Moments on Beam-Columns with Flexible Connections in Braced Frames

JACK H. BROWN

The theoretical behavior of beam-columns in braced frames with AISC Type 2 connections is examined here to determine the validity of ignoring eccentricity in the design of columns with flexible connections. The AISC *Specification for the Design, Fabrication and Erection of Structural Steel for Buildings*[5] is not specific on whether eccentricity should be considered, but the British Standard 449 has specific requirements on the eccentricity to be taken. Neither specification provides any relief for restraint offered by the flexible connection as shown in Fig. 1, nor for the stiffness due to the difference in loading of an upper and lower column, continuous through two floors.

This paper is a revised version of the one presented at the Structural Stability Research Council's 1986 Tehnical Session in Washington, D.C.

EFFECTIVE COLUMN LENGTH

Recent papers have recommended the effective column length be reduced when the column is restrained by flexible connections without taking into account effects of the moment from partial fixity of the beam. This moment can have a significant effect on the strength of a column when the beam frames into one side of the column web. Under these circumstances, the beam connection can only restrain the column after the column joint has rotated to completely relieve the partial fixity. Beams with flexible connections that frame in either side of a column offer restraint to the column and may justify a reduction in the effective column length.

BASIS OF ANALYSIS

The experimental data is based on the University of Illinois Bulletin 500, entitled "Characteristics of Flexible Riveted and Bolted Beam-to-Column Connections," by C. W. Lewitt, E. Chesson, Jr. and W. H. Munse.[1] In particular, use of data for a W18 × 50 beam and the characteristics of the beam-connection are shown in Fig. 15 of that publication.

The governing equations for the buckling load on columns AB and BC (Fig. 1) will be derived for two end conditions at A and C. These end conditions are pinned and fixed, but only the pinned case will be used for the analysis of beam connection restraint in this paper. The frame shown in Fig. 1 illustrates behavior of a W18 × 50 beam with web angle connections framing into a W12 × 45 column. The beam connections offer variable restraint against column joint rotation.

Modified versions of Fig. 1 are used in the subsequent analysis for single columns and for seat connection that offers no restraint against column rotation. For single col-

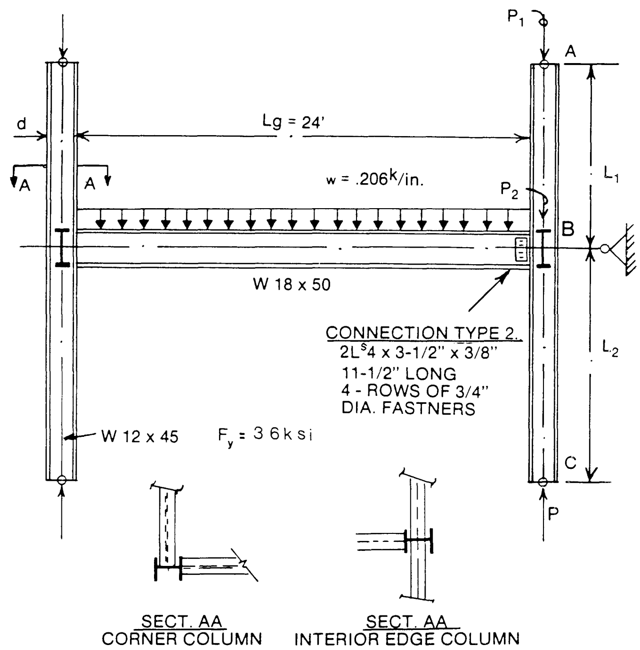

Fig. 1. Double-column frame

Jack H. Brown, P.E., is Senior Research Associate, Research & Development, The Austin Company, Cleveland, Ohio.

umns, the dimension L_1 is taken as zero. Only the elastic characteristics of the frame are considered and a simplified mode of failure is assumed; this is that failure occurs when the yield stress in the column is reached or exceeded.

The paper assumes connections will follow the moment rotation curve (Fig. 2) without the inelastic behavior of the connection affecting the curve. In the analysis of all double columns, the axial load in the lower column is taken as twice the load in the upper column.

CONNECTION RESTRAINT

The beam connection restraint or partial end moment will be referred to as M_r, which is a nonlinear function of the beam's end rotation.

$$M_r = -(\phi - \theta)C_f(\phi - \theta) \tag{1}$$

The argument for C_f is always taken as the absolute value.

Where ϕ = the free end slope of the beam, radians

θ = the slope of the column at B, radians

C_f = the rotational stiffness of the beam and connection, kip-in./radian

The following equation gives a very good fit to the W18 × 50 beam connection for[1] Fig. 2 curve. The equation is:

$$C_f(\psi) = \frac{10^4 \times (11 - 110\psi)}{(1 + 1250\psi)^{0.563}} \tag{2}$$

where ψ = the rotation of the connection = $(\phi - \theta)$

The general form of Eq. 2 is

$$C_f(\psi) = \frac{(a - b\psi)e}{(1 + c\psi)^d} \tag{3}$$

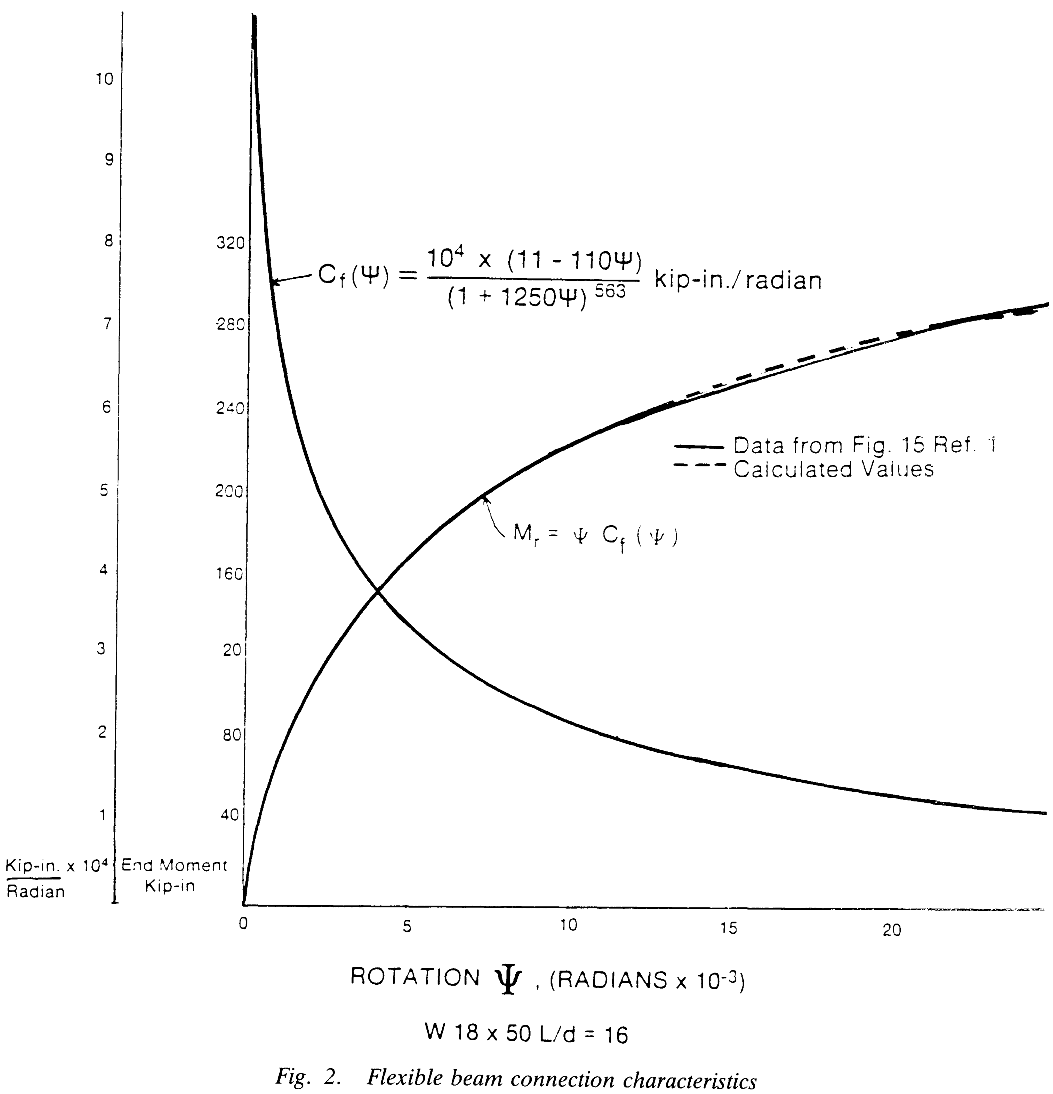

Fig. 2. Flexible beam connection characteristics

The values of the variables a, b, c and d must be determined for each type of connection. The data for different types of connections is in Ref. 1.

A comparison of the experimental data and Eq. 2 is shown in Fig. 2.

METHOD OF ANALYSIS

The method of analysis will be based on stability functions.[2,3]

The stability functions are generalized slope deflection equations that take into account the axial load and are given by Eqs. 4–8.

$$M_{AB} = \frac{EI}{L}\left(s\theta_A + sc\theta_B - \frac{s(1+c)\delta}{L}\right) \tag{4}$$

Where

$$s = \frac{u(1 - 2u \cot 2u)}{\tan u - u} \tag{5}$$

$$c = \frac{\sin 2u - 2u}{2u \cos 2u - \sin 2u} \tag{6}$$

$$u = \frac{\pi}{2}\sqrt{\rho} \tag{7}$$

$$\rho = \frac{PL^2}{\pi^2 EI} \tag{8}$$

The governing equations for the no-sway double column shown in Fig. 1 will be derived using Eq. 4 (see Fig.3).

Double Column

Pinned ends at "A" and "C". No sway (see Fig. 1).

Boundary conditions:

$$M_{BA} = 0$$
$$M_{CB} = 0$$
$$\delta = 0$$

For equilibrium at Joint B

$$k_1 = \frac{I_1 E}{L_1}$$

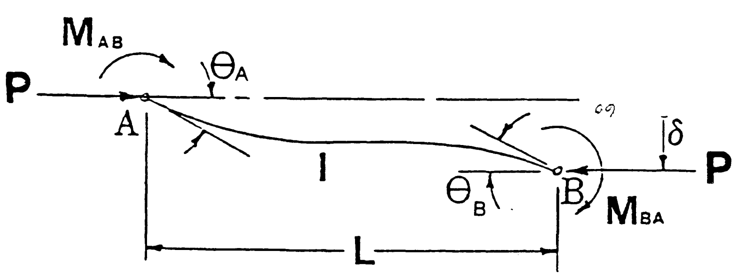

Fig. 3. Stability functions

$$k_2 = \frac{I_2 E}{L_2}$$

$$M_{BA} + M_{BC} + M_r = M_b \tag{9}$$

$$M_{AB} = k_1(s_1\theta_A + s_1c_1\theta_B) = 0$$

$$\theta_A = c_1\theta_B \tag{10}$$

$$M_{BA} = k_1(s_1c_1\theta_A + s_1\theta_B)$$

$$M_{BA} = k_1 s_1(1 - c_1^2)\theta_B \tag{11}$$

Similarly,

$$M_{BC} = k_2 s_2(1 - c_2^2)\theta_B \tag{12}$$

Substituting Eqs. 11 and 12 into Eq. 9 and solving for θ_B, letting $\theta = \theta_B$

$$\theta = \frac{M_b}{k_1 s_1(1 - c_1^2) + k_2 s_2(1 - c_2^2) + M_r/\theta} \tag{13}$$

Where M_b is the applied moment at joint B and M_r is the connection restraint from the beam.

Fixed ends at "A" and "C". No sway.

The boundary conditions are:

$$\theta_A = 0$$
$$\theta_B = 0$$
$$\delta = 0$$

The slope at "B" is found to be:

$$\theta = \frac{M_b}{k_1 s_1 + k_2 s_2 + M_r/\theta} \tag{14}$$

Only variations on Eqs. 9 and 13 will be used in subsequent analysis.

EQUILIBRIUM OF A DOUBLE BEAM-COLUMN

Fig. 4 shows the three stages of equilibrium of a double column with a one-sided connection, restrained against sway, with $L_1 = L_2$.

Stage 1

The beam-column is initially in an unloaded state. When the beam is loaded, rotation occurs at the ends of the beam and column joint "B", causing the column to rotate through an angle θ, while the beam ends rotate through angle ϕ. The resulting rotation of the flexible connection is $(\phi - \theta)$.

During the transition from Stage 1 to Stage 2, the beam imposes a moment on the column.

Stage 2

As the column load P_1 increases, Stage 2 is reached when $\theta = \phi$. At this point, the beam imposes no moment or restraint to the column.

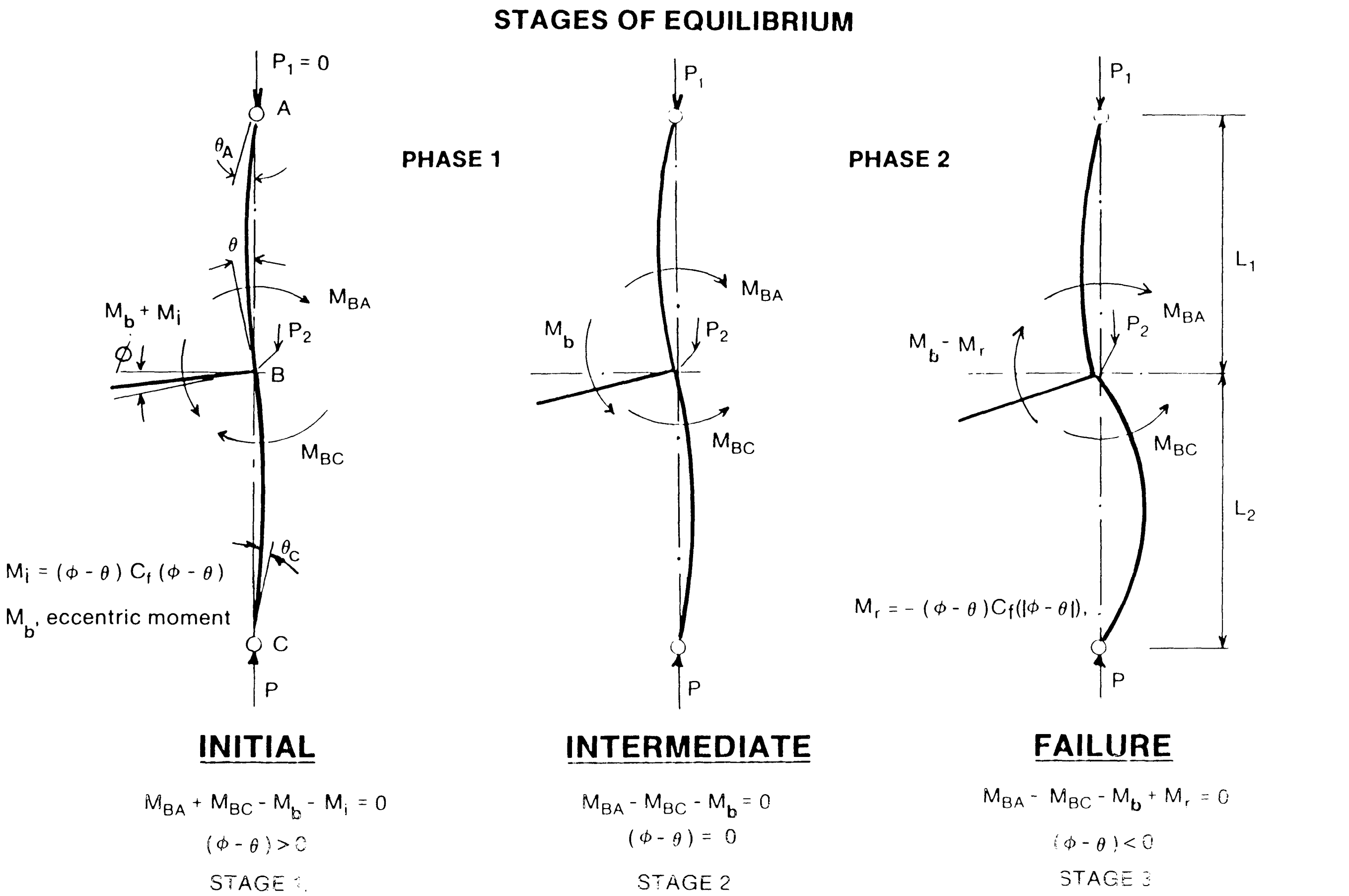

Fig. 4. Stages of equilibrium for double column

Stage 3

As the column's axial load is further increased, failure occurs at Stage 3. At failure, a plastic hinge forms below joint "B". Between Stage 2 and 3, the lower column imposes a moment on the beam and the upper column.

The column stiffness decreases with increasing axial load and has no flexural rigidity at failure.

TYPE 2 CONSTRUCTION

The assumption used for Type 2 construction[4] occurs at Stage 2, when the beam is simply supported, imposing no moment on the column. Fig. 4 shows that before these assumptions are reached the column will have rotated through an angle ϕ, equal to the free end rotation of a simply supported beam. At this stage, the connection will not offer any restraint to the column until further rotation has occurred.

TYPES OF CONNECTIONS

Three types of connections will be examined with various combinations of moments applied about each axis of the column. The ultimate capacity of the lower column will be determined for the double-column frame (Fig. 1) and for the single column frame (Fig. 5).

The column framing configuration and connection classification will be as follows:

Case 1. Column with angle seat; the connection is assumed to offer no restraint to column rotation.

Case 2. A corner column with angles attached to the beam web; the connections offer restraint to column rotation about both axes.

Case 3. An interior edge column with Type 2 connections attached to one flange and edge beams attached to each side of the web; the connections offer restraint to column rotation.

For Cases 2 and 3, it is assumed that the beams impart their partial restraint and partial fixed end moments to the column at joint "B". In Case 3, the partial end moments about the weak axis balance, and the resulting moment from the beams is zero.

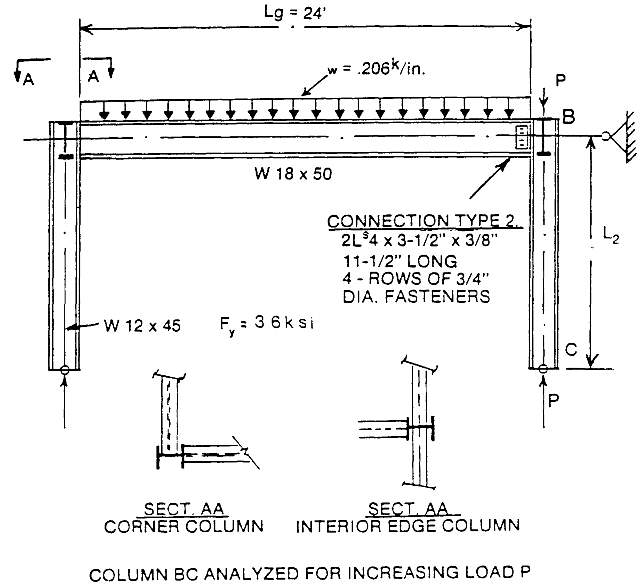

Fig. 5. Single-column frame

ANALYSIS

Modifying Eqs. 9 and 13 to determine the failure modes for Cases 1, 2 and 3.

Case 1: No restraint from connection, $M_r = 0$. Eq. 9 becomes:

$$M_{BA} + M_{BC} = M_b$$

$$\theta = \frac{M_b}{k_1 s_1(1 - c_1^2) + k_2 s_2(1 - c_2^2)} \tag{15}$$

$$M_{BC} = \theta k_2 s_2(1 - c_2^2)$$

Limiting the failure to the point where the total column stress $F_c \geq F_y$ (yield stress), where F_c is given by:

$$F_c = \frac{P}{A} + \frac{M_{BX}}{S_x} + \frac{M_{BY}}{S_x} \tag{16}$$

Failure occurs as:

$$k_1 s_1(1 - c_1^2) + k_2 s_2(1 - c_2^2) \Rightarrow 0$$

$$\theta \Rightarrow \infty \text{ and } M_B \Rightarrow \infty$$

Case 2: Since this is a corner column, the connections are unsymmetrical about the column.

Phase 1. $(\phi - \theta) > 0$

Phase 2. $(\phi - \theta) \leq 0$

The loading phases can best be described by referring to Fig. 4.

During phase 1 the beam imposes both a partial end moment and an eccentric moment on the column.

During phase 2 the connection offers restraint against column rotation, with the column transferring some of its moment to the beam, while the eccentric moment remains constant.

Equation 9 becomes:

$$M_{BA} + M_{BC} + M_r = M_b \tag{17}$$

Substituting Eq. 1 for M_r in Eq. 9 and solving for θ.

$$M_{BA} + M_{BC} - (\phi - \theta)C_f(|\phi - \theta|) = M_b$$

$$M_{BA} + M_{BC} + \theta C_f(|\phi - \theta|) = M_b + \phi C_f(|\phi - \theta|)$$

$$\theta[k_1 s_1(1 - c_1^2) + k_2 s_2(1 - c_2^2) + C_f(|\phi - \theta|)] = M_b + \phi C_f(|\phi - \theta|)$$

$$\theta = \frac{M_b + \phi C_f(|\phi - \theta|)}{k_1 s_1(1 - c_1^2) + k_2 s_2(1 - c_2^2) + C_f(|\phi - \theta|)} \tag{18}$$

The moment acting at the top of the lower column will be given by Eq. 12.

Failure occurs as $F_c \geq F_y$, where F_c is given by Eq. 16.

Case 3: The connections are to one face of the flange and either side of the column web, with the assumption that the edge beams on either side of the columns are initially equally loaded. The behavior of flange connections is the same as connections for Case 2. The behavior of the web connections will be investigated. The partial fixities from the beams balance and the resulting moment on the column is zero. Let an unbalanced beam end moment be applied to the column, let this moment be M_b. Then Eq. 9 becomes:

$$M_{BA} + M_{BC} + \text{(resistance of connections)} = M_b \tag{19}$$

The rotation of column joint "B" is given by:

$$\theta = \frac{M_b}{k_1 s_1(1 - c_1^2) + k_2 s_2(1 - c_2^2) + \dfrac{(\phi + \theta)C_f(\phi + \theta) - (\phi - \theta)C_f(|\phi - \theta|)}{\theta}}$$

$$\theta = \frac{M_b + \phi[C_f(|\phi - \theta|) - C_f(\phi + \theta)]}{k_1 s_1(1 - c_1^2) + k_2 s_2(1 - c_2^2) + C_f(|\phi - \theta|) + C_f(|\phi + \theta|)} \tag{20}$$

SINGLE COLUMN

The equation derived for the double column with flexible connections can also be used for the analysis of a single column by dropping the terms for the upper column. To analyze the single column, see Fig. 6. With applied moment at the top, without connection restraint, the following equations were used:

$$y = \frac{M}{P}\left(\frac{X}{L} - \frac{\sin kx}{\sin kL}\right) \text{(Ref. 6)} \tag{21}$$

where

$$k^2 = \frac{P}{EI}$$

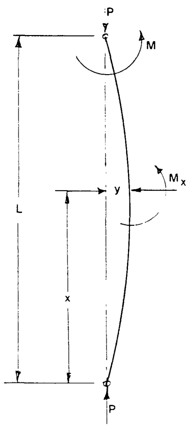

Fig. 6. Pin-ended column subjected to end moment

$$\frac{dy}{dx} = \frac{M}{P}\left(\frac{I}{L} - \frac{k \cos kx}{\sin kL}\right) \tag{22}$$

$$\frac{d^2y}{dx^2} = \frac{M}{P}k^2\frac{\sin kx}{\sin kL} \tag{23}$$

$$M_x = EI\frac{d^2Y}{dx^2} = M\frac{\sin kx}{\sin kL} \tag{24}$$

Equating $\frac{dy}{dx} = 0$, then

$$x = \cos^{-1}\left(\frac{\sin kL}{kL}\right)\Big/k \tag{25}$$

The maximum stress in the column can be determined by Eqs. 16, 24 and 25.

PARTIAL END MOMENT

When a uniformly loaded beam is simply supported, the ends of the beam will rotate through an angle,

$$\phi = \frac{wLg^3}{24EIg} \tag{26}$$

Where w = Load/unit length

L = Length of beam

I = Moment of inertia of the beam

E = Modulus of elasticity

The effect of the beam framing into the column will induce a moment at joint "B" which is proportional to the rotation of the connection. The total external moment acting at the joint will be the partial fixity of the beam plus the beam reaction times its distance to the column centroid.

For W18 × 50 beam shown in Fig. 1, the following data will be used to calculate the partial end moment.

$Lg/d = 16$, $Lg = 16 \times 18$ in. $= 288$ in.

$f_b = 24$ ksi $\quad S_x = 88.9$ in.^{3}

For a simply supported W18 × 50 beam,

$$f_b = \frac{wLg^2}{8S_x} \text{ and } w = \frac{8f_bS_x}{Lg^2} = 0.206 \text{ kip-in,}$$

Beam reaction = 0.206 kip/in. × 288/2 = 29.7 kips

The free rotation of the beam under uniformly distributed load will be given by

$$\phi = \frac{wLg^3}{24EIg} = \frac{0.206 \times 288^3}{24 \times 29000 \times 800} = 8.84 \times 10^{-3} \text{ rad}$$

$$\phi C_f(\phi) = 8.84 \times 10^{-3} \times \frac{(11 - 110 \times 8.84 \times 10^{-3}) \times 10^4}{(1 + 1250 \times 8.84 \times 10^{-3})^{.563}}$$

$$= 218.3 \text{ kip-in.}$$

Applying the load at 2.5 in. from the face of the column, the eccentric moments will be:

$e_y = 2.5 + .334/2 \quad = 2.7$ in.
$M_y = 2.7 \times 29.7 \quad = 80$ kip-in.
$e_x = 2.54 + 12.06/2 = 8.6$ in.
$M_x = 8.6 \times 29.7 \quad = 255$ kip-in.

Computer programs were written to solve the equations for Cases 1, 2 and 3 for both the single and double columns. A summary of results are shown in Figs. 7, 8 and 9.

The loading for each case, was incremented from zero to failure, with a constant applied moment M_x of 255 kip-in. being applied at joint "B" for all cases. The moment about the y-y axis of the column was varied from 0.1 to 300 kip-in. for each failure load. For the cases where the connections offered restraint against joint rotation, an additional partial fixed end moment of 218.3 kip-in. from the framing beam was included in the calculation, but not shown in Fig. 8. This was used for curves 3 and 4.

For the double column, the load in the lower column was taken as twice the load in the upper column.

The curves for Fig. 7 are as follows:

1. Single column, no joint restraint, applied moment only
2. Double column, no joint restraint, applied moment only
3. Single corner column, joint restraint, with beam end moment effect
4. Double corner column, joint restraint, with beam end moment effect
5. Single interior edge column, joint restraint, with double beams and joint moment
6. Double interior edge column, joint restraint, with double beams and joint moment

ANALYSIS OF COMPUTER RESULTS

When a beam with flexible connections frames into the web of a column, it has two distinct modes of behavior. A lightly loaded beam offers considerable restraint to the column, whereas, a long span heavily loaded beam imposes moments from both partial fixity and eccentricity of connec-

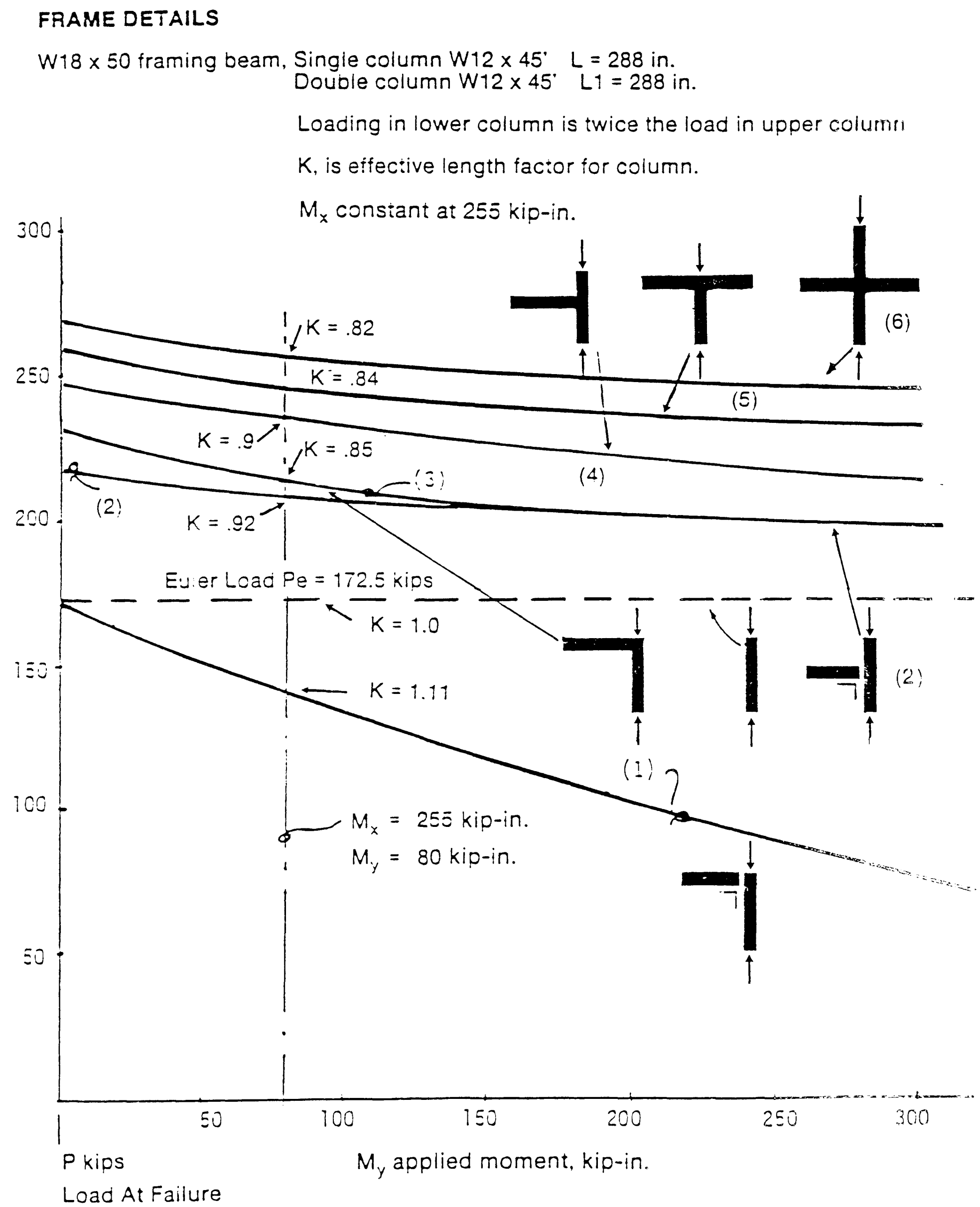

Fig. 7. Ultimate capacity of columns subjected to varying end moments about weak axis

tion on the column. The small eccentric moment is usually ignored in design, while existence of the much larger moment from partial fixity is not even considered. Fortunately, the analysis of the case investigated indicates the flexible web connection does increase the capacity of column at failure.

Simply supported beams with flexible connections should not be designed for partial fixity since the moment is relieved or reversed when the column is loaded. Similarly, there does not appear to be any valid reason for reducing the effective length of the column for the restraint offered by the beams with flexible connections if these beams can impose moments on the column and these moments are ignored in the column design.

In all cases where the flexible connection offered restraint against rotation, the column with applied partial moments had a higher load capacity than the unrestrained columns with no applied moments. The effect of the partial fixity from the beam causes the column to rotate under increasing axial load, with the connection offering no restraint until the end moment in the beam is zero. When this point is reached, any further increase in rotation is resisted by the connection, with the moment being reversed at the joint. This moment increases until failure occurs. Figs. 8

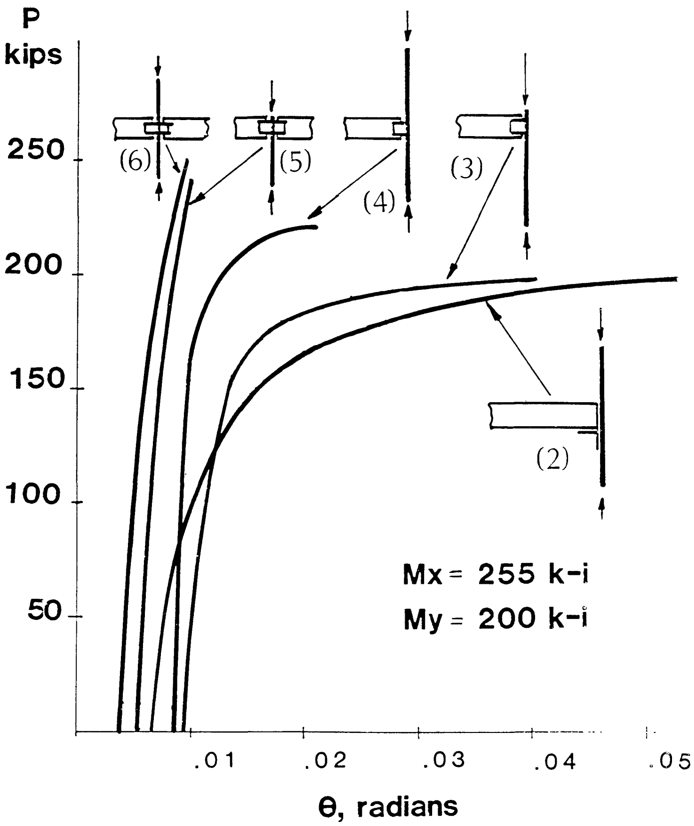

Fig. 8. Column joint rotation characteristics with constant applied moments and varying axial loads

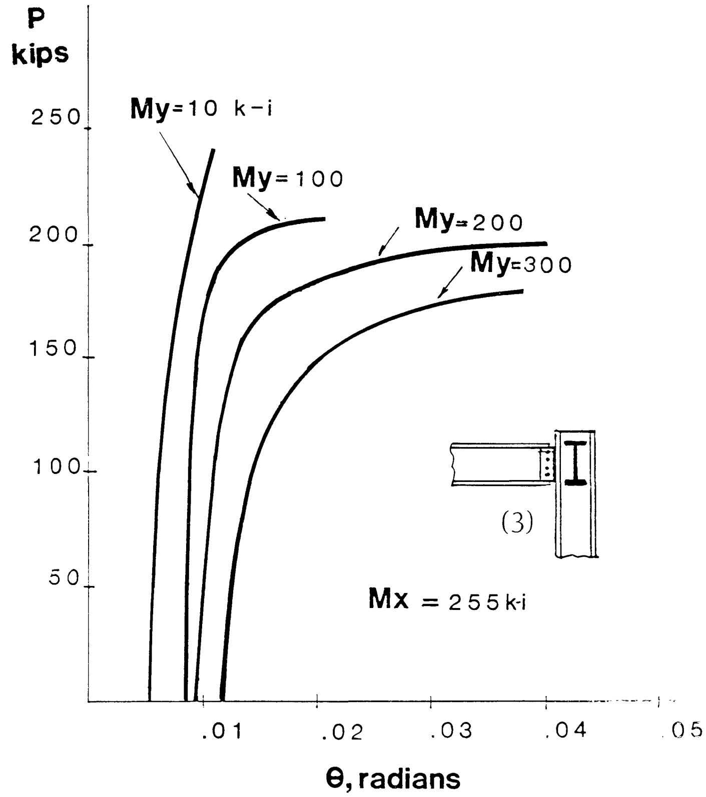

Fig. 9. Single-column joint rotation characteristics for varying moments and axial load

and 9 show the joint rotation characteristics for various frame configurations and loading.

The analysis indicates that if the column had been designed using an effective column length factor of $K = 1$ and ignoring the moments due to eccentricity, it would have had a reserve of strength greater than the axial loaded column. This reserve of strength can be attributed to the restraint of the connection to rotation, and in the case of the double column, to the additional restraint from the stiffness of the smaller loaded upper column.

The present AISC Specification[5] does not consider the relief from flexible joint restraint or the effect of different loading in adjoining columns, nor does it provide clear directions for the design of eccentrically loaded columns. In this analysis, the effect of torsional stability was not considered, but the results indicate that columns tend to fail about the weak axis for the column shown in Fig. 1, irrespective of the moment applied to the *x-x* axis for values used.

If full-depth, double-angle connections such as those listed in the AISC *Manual of Steel Construction*, 7th Edition,[4] are used with a $K = 1$, it seems the strength of the column is not impaired by ignoring normal eccentricities. This is especially true when the same column section is continuous through two or more floors.

When seated beam connections or similar types of construction that offer no restraint against rotation are used, the single column has only 142.3 kips capacity with $K = 1.1$. This is only 82% of the capacity of the axially loaded column and shows that the eccentric moment should be considered in this type of design.

Only the test data for a W18 × 50 beam is used in this analysis. Other test curves for flexible connections have similar characteristics. It is likely that beam columns with these connections will act in a similar manner to the beam-column analyzed in this paper.

The equations derived in this paper are not intended for use in design practice since the equations are highly nonlinear and require a computer for their solution.

CONCLUSION

The current AISC Specification[5] does not specify requirements as to whether columns should be designed for eccentricity. Future codes should clarify this requirement.

The investigation shows that the eccentric moment about the major axis of the column did not materially influence its ultimate capacity. Type 2 connections at one side of the web of a column have a small eccentricity, but impose a large partial moment to the column's weak axis that ultimately determines its capacity.

This paper shows that there is some justification for ignoring the effects of normal eccentric moments when the beam column has Type 2 full depth shear flexible connections and the column is designed for axial loading with an effective column length factor of $K = 1$.

Eccentrically loaded columns without rotational restraint should be designed for the eccentric moments. When partial fixity from the beam is used to reduce the effective length of the column, it should be remembered that the connection also imposes a substantial moment on the column.

NOMENCLATURE

c = Given in Eq. 4

C_f = Rotational stiffness of beam and connection, kip-in./radian

d = Depth of beam or column

E = Modulus of elasticity

f_b = Bending stress

I = Moment of inertia

k $= \sqrt{\dfrac{P}{EI}}$

L = Column or beam length

M_b = Applied moment at column joint "B"

M_{BX} = Bending moment below joint "B" about the x-x axis

M_{BY} = Bending moment below joint "B" about the y-y axis

M_i = Moment due to partial fixity of beam $= (\phi - \theta)C_f(|\phi - \theta|)$

M_r = Beam connection restraint $= -(\phi - \theta)C_f(|\phi - \theta|)$

P = Axial load on column L

P_1 = Axial load on column L_1

P_2 = Axial load applied at top column

s = Given in Eq. 4

S = Section modulus

S_x = Section modulus of the column about the x-x axis

S_y = Section modulus of the column about the y-y axis

u $= \dfrac{\pi}{2}\sqrt{\rho}$

w = Load/unit length

θ = Rotation of column at joint "B"

ϕ = Free rotation of a simply supported beam

ψ = Rotation of connection, radians

δ = End displacement of beam or column. Taken as $\delta = 0$ in this paper

ρ $= \dfrac{PL^2}{\pi^2 EI}$

REFERENCES

1. *Lewitt, C. W., E. Chesson, Jr. and W. H. Munse* Restraint Characteristics of Flexible Riveted and Bolted Beam-To-Column Connections *College of Engineering, University of Illinois, 1969, Urbana, Ill.*
2. *Kirby, P. A. and D. A. Nethercot* Design for Structural Stability *John Wiley & Sons, 1979, New York, N.Y.*
3. *Livesley, R. K. and D. B. Chandler* Stability Functions for Structural Frameworks *Manchester University Press, 1956.*
4. *American Institute of Steel Construction, Inc.* Manual of Steel Construction *7th Ed., 1970, New York, N.Y.*
5. *American Institute of Steel Construction, Inc.* Manual of Steel Construction *8th Ed., 1980, Chicago, Ill.*
6. *Chajes, Alexander* Principles of Structural Stability Theory *Prentice-Hall, Inc., 1974, Englewood Cliffs, N.J.*