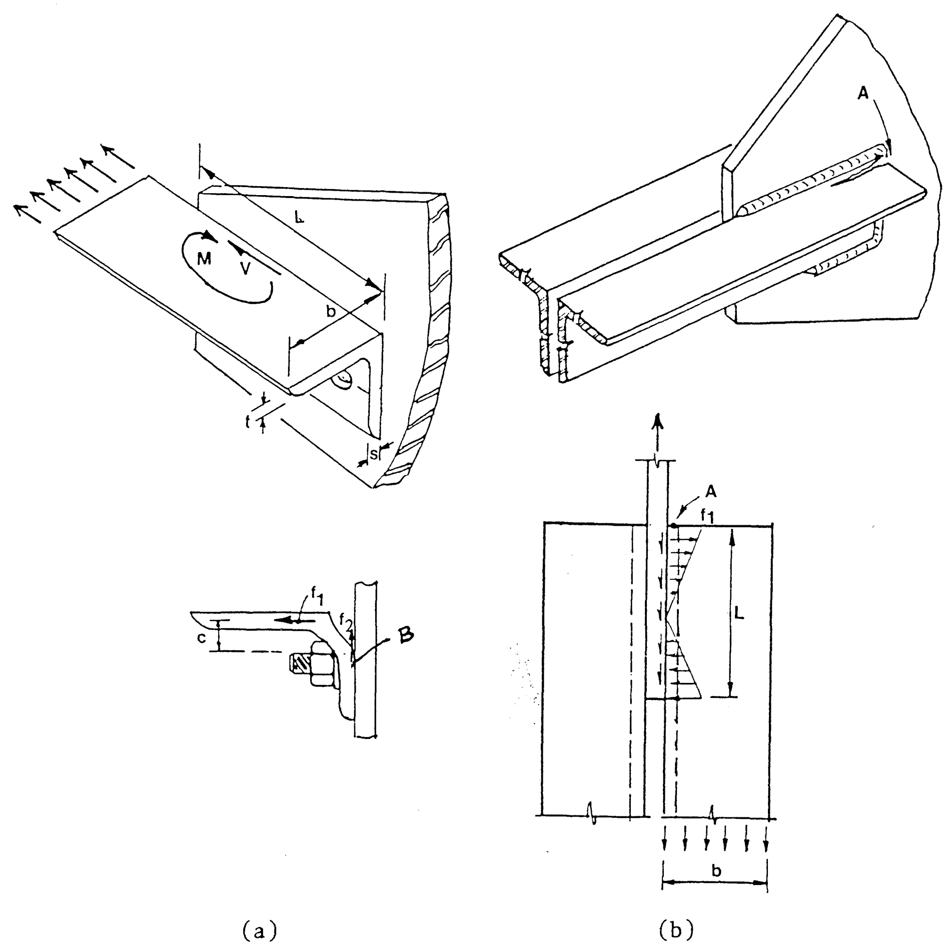

Fig. 8. Behavior of welded and bolted connections

moment is given as,

$$M = F_y \, t \, b \frac{b}{2} = F_y \frac{t \, b^2}{2} \tag{1}$$

and maximum tensile stress f_1 will occur at point A;

$$f_1 = \frac{F_y \, t \, b^2}{2 \dfrac{t \, L^2}{6}} = \frac{3 \, b^2}{L^2} F_y \tag{2}$$

To ensure elastic behavior:

$$f_1 \leq F_y \tag{3}$$

$$\frac{3b^2}{L^2} \leq 1.0 \tag{4}$$

which reduces to

$$\frac{b\sqrt{3}}{L} \leq 1.0 \tag{5}$$

Satisfying the above approximate condition (it neglects the effect of shear stress) in test specimens resulted in elastic behavior of the outstanding legs. In specimen AW11 the ratio $b\sqrt{3}/L$ was equal to 1.44 and this specimen fractured during second cycle of loading through point A as shown in Fig. 8a.

For bolted specimens the behavior model is slightly different since the outstanding leg is not directly connected to the support. Instead, the back-to-back leg is bolted to the gusset plate. As a result, point B, in Fig. 8b is the critical point of maximum tensile stress.

Maximum stress at point B may be approximately calculated as follows:

$$f_1 = \frac{3b^2}{L^2} F_y \tag{6}$$

and

$$f_2 = f_1 \frac{t \, c}{s^2/6} = \frac{18 \, b^2 c \, t}{L^2 \, s^2} F_y \tag{7}$$

Limiting f_2 to yield stress results in

$$\frac{b\sqrt{18 \, c \, t}}{sL} \leq 1.0 \tag{8}$$

For test specimen AB1 the ratio at left side of above equation was equal to 1.5 and specimen failed in early cycles of loading. In other specimens this value was less than 0.94 and all behaved satisfactorily.

Satisfying Eqs. 5 and 8 in welded and bolted connections respectively, should prevent premature failure of angle to gusset plate connections.

Another failure mode that can occur in the angles to gusset plate connection is fracture at the net area of the angles. Formation of a plastic hinge at the net section of the first bolt in the connection causes rapid deterioration and premature fracture of net area, particularly for in-plane buckling members. To prevent such failures, the net section should be reinforced to move the plastic hinge into the gross section of double angles. An effective way to provide this reinforcing is to weld plates to the back-to-back angle legs (Fig. 9). The reinforcing plate in Fig. 9 can be extended to cover other bolt holes and will prevent failure within bolt spacing as discussed earlier. To calculate the area required for the reinforcing plates the yield conditions of double angle sections can be used as explained in Ref. 1. However, for practical design purposes the dimensions of reinforcing plates can be selected such that the tension yield capacity as well as the yield moment capacity of the reinforced net section is larger than the corresponding values for the double angle gross section.

GUSSET PLATE

In current design practice the stresses in gusset plates are checked at critical sections by applying simple beam theory. In applying this procedure the effective area of a gusset plate with bolted connections is calculated using Whitmore's method.[6] Following this method, the effective area is found by multiplying the effective width by thickness of the gusset. The effective width is obtained by drawing 30% lines from the outer fastener in the first row to their intersection with a line passing through the last row of the fasteners and perpendicular to the line of action of the force (Fig. 10a). Whitmore derived this effective area concept for bolted gussets. In this study, however, a similar definition is adopted for welded gussets as shown in Fig. 10b.

The behavior of gusset plates designed by using this procedure was found to be satisfactory for in-plane buckling specimens. The gusset plates of these members during the tests remained generally elastic. However, the gusset plates of out-of-plane buckling bracing members generally showed poor ductility and early fractures. The failure in these gusset plates was caused by undesirable constraint which prevented plastic hinge free rotation at the ends

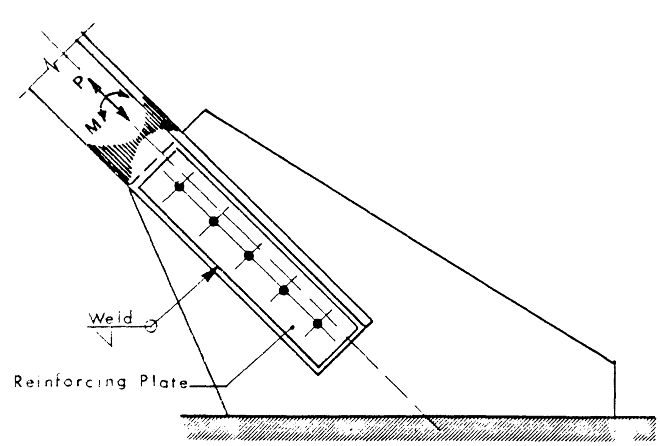

Fig. 9. Reinforcement of net section

during post buckling stage. Gusset plate failures are shown in Figs. 4b, 4c and 4d. Further study of the behavior of gusset plates[1] indicated an adequate free length of gusset plate between the end of the angles and inner corner of gusset plate is necessary to ensure free formation of plastic hinge for improved ductility. A minimum free length equal to twice the thickness of the gusset plating proved to be adequate (Fig. 11).

SUMMARY OF RECOMMENDED DESIGN PROCEDURE

1. Obtain ultimate (factored) axial load T applied to the bracing from analysis of braced frame.
2. Calculate required area of double angles from following equation and select double angles.

$$A_{req.} = T/F_y$$

3. Calculate effective slenderness ratios and determine direction of buckling. Use effective length factor of 0.5 and 1.0 for in-plane and out-of-plane buckling, respectively.
4. For in-plane buckling double angles, limit b/t ratio of back-to-back legs to the values given in Part 2 of AISC Specification. For out-of-plane buckling double angles, b/t ratio of outstanding leg should be limited to those of Part 2 of AISC Specification. In both cases the b/t ratio of the other leg should be less than $76/\sqrt{F_y}$ given in Sect. 1.9.1.2 of the Specification.
5. Calculate spacing of stitches such that single angle buckling between the stitches is prevented. Avoid placing bolted stitch at mid-length of the bracing member. For in-plane buckling bracing members use nominal stitches. For out-of-plane buckling bracing members design the stitches to transfer a force equal to $AF_y/4$ from one angle to the other where A is total area of member. This force should be considered acting at the centroid of one angle and parallel to the longitudinal axis of the member.
6. Design end connections of the bracing member for an axial force equal to AF_y. For in-plane buckling specimens the end connections should also be capable of transferring an axial force of $\frac{1}{2} A\, F_y$ in combination with an in-plane bending moment of $2.5\, M_y$.
7. To avoid tearing failure in connections of angles to gussets, satisfy the following requirements:

   a) for welded connections: $\dfrac{b\sqrt{3}}{L} \leq 1.0$

   b) for bolted connections: $\dfrac{b\sqrt{18\, t\, c}}{sL} \leq 1.0$

8. For out-of-plane buckling bracing members, provide sufficient free length of gusset plate for plastic hinge formation. A free length equal to twice the thickness of the gusset plate is recommended.

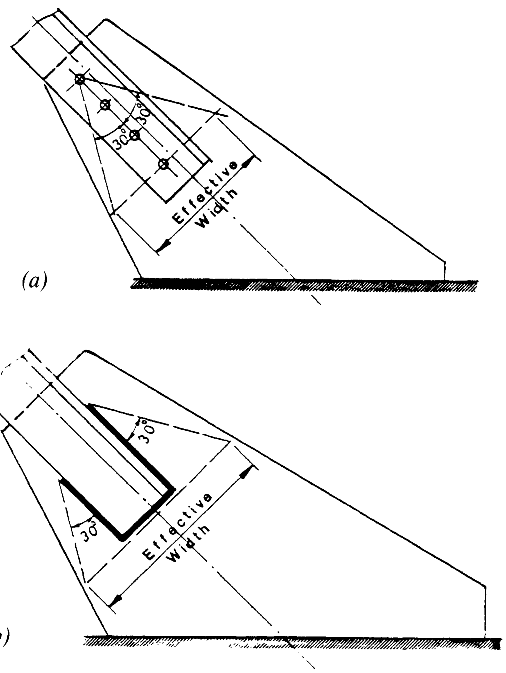

Fig. 10. Effective width of gusset plate in bolted and welded connections

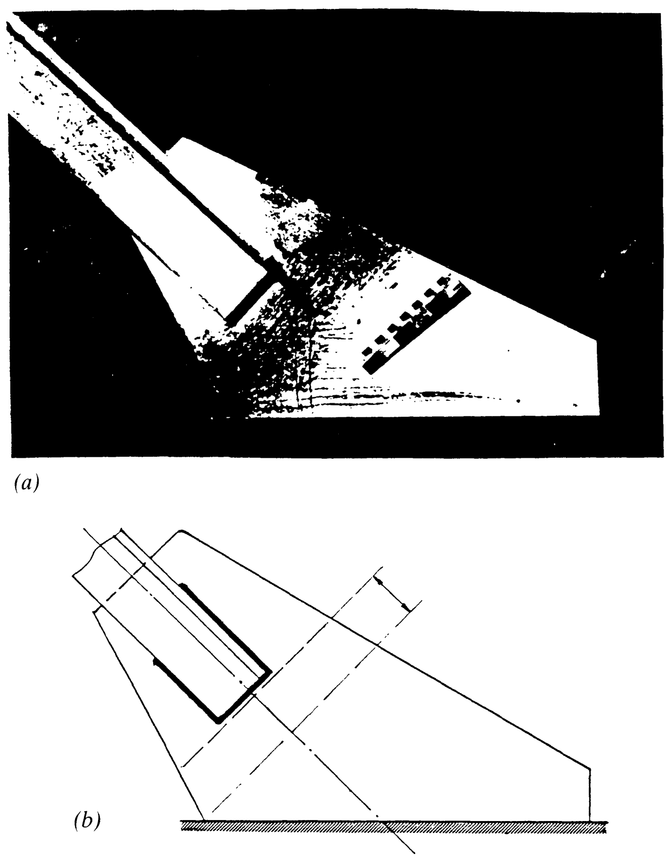

Fig. 11. Plastic hinge and free length of gusset plate

DESIGN EXAMPLES

The following examples illustrate application of the recommended design procedure for double angle bracing members in earthquake resistant structures.

Example 1: Bolted bracing member (in-plane buckling)

Given:

Design force in bracing member from analysis: 137 kips
Length of bracing member: 142 in.
Angle between diagonal bracing, horizontal beam: 45°
Steel: A36 (F_y = 36 ksi; F_u = 58 ksi)
Fasteners: ⅞-in. dia. A325 bolts (bearing-type)

Design the bolted connections and stitches of the bracing member to withstand severe cyclic loading.

Solution:

1. Ultimate load = 137 kips × 1.3 = 178 kips
   Load factor = 1.3 (as per Sect. 2.1 of AISC Specification[4])
2. Select double angles:

   $A_{req.}$ = 178/36 = 4.95 in.^{2};
   try 2 L − 4 × 3 × ⅜ (short legs back to back)

3. *Determine direction of buckling:*

   Assume back-to-back of angles to be ½ in. apart.

   $K_x L/r_x$ = 0.5(142 in.)/(0.879 in.) = 81 **(governs)**

   $K_y L/r_y$ = 1.0(142 in.)/(1.990 in.) = 71

   Buckling will occur in the plane of gusset plate.

4. *Check for local buckling:*

   Since buckling is in the plane of gusset plate, b/t of back-to-back legs is critical.

   b/t of back-to-back legs:
   (3.0 in.)/(0.375 in.) = 8 < 8.5 **o.k.** (AISC Spec. Part 2)

5. *Design stitches:*

   Place the stitches so that single-angle buckling between the stitches is prevented; (AISC Spec. Sect. 1.18.2.4)

   $(KL/r)_{member} = 81$

   $(\ell/r_z)_{angle} \leqq (KL/r)_{member}$

   $\ell_{single\ angle} \leqq (r_z)(KL/r)_{member}$

   $\ell_{single\ angle} \leqq (0.644\text{ in.})(81) = 52.0\text{ in.}$

   Use two stitches at 1/3 points resulting in $\ell_{single\ angle}$ equal to 142/3.

Since bracing member buckles in the plane of gusset plate, nominal stitches are sufficient.

To meet the edge distance requirement of AISC Specification for ⅞-in. bolt, a stitch plate 3 × 3 × ½ is used.

6. *Determine design forces for end connections:*

   Two loading conditions must be considered:

   a) Direct tension with no bonding moment.

   $F_1 = P_y = AF_y = (4.97\text{ in.}^2)(36\text{ ksi}) = 178.9\text{ kips}$

   $M_1 = 0$

   b) Direct tension combined with bending moment.

   $F_2 = 0.5\ P_y = 178.9/2 = 89.5\text{ kips}$

   $M_2 = 2.5\ M_y = (2.5)(S_x)(F_y) = 2.50(1.73\text{ in.}^3)(36) = 155.7\text{ kip-in.}$

7. *Design connections of angles:*

   Due to limited width of back-to-back legs, only one line of bolts is used to connect angles to the gussets.

   First let us consider loading case (a).

The shear force acting on the bolts of each angle:

T_a = (178.9 kips)/2 = 89.5 kips

Shear capacity of a ⅞-in. dia. bolt:

S = (0.601)(21)(1.7) = 21.46 kips/bolt

Number of bolts required:

$n = T_a/S$ = (89.5 kips)/(21.46 kips/bolt) = 4.2 bolts

Use 5 ⅞-in. dia. bolts

Following Sect. 1.16 of the AISC Specification,[4] the edge distance and bolt spacing of Fig. 12 are adequate.

Check the connection for combined effect of axial load and bending moment:

Forces acting on one angle are:

$T_2 = F_2/2 = 89.5/2 = 44.7$ kips

$M_2 = M_2/2 = 155.7/2 = 77.9$ kip-in.

By applying ultimate strength method outlined in Ch. 4 of the AISC Manual[5] and using corresponding tables:

$n = 5$

$b = 3$ in.

$\ell = 77.9/44.7$
$= 1.74$ in. < 3.0 in. (conservatively use 3.0 in.)

$T_{ult.}$ = (3.90) (21.48 kips/bolt)
= 83.8 kips > 44.7 kips **o.k.**

Check connection against tearing failure:

The proposed design procedure is applied herein;

$$b\sqrt{\frac{18ct}{sL}} \leq 1.0$$

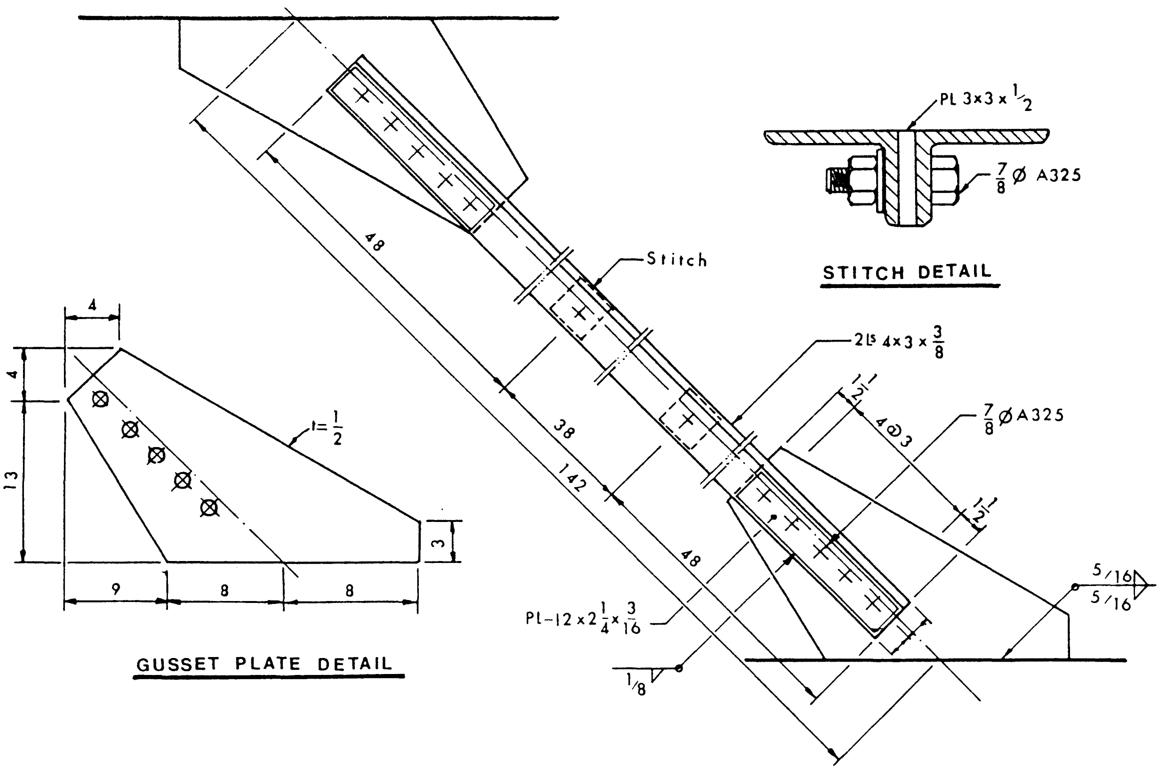

Fig. 12. Bolted double angle bracing member of Example 1

where

$L = 4(3.0) + 2(1.5) = 15.0$ in.

$t = s = 3/8$ in.

$c = 1.5 - 1/2(3/8) - 1/2(7/8) + (1/16) = 0.84$ in.

$b = 4$ in.

$$\frac{b\sqrt{18ct}}{sL} = \frac{4\sqrt{18 \times 0.84 \times 0.375}}{0.375 \times 15} = 1.69 > 1.0 \quad \textbf{n.g.}$$

The above calculations indicates that the connection is not sufficient and needs to be reinforced. The suggestion made earlier in this paper was to reinforce the connection by welding plates to the back-to-back legs. This solution, in fact, amounts to increasing the value of s in above expression.

According to the proposed method, the yield capacities of the reinforced net section of angles in axial tension and bending should be greater than those of the gross section,

$P_y = 178.9$ kips

$M_y = 62.2$ kip-in.

Considering the net section of angle at section *a-a* in Fig. 13,

$A_n F_y > P_y$

Therefore,

$A_n > 4.97$ in.^{2}

Try a ¼-in. thick reinforcing plate on each back-to-back leg:

$A = 4.97 - 2(0.625) + 2(1/4)(2.5) = 4.97 = 4.97$ in.^{2} **o.k.**

The yield moment of reinforced section is:

$M_y = 96.6$ kip-in > 62.2 kip-in. **o.k.**

Now check Eq. 8 or the reinforced section:

$L = 15.0$

$t = 0.375$

$s = 0.625$

$c = 0.84$ in.

$b = 4$ in.

$$\frac{b\sqrt{18ct}}{sL} = \frac{4\sqrt{18 \times 0.84 \times 0.375}}{0.625 \times 15} 1.02 \simeq 1.0 \quad \text{Say } \textbf{o.k.}$$

8. *Design gusset plates:*

The force acting on the gusset plate,

$T = 178.9$ kips

The maximum effective width W_{eff} of the gusset plate along the section perpendicular to the axis of member (using Whitmore's method) is obtained as follows:

$W_{eff} = [2(4 \times 2.33)(\tan 30°)] = 10.8$ in.

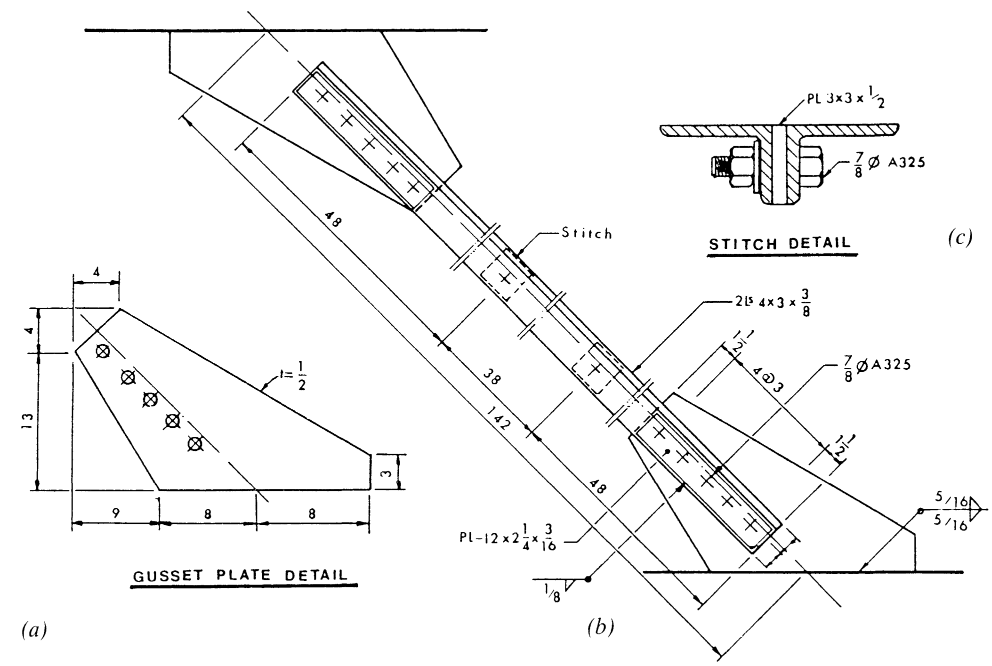

Fig. 13. Details of connections in Example 1

Obtain the area of the gusset plate required to resist the tension force T

$A_{req} = T/F_y = (178.9)/(36) = 4.97 \text{ in.}^2$

$t_{g(\min)} = A_{req}/W_{eff} = (4.97)/(10.8) = 0.46$ in.

A thickness of ½ in. is used for the gusset plates.

Therefore, the required width of the gusset is,

$W_g = A_{req}/t_g = (4.97)/(0.5) = 9.94$ in.

A width of 10 inches ($< W_{eff} = 10.8$ in. **o.k.**) is provided symmetrically about the longitudinal axis of the bracing, in order to eliminate eccentricity in the plane of the gusset plate.

9. *Check ductility of gusset plate:*

Since buckling occurs in the plane of gusset plate, the gusset plates are expected to remain generally elastic. The geometry of the gusset plate is shown in Fig. 14.

Check stresses along the horizontal section at the base of the gusset plate:

By using Von Mises' yield criteria and assuming uniform distribution of axial and shear stresses, the shear stress:

$f_v = T \cos \alpha / L_s t_g$

$f_v = (178.9)(\cos 45°)/(14)(0.5) = 18.1$ ksi

and the axial stress:

$f_a = T \sin \alpha / L_s t_g$

$f_a = (178.9)(\sin 45°)/(14)(0.5) = 18.07$ ksi

The Von Mises' criteria may be expressed as:

$(f_1 - f_2)^2 + (f_2 - f_3)^2 + (f_3 - f_1)^2 < F_y^2$

where f_1, f_2 and f_3 are principal stresses. Applying the criteria to the elements along the base of gusset plate and rearranging the equation,

$(3f_v^2 + f_a^2)^{1/2} < F_y$

$(3 \times 15.8^2 + 18.07^2)^{1/2} = 31.6 \text{ ksi} < 36 \text{ ksi}$ **o.k.**

Example 2: Welded bracing member (in-plane buckling)

Given:

Same as Ex. 1 but connections are welded using E70 electrodes.

Solution:

Following steps 1 through 5, as in Ex. 1:

1. Ultimate load = 178 kips
2. Try 2Ls − 4 × 3 × 3/8 (short legs back-to-back)
3. Buckling will occur in plane of gusset plate
4. b/t of back-to-back leg = 3/0.375 = 8 < 8.5 **o.k.**
5. Two stitches are required. Since, buckling occurs in plane of gusset plate nominal stitches are sufficient.

A minimum size of stitch which is practical for fabrication is selected.

Use 2 × 2 × 1/2

According to AISC Specification a minimum weld size of 3/16 is required to connect stitches to angles.

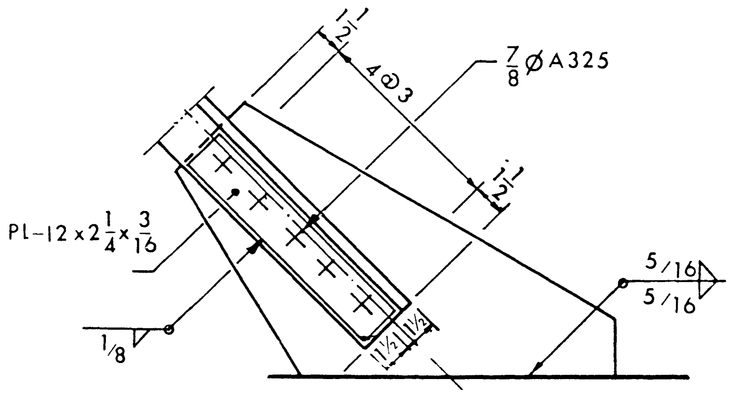

Fig. 14. Reinforced connection

6. Referring to step 6 of Ex. 1, design forces for end connections are:

   a) $F_1 = 178.9$ kips

   $M_1 = 0$

   and

   b) $F_2 = 89.5$ kips

   $M_2 = 155.7$ kips

7. *Design Connections of Angles*

   Balanced fillet welds are used to connect angles to gussets.

   The shear force acting on the welds of each angle,

   $T_a = (178.9)/2 = 89.5$ kips

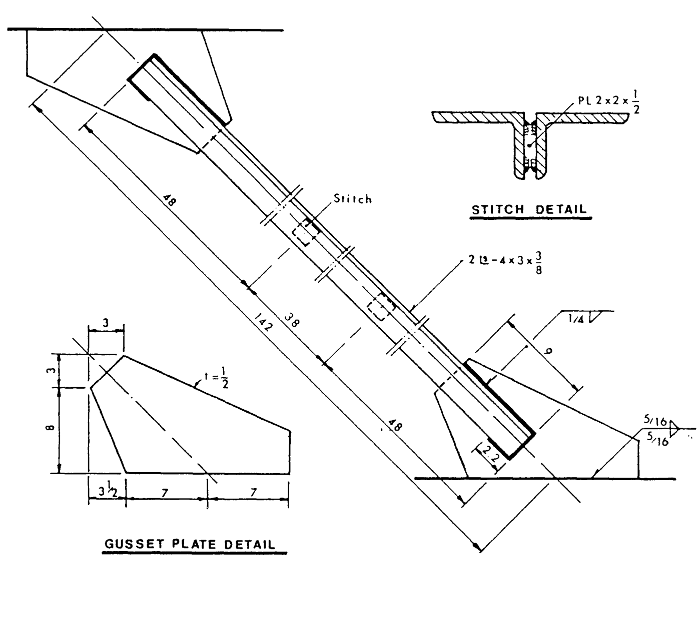

Fig. 15. Welded double angle bracing member of Example 2

Using 1/4-in. weld size and following procedures to balance welds,[7] weld lines shown in Fig. 15 are obtained.

Shear capacity of 1/4-in. weld = $(1/4)(\sqrt{2}/2)(0.3 \times 70)(1.7) = 6.3$ kips/in.

Total length of weld required = $89.5/6.3 = 14.2$ in.

Length of weld on toe of back-to-back leg = $0.782(14.2/3) - (3/2) = 2.2$ in. length of weld on heel of back-to-back leg = $[(3 - 0.782)(14.2)/3] - (3/2) = 9$ in.

The connection checks out **o.k.** for combined effects of axial force and bending moment.

Check connection against fracture in outstanding leg: The proposed design criteria is applied, i.e.,

$$\frac{b\sqrt{3}}{L} \leq 1.0$$

where

$L = 9$ in.

$b = 4$ in.

$$\frac{b\sqrt{3}}{L} = \frac{4\sqrt{3}}{9} = 0.77 < 1.0 \quad \textbf{o.k.}$$

It may be noted that if unbalanced welds were used L would be 5.5 in. and $b\sqrt{3}/L = 1.26$ would indicate unsatisfactory behavior.

8. *Design gusset plates*

   Design of gusset plates is similar to Step 8 in Ex. 1. Dimensions are shown in Fig. 15.

9. *Check ductility of gusset plate*

   Similar to Step 9 of Ex. 1. Gusset plates shown in Fig. 15 will be adequate.

Ex. 3: Welded bracing member (out-of-plane buckling)

Given:

Design force in bracing member from analysis: 137 kips

Length of bracing member: 142 in.

Angle between diagonal bracing and the horizontal beam: 45°

Steel: A36

Design welded connections and stitches of the bracing members to withstand severe cyclic loading.

Solution:

1. Ultimate load = $137 \times 1.3 = 178$ kips
2. Select double angles:

$$A_{req} = \frac{178}{36} = 4.95 \text{ in.}^2\text{; try}$$

2L − 4 × 3 × 3/8 (long legs back to back)

3. *Determine direction of buckling*

Assume back to back angles to be ½ in. apart.

$K_x L/r_x = 0.5\ (142)/(1.26) = 56$

$K_y L/r_y = 1.0\ (142)/(1.35) = 105$ **(governs)**

Buckling will occur out of plane of gusset plate.

4. *Check for local buckling*

Since buckling is out of the plane of the gusset plate, b/t of outstanding legs is critical.

b/t of outstanding leg $= (3.0)/(0.375) = 8 < 8.5$ **o.k.**

5. *Design stitches*

Place the stitches such that single angle buckling between the stitches is prevented.

$(KL/r)_{\text{members}} = 105$

$(\ell/r)_{\text{single angle}} \leq (KL/r)_{\text{member}}$

$\ell_{\text{single angle}} = 105\ (0.644) = 68$ in.

Use two stitches at $\ell/3$ points.

Since the member buckles out-of-plane, as discussed in this paper earlier, the stitches are expected to be subjected to quite large forces and should be designed to resist these forces. Maximum force expected to be transferred by the stitches from one angle to the other is:

$F_{st} = P_y/4 = (4.97)(36)/4 = 44.7$ kips

Eccentricity of the force $= 0.50 + 0.782 = 1.28$ in.
Therefore, bending moment acting on stitch welds:

$M_{st} = 44.7\ (1.28) = 57.2$ kip-in.

$$f_b = \frac{M_{st}\,C}{I} = \frac{57.2/2}{(L_{st})^2/6} = \frac{171.6}{(L_{st})^2}$$

$$f_v = \frac{F_{st}}{L_{st}} = \frac{44.7}{L_{st}}$$

$$f = \sqrt{f_b^2 + f_v^2} = \sqrt{29446/L_{st}^4 + 1998/L_{st}^2}$$

This stress should be less than or equal to the weld capacity. Using weld size of ⁵⁄₁₆ in.

$f \leq 0.707\ (5/16)(0.3)(70) \times 1.7$

or

$$\sqrt{29446/L_{st}^4 + 1998/L_{st}^2} = 7.9$$

Solving the above equation by iteration

$L_{st} = 6.5$ in.

Use PL 6½ × 3 × ½ stitches

6. *Determine design forces for end connections*

a) Direct tension

$F_1 = P_y = (4.97)(36) = 178.9$ kips

$M_1 = 0$

b) Tension and bending

Since, buckling is out-of-plane bending moment at the ends is negligible.

7. *Design connections of angles*

Balanced welds will be used to connect angles to gusset plates.
The shear force acting on each angle,

$T_a = 89.5$ kips

Using ¼ weld size;

Shear capacity $= 1/4\ (\sqrt{2}/2)\ (0.3 \times 70)(1.7)$
$= 6.3$ kip/in.

Total length of weld $= 89.5/6.3 = 14.2$ in.

Length of weld on toe of back-to-back leg
$= 1.28\ (14.2/4) - (4/2) = 2.5$ in.

Length of weld on heel of back-to-back leg
$= [(4 - 1.28)(14.2)/4] - (4/2) = 7.7$ in.

8. *Design gusset plates*

Using procedures similar to Ex. 1, the required cross section area of gusset plate is 4.97 in.² Detail of gusset plate is shown in Fig. 16. The thickness of gusset is equal to ½ in.

9. *Check ductility of gusset plate*

Since buckling occurs out-of-plane of the gusset plate a plastic hinge will form in the gusset. Therefore, it is necessary to provide sufficient free length of gusset plate for plastic hinge formation.

Free length of gusset $= L_{fg} = 2\,t_g = 2 \times 1/2 = 1$ in.

The free length is shown in Fig. 16.

Check connection against fracture in outstanding leg, using the proposed criteria:

$$\frac{b\sqrt{3}}{L} \leq 1.0$$

where

$L = 7.7$

$b = 3$

$$\frac{b\sqrt{3}}{L} = \frac{3\sqrt{3}}{7.7} = 0.74 < 1.0 \quad \textbf{o.k.}$$

ACKNOWLEDGMENTS

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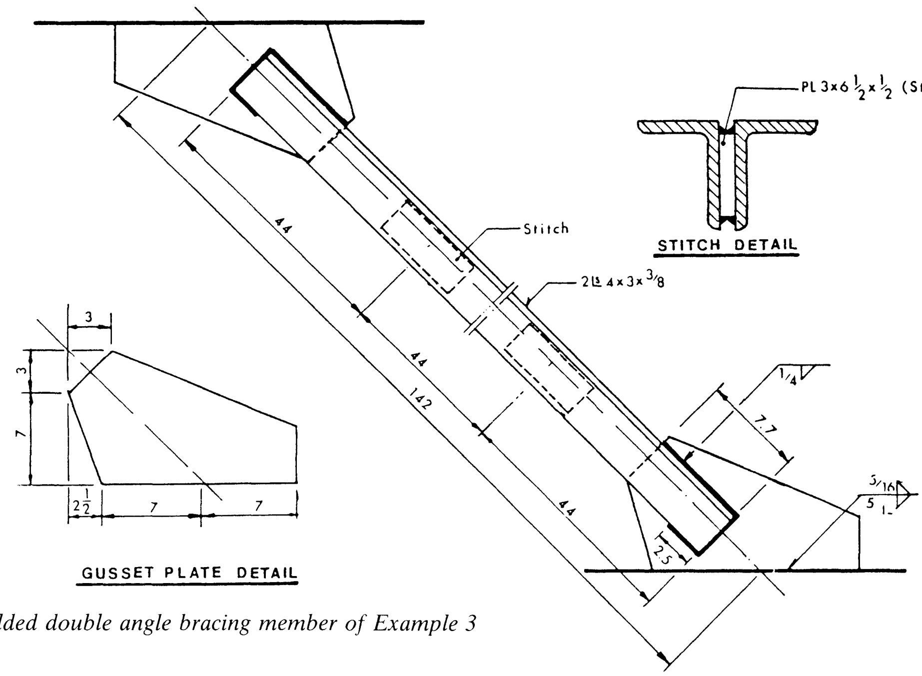

Fig. 16. Welded double angle bracing member of Example 3

NOTATION

A Cross-sectional area

A_{eff} Effective area in gusset plate

A_n Net area of an axially loaded tension member

A_{req} Required cross-sectional area

b Actual width of stiffened and unstiffened compression elements—dimension normal to the direction of stress

b Overall width of one leg in unequal leg angles

b/t Width-thickness ratio of a leg of angle

c Distance from centroid to extreme fiber

F External axial force

F_{st} Force on stitch

f Tensile or compressive stress on an element

f_b Bending stress

f_v Shear stress

K Effective length factor for a prismatic member

L Length of bracing member

L Total length of welds

L_{fg} Free length of gusset in which plastic hinge forms

L_{st} Length of stitch

ℓ Length

M Bending moment

M_{st} Moment on stitch

P_y Plastic axial load equal to profile area times specified minimum yield stress

r Radius of gyration

S Ultimate shear strength per unit length of welds

s Thickness of back-to-back leg

T Tensile force in bracing member

t_g Thickness of gusset plate

W_{eff} Effective width of gusset

REFERENCES

1. *Astaneh-Asl, A., S. C. Goel and R. D. Hanson* Cyclic Behavior of Double Angle Bracing Members with End Gusset Plates *Department of Civil Engineering, Report No. UMEE 82R7, University of Michigan, August 1982, Ann Arbor, Mich.*
2. *Astaneh-Asl, A. and S. C. Goel* Cyclic In-plane Buckling of Double Angle Bracing *ASCE Journal of the Structural Division, Vol. 110, No. 9, September 1984 (pp. 2036–2055).*
3. *Astaneh-Asl, A., S. C. Goel and R. D. Hanson* Cyclic Out-of-plane Buckling of Double Angle Bracing *ASCE Journal of the Structural Division, Vol. 111, No. 5, May 1985 (pp. 1135–1153).*
4. *American Institute of Steel Construction, Inc.* Specification for the Design, Fabrication and Erection of Structural Steel for Buildings *1978, Chicago, Ill.*
5. *American Institute of Steel Construction, Inc.* Manual of Steel Construction *8th Ed., 1980, Chicago, Ill.*
6. *Salmon, C. G. and J. E. Johnson* Steel Structures *2nd Ed., 1980, Harper and Row, New York, N.Y.*