

Least Cost Computer-aided Design of Steel Girders

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Hybrid steel beams, used for over a quarter of a century, have evolved through many theories of design criteria. In this paper, emphasis is placed on stiffened hybrid girders, although unstiffened and homogeneous girders are also included as special cases. AISC has developed specifications to which a steel member must perform. The formulas in the specifications, however, do not define explicitly the best girder to do a particular structural task. Instead, a "region" is defined which contains a large number of acceptable girders from which the designer must choose. Even when a computer is used for the design, unless special programming is implemented, the output is an acceptable non-optimized region from which the final choice is made.

By equipping the design procedure with the ability to rank the acceptable choices by cost or weight, an optimized girder can be located within the region. This ranking process should include the cost and availability factors for several steel grades so that a hybrid girder, having the lowest cost, can be found. Likewise, by including labor costs for manufacturing steel girders, the feasibility of using web stiffeners can also be determined.

The innovative design method which follows incorporates the AISC Specifications⁵ and extends the techniques developed by Ken P. Chong for unstiffened built-up steel sections.² The goal of the design process is to define the geometry of the girder (Fig. 1) by optimizing a particular function, which, in this case, is the depth of the girder.

THEORY AND APPLICATIONS

The following assumptions made for this study are based upon criteria given in the *AISC Manual of Steel Construction*, 8th Ed.⁵

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1. Stiffened girders utilize vertical web stiffeners to carry the shear forces to assure web shear stresses remain below allowable limits. The aspect ratios a/h , are less than 3.0 or are less than $[260/(h/t)]^2$, where a is the spacing between stiffeners h is the web depth and t is the web thickness.
2. Web slenderness ratio h/t , should be small for unstiffened beams. However, for efficient stiffened hybrid beams, the web slenderness ratio should be large. Hence, the coefficient $C_v \leq 0.8$.
3. Web slenderness ratio is usually less than $760/\sqrt{F_{bf}}$, and no reduction in flange stress is required. (F_{bf} is the allowable bending stress).
4. Tension field effect and shear/tension interaction are neglected for both homogeneous and hybrid beams, with and without web stiffeners.
5. Axial forces must be less than $0.15 F_{yf}$ times the area of the gross section. (F_{yf} is the yield stress of flanges).
6. Flanges must be of equal area and of the same grade of steel.

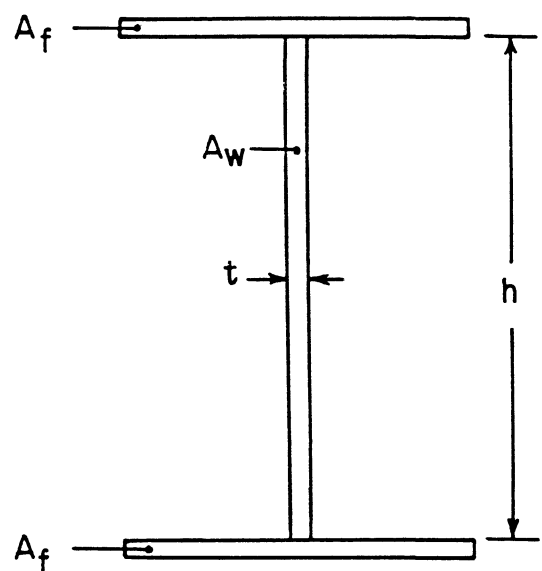


Fig. 1. Geometry of girder

7. The flange thickness is very small compared with the clear distance h between the flanges.
8. The yield strength of the flange steel is equal to or greater than the yield strength of the web steel.
9. The bearing and intermediate stiffeners use the same yield strength steel as the web. Also, the stiffener thickness is the same as the web.

Parameters

Optimization partly incorporates the following parameters:

V = governing shear; M = maximum loading moment; $\alpha = F_{yw}/F_{yf}$, the ratio of web and flange steel yield strengths; $c = P_w/P_f$, the ratio of web to flange material prices; the bending stress correction factor, $\bar{K} = \alpha(3 - \alpha^2)/2 \leq 1.0$; A_w and A_f , the web and flange areas; and $F_{bf} = 0.6 F_{yf}$ is the allowable bending stress.

Derivations

The equation for the area of one flange, which was derived by Chong,^{3,4} is used in a design example in the AISC Manual⁵

$$A_f = \frac{M}{F_{bf} h} - \frac{\bar{K} t h}{6} \quad (1)$$

The cross-sectional area of an unstiffened girder is the sum of the web and flange areas.

$$A = 2A_f + A_w = 2\left(\frac{M}{F_{bf} h} - \frac{\bar{K} t h}{6}\right) + t h \quad (2)$$

Dividing the flange area by the ratio of web material price to flange material price c gives the price weighted area equivalent to the web material,²

$$\bar{A} = \frac{2}{c} \left(\frac{M}{F_{bf} h} - \frac{\bar{K} t h}{6} \right) + t h \quad (3)$$

The AISC Manual⁵ allows the height to thickness ratio to reach the value

$$\frac{h}{t} = 14000 / \sqrt{F_{yf}(F_{yf} + 16.5)} \quad (4)$$

before transverse stiffeners, at not more than 1.5 times the girder depth, are required. Therefore, substituting the value of web thickness

$$t = h \sqrt{F_{yf}(F_{yf} + 16.5)} / 14000 \quad (5)$$

into Eq. 3 for the price weighted area, \bar{A} , gives

$$\bar{A} = \frac{2}{c} \left(\frac{M}{F_{bf} h} - \frac{\bar{K} h^2 \sqrt{F_{yf}(F_{yf} + 16.5)}}{6 (14000)} \right) + \frac{h^2 \sqrt{F_{yf}(F_{yf} + 16.5)}}{14000} \quad (6)$$

By optimizing this function, the optimal depth of section may be found since Eq. 6 is in terms of web depth only.

When the derivative of the price weighted area is taken with respect to the girder depth and set equal to zero, the optimal depth for minimum cost may be found.

$$\frac{d(\bar{A})}{dh} = 0 = \frac{-2 M}{c F_{bf} h^2} + 2h \left(1 - \frac{\bar{K}}{3 c} \right) \frac{\sqrt{F_{yf}(F_{yf} + 16.5)}}{14000} \quad (7)$$

Solving for h yields

$$h = \left\{ 14000 M / \left[c F_{bf} \left(1 - \frac{\bar{K}}{3 c} \right) \sqrt{F_{yf}(F_{yf} + 16.5)} \right] \right\}^{1/3} \quad (8)$$

This gives the web thickness as either $h \sqrt{F_{yf}(F_{yf} + 16.5)} / 14000$ or $V / (F_{yw} h)$, whichever is larger. F_{yw} is the allowable shear stress of the web. By using the larger value, it is certain that the web thickness is sufficient for shear and that the height to thickness ratio does not exceed the allowable value.

The flange proportions are then based upon local buckling criteria which states the ratio of flange width to two times the thickness must not exceed $95 / \sqrt{F_{yf}}$ for a maximum allowable bending stress of $0.6 F_{yf}$.

Applications

Given the loading and support conditions for the girder and a list of available steel strengths and costs, the foregoing theory can be used to design cost efficient, hybrid or homogeneous girders. A programmable calculator or computer aid tremendously for checking multiple solutions if many steel combinations are available. Also, all other AISC criteria⁵ may be easily checked, proportions adjusted and stiffeners designed. Graphs can be generated using a plotting routine to show basic trends of which combination of steels is usually the most cost effective for general loading and span conditions. This allows a periodic regeneration of tables rather than an in-depth search for material combinations for every girder being designed.

Procedure

Two factors will raise the cost of a stiffened hybrid girder above that of an unstiffened homogeneous girder. First, the stiffeners involve a labor-related expense of cutting and welding the additional material. Second, the higher strength flange still will be more expensive than the material used for the web.

The labor-related factors are found by defining dimensionless ratios for both the bearing and intermediate stiffeners. Multiplying the amount of stiffener material by this factor results in a representative amount of web material having the same cost. The basis used for this labor cost weighting is the additional weld length required for the attachment of a stiffener to the web and flange(s). The web and flanges are joined by a fillet weld on each side of the web at both flanges, giving a web-to-flange weld length of four times the span length (L).

The factor B_m for the beam is defined as the weld length

per volume of the flanges and web (V_{fw}). Thus,

$$B_m = \frac{4L}{V_{fw}} \quad (9)$$

Likewise, the length of weld for the bearing stiffeners, the height h_b plus the top and bottom widths w_b , when divided by the volume of material V_B gives the factor

$$B_s = \frac{h_b + 2w_b}{V_B} \quad (10)$$

The intermediate stiffener is welded only along the top flange and web so its factor may be defined as

$$I_s = \frac{h_i + w_i}{V_I} \quad (11)$$

where h_i , w_i and V_I are the height, width and volume of the intermediate stiffener, respectively.

These factors are used to define the following weighting ratios for each type of stiffener:

$$F_B = \frac{B_s + B_m}{B_m} \quad (12)$$

$$F_I = \frac{I_s + B_m}{B_m} \quad (13)$$

Multiplying the stiffener volumes by these values reflects the additional expense of welding on the stiffeners. Therefore, the price weighted area of the bearing stiffeners is given as

$$\bar{A}_B = \frac{V_B}{L} \times N_B \times F_B \quad (14)$$

where

N_B = Number of bearing stiffeners.

Likewise, the price weighted area of the intermediate stiffeners is

$$\bar{A}_I = \frac{V_I}{L} \times N_I \times F_I \quad (15)$$

where

N_I = Number of intermediate stiffeners.

Combining Eqs. 14 and 15 with the flange and web contributions gives the total price weighted area, with respect to the web steel, as

$$\bar{A} = \frac{2}{c} A_f + A_w + \bar{A}_B + \bar{A}_I \quad (16)$$

The price weighted area, with respect to A36 steel, is then

$$\bar{A}_{36} = \bar{A} \left(\frac{P_w}{P_{36}} \right) \quad (17)$$

PRACTICAL EXAMPLES

The following two examples show a significant cost savings using the presented theory versus the conventional design method presented in the AISC Manual.⁵

Example 1: AISC Manual,⁵ 8th Ed., pp. 2-79 to 2-84.

Design a welded plate girder to support a uniform load of 3 kips per ft (43.8 kN/m) and two concentrated loads of 70 kips (311.5 kN) located 17 ft (5.2 m) from each end. The compression flanges of the girder are laterally supported only at points of concentrated load.

Given: Maximum bending moment: 2054 kip-ft
(2,788 kN-m)

Maximum vertical shear: 142 kips (632 kN)

Span: 48 ft (14.6 m)

Maximum depth: 72 in. (3.7 m)

Steel: $F_{yf} = F_{yw} = 36$ ksi (248 MPa)

Results: Web plate: $\frac{5}{16}$ in. \times 70 in. (0.79 cm \times 178 cm)

Flange plates: $\frac{3}{4}$ in. \times 18 in. (1.91 cm \times 45.7 cm)

Stiffeners: $\frac{9}{16}$ in. \times 8 in. \times 69- $\frac{3}{4}$ in.

(1.42 cm \times 20.3 cm \times 177.2 cm)

Bearing stiffeners: 4 pairs

Intermediate stiffeners: 4

The above results, which are calculated and published on pp. 2-79 to 2-84 of Ref. 5, are shown in Column 2 of Table 1 as a basis for comparison. Optimal designs are also shown in Table 1, with various yield stress combinations for flanges and webs. The price ratio c is taken from Ref. 6. Cost savings of up to 15.4% are possible.

Table 1. Optimal Girders vs. Conventional Design for Ex. 1

Item (1)	AISC Design (2)	Optimal Hybrid Girders	
		(3)	(4)
Flange			
F_{yf} , ksi	36	50	65
A_f , in. ²	13.50	13.64	15.30
Web			
F_{yw} , ksi	36	36	36
h , in.	70	55	40
t , in.	0.3125	0.300	0.2465
$c = p_w/p_f$	1	0.888	0.847
No. of bearing stiffeners	8	8	8
Factor, F_B	7.009	6.893	8.194
No. intermed. stiffeners	4	8	8
Factor, F_I	6.454	6.262	7.237
Price-weighted web equivalent area, A	73.576	70.651	62.223
% of savings	Basis	4.0	15.4

Notes: 1 ksi = 6.9 MPa; 1 in. = 2.54 cm.

Example 2: AISC Manual,⁵ 8th Ed., pp. 2-85 to 2-90.

Design a hybrid girder to support a uniform load of 2 kips per ft (29.2 kN/m) and three concentrated loads of 200 kips (890 kN) located at the quarter points. The girder depth must be limited to 5 ft (1.5 m). For comparison, two of the optimal girders are allowed to exceed the girder depth of 5 ft (1.5 m). The compression flange is laterally supported throughout its length.

Given: Maximum bending moment: 9600 kip-ft
(13,030 kN-m)
Maximum vertical shear: 380 kips (1,691 kN)
Span: 80 ft (24.4 m)
Maximum depth: 60 in. (1.52 m)
Steel: Flanges: $F_{yf} = 50$ ksi (345 MPa)
Web: $F_{yw} = 36$ ksi (248 MPa)

Results: Web plate: $\frac{1}{16}$ in. \times 54 in. (1.42 cm \times 137.2 cm)
Flange plates: $2\frac{7}{8}$ in. \times 24 in.
(7.30 cm \times 61.0 cm)
Stiffeners: $\frac{3}{4}$ in. \times 11 in. \times $53\frac{3}{4}$ in.
(1.91 cm \times 27.9 cm \times 136.5 cm)
Bearing stiffeners: 5 pairs, $\frac{1}{4}$ in. \times 4 in. \times 51 in.
(0.64 cm \times 10.2 cm \times 129.5 cm)
Intermediate stiffeners: 6

Again, the above results are published in Ref. 5 (pp. 2-79 to 2-84) and shown in Table 2 with optimal designs. Cost savings ranging up to 25.2% are achieved.

Using Ex. 2, the input data for the material and labor fractions is varied to illustrate the effects of higher and lower ratios of the cost of material to the cost of labor. Figure 2 shows graphically the influence of the material/

Table 2. Optimal Girders vs. Conventional Design for Ex. 2

Item (1)	AISC Design (2)	Optimal Hybrid Girders			
		(3)	(4)	(5)	(6)
Flange					
F_{yf} , ksi	50	50	50	65	65
A_f , in. ²	69.0	67.2	28.1	51.4	26.1
Web					
F_{yw} , ksi	36	36	36	36	36
h , in.	54	54	116.5	54	96
t , in.	0.5625	0.4887	0.5036	0.4887	0.500
$c = p_w/p_f$	0.888	0.888	0.888	0.847	0.847
No. of bearing stiffeners	10	10	10	10	10
Factor, F_B	8.191	11.534	7.161	9.471	6.597
No. intermed. stiffeners	6	6	12	4	12
Factor, F_I	46.377	23.168	15.745	18.825	14.054
Price-weighted web equivalent area, A	238.382	226.809	215.064	184.085	178.376
% of savings	Basis	4.9	9.8	22.8	25.2

Notes: 1 ksi = 6.9 MPa; 1 in. = 2.54 cm.

labor ratio on the optimum 36-web/65-flange girder in Ex. 2. The proportions considered are 30% material vs. 70% labor; 50% material vs. 50% labor; 70% material vs. 30% labor; 80% material vs. 20% labor; and 90% material vs. 10% labor.

CONCLUSIONS

The theory presented has applications for both stiffened and unstiffened, homogeneous and hybrid girders. Factors accounted for in the minimum cost optimization include the variety of steels available and their respective costs, labor factors related to web stiffeners to determine their economic feasibility, and the correction factor \bar{K} which eliminates the reduction of the allowable bending stress for the flanges. The resulting hybrid, stiffened girder is efficient from the standpoint of bending and shear stresses, as well as minimization of the cost.

NOMENCLATURE

- A = Area of cross section
- A_B = Price-weighted area of bearing stiffeners
- A_I = Price-weighted area of intermediate stiffeners
- A_f = Area of one flange
- A_w = Area of web
- \bar{A} = Price-weighted area of cross section
- B_m = Ratio relating flange to beam welds and the volume of flanges and web (Eq. 12)
- B_s = Length of weld for the bearing stiffener divided by its volume (Eq. 13)
- c = P_w/P_f
- C_v = Shear coefficient
- F_B = Factor relating bearing stiffeners (Eq. 15)
- F_I = Factor relating intermediate stiffeners (Eq. 16)
- F_{bf} = Allowable bending stress
- F_{yf} = Yield stress of flange
- F_{yw} = Yield stress of web
- F_{vw} = Allowable shear stress in web
- h = Depth of web
- h_i = Height of intermediate stiffener
- h_b = Height of bearing stiffener
- I_s = Intermediate stiffener height and width divided by its volume (Eq. 14)
- K = Bending stress correction factor
- L = Span length

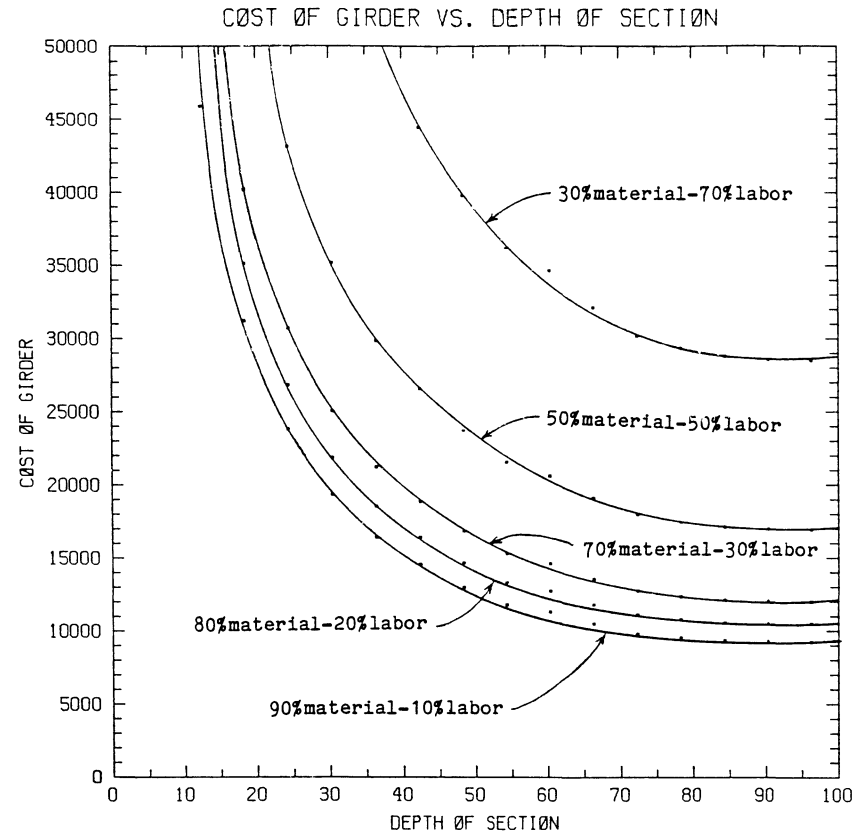
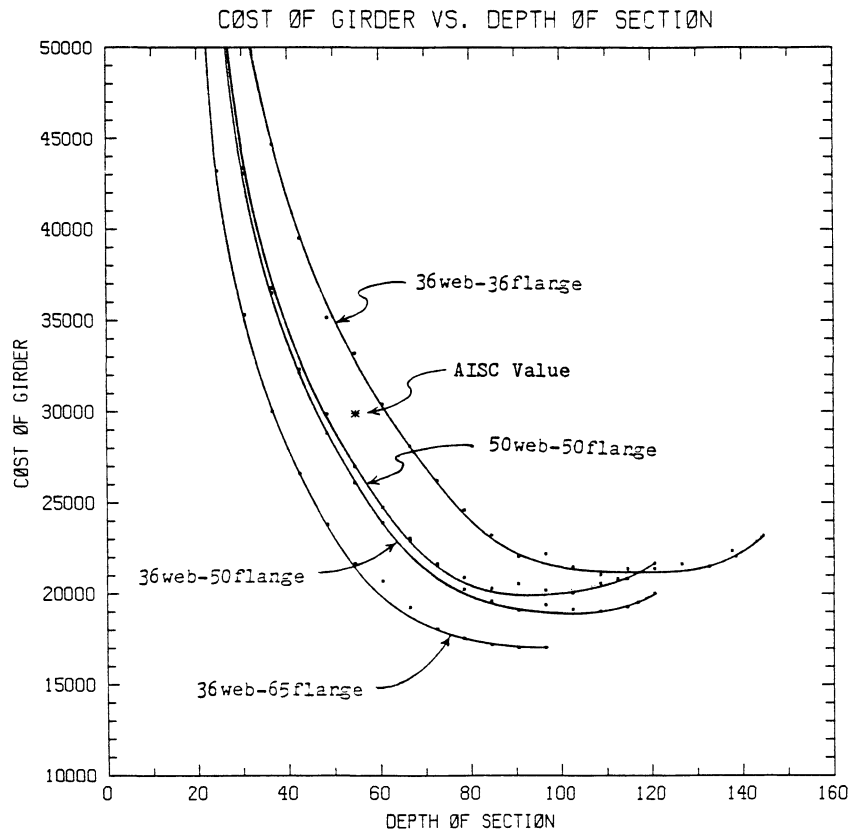


Fig. 2. Relative cost of girder vs. depth of section (in inches) for different ratios of material/labor costs

M = Governing moment
 N_B = Number of bearing stiffeners
 N_I = Number of intermediate stiffeners
 p_f = Unit price of flange material
 p_w = Unit price of web material
 p_{36} = Unit price of A36 steel
 t = Web thickness
 V = Governing shear
 V_B = Volume of bearing stiffener
 V_I = Volume of intermediate stiffener
 V_{fw} = Volume of flanges and web for the whole girder
 w_b = Width of bearing stiffener
 w_i = Width of intermediate stiffener
 $\alpha = F_{yw}/F_{yf}$

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