Steel Shell Roof Structures

ARTHUR H. NILSON

THE ADVANTAGES of shell structures for enclosing large, column-free areas are well known. While reinforced concrete has dominated the field of shell construction for many years, recent developments have indicated the feasibility of using steel for certain singly- and doubly-curved roofs. Steel shells are attractive and highly versatile. They are extremely light in weight, well suited to the preassembly of parts, and do not require the elaborate formwork and scaffolding during construction that is associated with concrete shells.

Steel shells may be constructed using the standard light gage steel roof deck panels available from many sources in the United States and abroad. A few of the more common panels are shown in cross section in Fig. 1. Used in shell structures, the panels carry normal loads by their flexural strength, but in addition are designed to resist membrane shear loads acting in the plane of their surface.

While various shell forms have been built using such panels, there are certain limitations. Standard panels can not be curved much in the direction of their length, and it is not advisable to curve them transversely, except for the most elementary corrugated sections. However, plane subelements can be used as components of curved shells. In addition, most panels can be warped to a certain degree about an axis parallel to their length.

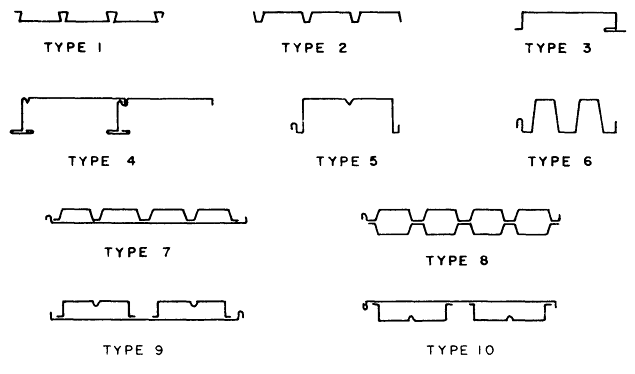

Fig. 1. Typical panel cross sections

Two shapes particularly suited to the use of steel panels are the folded plate and the hyperbolic paraboloid. In the first case, flat or inclined panels span transversely between ridge and valley lines, and provide transverse flexural strength, as well as membrane shear strength for the longitudinal plate elements. For hyperbolic paraboloids, slight warping of individual panels is required. Welded to adjacent panels in the warped state, each panel contributes to the formation of a doubly-curved shear membrane which maintains the equilibrium of the shell.

MEMBRANE SHEAR STRENGTH OF PANEL ASSEMBLIES

The resistance of light gage steel panel assemblies to shear loads applied in their own plane has been the subject of active investigation for a number of years. Such loads may come about due to wind or seismic forces acting on buildings, for example. While separate horizontal bracing in the plane of a floor or roof may be provided to transmit horizontal loads into braced bents or end walls, it has been discovered that even the rather nominal welding pattern used to hold floor and roof panels in position is capable also of transmitting substantial shear. With special attention to the design of welds, supplementary bracing can in most cases be entirely eliminated.

The performance of such shear-resisting membranes, or diaphragms, can not be predicted with confidence on the basis of theoretical analysis alone. They are composed of a large number of relatively small parts, each subject to individual movement. Under certain circumstances connections undergo elasto-plastic deformation. Also, it has been found that the strength of many of these fastenings is strongly dependent upon the configuration of the surrounding panel material. These and other factors have led investigators to study such systems on a primarily experimental basis.

The first full scale tests of light gage steel shear membranes were performed in California in the early 1950's. An extensive program of testing began at Cornell University in 1955, and a large body of knowledge was developed, based on tests of more than 110 diaphragms, covering a multitude of panel types, spans, and fastening systems.[1, 2]

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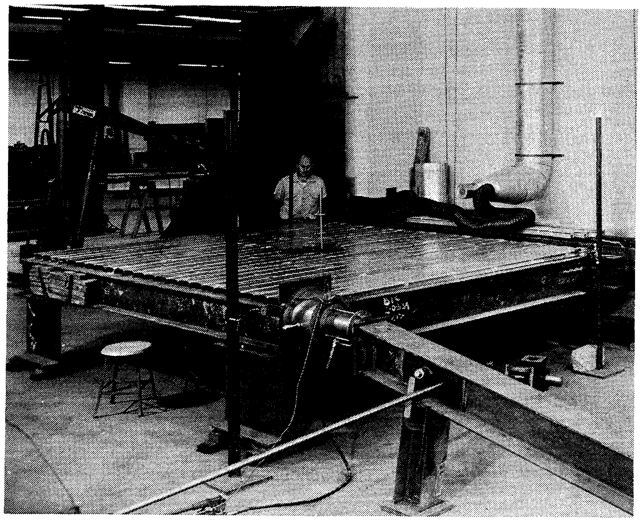


Fig. 2. Membrane shear test frame

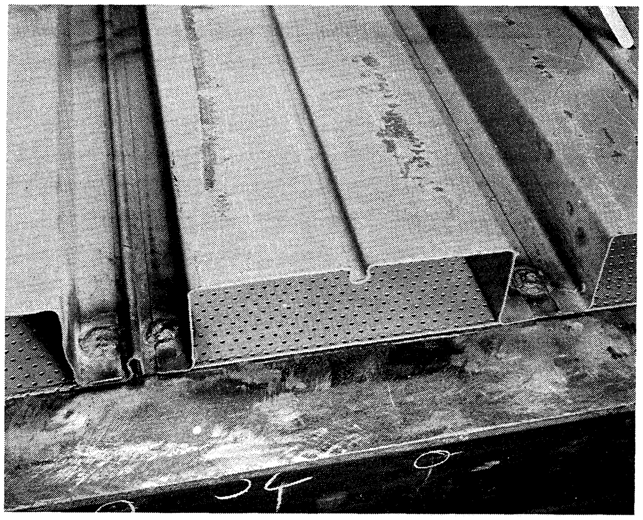


Fig. 3. Puddle welds

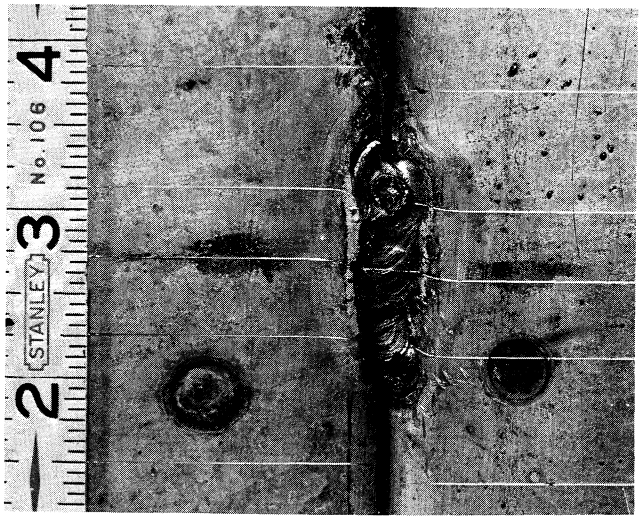


Fig. 4. Seam fillet weld

Figure 2 shows an overall view of one of the smaller shear frames that was used for the Cornell tests. The frame measured 10 x 12 ft in plan and was fixed in two directions at the right hand corner, with the connection at the far corner permitting free movement parallel to the wall only. Horizontal load was applied using the hydraulic jack shown in the foreground. The entire frame was supported on rollers, and connections were designed for low moment resistance, so that the only substantial resistance to shear distortion of the assembly was provided by the steel membrane. Other frames as large as 30 x 30 ft were used.

While light gage steel panels may be fastened using bolts or sheet metal screws, for heavily loaded membranes welds are more suitable. In fastening panels to the steel frame, a puddle weld is used, in which a hole is burned through the thin steel sheet and the weld made in one continuous operation. Such welds may be from about ½- to 1-in. in diameter. Typical puddle welds used as panel end welds are shown in Fig. 3. Panel edge welds joining the outside edge of the exterior panels to supporting steel are identical.

Panels normally terminate in some form of hook joint, with the recess of one panel engaging the lip of the next. If the hook joint is concave downward, a linear puddle weld is used, in which an elongated hole is burned through the top of the hook, fusing the hook to the upper part of the upstanding lip of the adjacent panel. If the hook is concave upward, a weld bead is laid along the slot between the panels, creating a fillet weld. Figure 4 shows such a fillet weld after testing of a diaphragm. The scribe lines clearly show the shear deformation at the weld. Seam welds in general may be of the order of 1½ in. in length, and spaced from about 6 to 48 in. on centers.

It is convenient in comparing the various shear diaphragms to express strength in terms of pounds of shear force per foot of diaphragm depth, the depth being measured parallel to the direction of the load. Strength will depend upon:

(a) Panel type
(b) Yield stress of the panel material
(c) Thickness of the panel steel
(d) Panel span
(e) Weld system

Table 1 summarizes the results of certain key tests of the Cornell diaphragm program. It will be observed that shear strengths vary from a minimum of 970 lb/ft, obtained with very nominal welding in conjunction with clinched seam connections, to a maximum of 7,170 lb/ft, obtained using very heavy welding with 14 gage cellular panels. A sufficient range of values is available to permit economical design.

FOLDED PLATES

The same shear strength properties that permit steel panel assemblies to resist horizontal loads can be utilized in particular forms of shell roofs. The folded plate structure is one example.

Much has been written relative to the analysis and design of folded plate roofs, mostly pertaining to reinforced concrete construction. It is not the purpose to develop this theory here, but only to review its salient points and to indicate some important modifications when that theory is applied to folded plates of light gage steel.

The structural action of a folded plate is separated into slab action and plate action (Figs. 5 and 6). The roof surface spans as a slab in the direction transverse to the fold lines, with the fold lines serving as lines of support for the transverse slab strips. The reaction of each slab strip, applied to any given fold line, resolves into components parallel to the two adjacent plates. The in-plane component for a plate at one fold line is added to that at the adjacent fold line of the same plate and to the in-plane component of the surface load to obtain the total in-plane load applied to the plate. The plate is then analyzed as a beam, carrying that load between end support frames (Fig. 6).

Figure 7 shows a cross section of a typical folded plate utilizing light gage steel panels. The panels are arranged to span in the transverse direction of the roof between fold lines. They are welded to one another intermittently

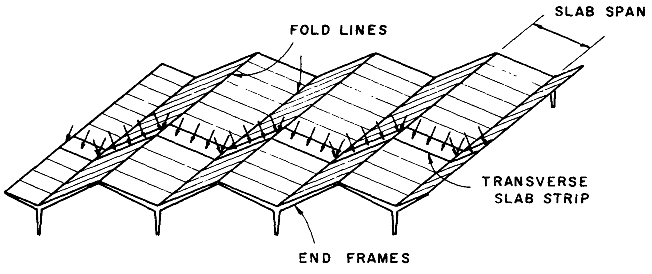


Fig. 5. Transverse slab action

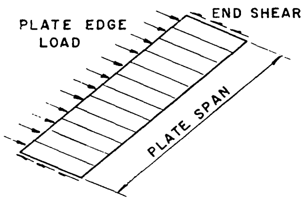


Fig. 6. Longitudinal plate action

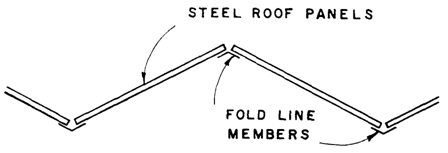


Fig. 7. Steel folded plate section

Table 1. Plane Shear Membrane Test Summary

Panel Type	Test Number	Panel Depth, in.	Gage of Shear Plate	Panel Span, ft	End Welds Dia, Type, Sp. (in.)	Edge Welds Dia, Type, Sp. (in.)	Seam Welds Dia, Type Sp. (in.)	Failure Shear, lb/ft
1	59-6	1.5	16	10	0.75P at 6	0.75P at 18	1.0P at 18	3580
	59-7	1.5	18	10	0.75P at 6	0.75P at 18	1.0P at 18	2320
	59-8	1.5	20	10	0.75P at 6	0.75P at 18	1.0P at 18	1330
	61-1	1.5	18	10	0.75P at 6	0.75P at 9	1.0P at 9	2880
4	58-1	7.5	18	30	1.0P at 12	1.0P at 18	1.0F at 18	1740
	58-2	6.0	18	26	1.0P at 12	1.0P at 18	1.0F at 18	1730
	58-3	4.5	18	20	1.0P at 12	1.0P at 18	1.0F at 18	2380
9	2a	3.0	16	10	1.0P at 8	1.0P at 48	C at 24	2030
	58-3	6.0	16	30	1.0P at 8	1.0P at 48	C at 24	970
	11	3.0	18	10	1.0P at 8	1.0P at 48	C at 24	1480
	1c	3.0	16	10	1.0P at 8	1.0P at 24	1.5P at 18	3230
	58-8	4.5	16	22	1.0P at 8	1.0P at 24	1.5P at 18	2910
	58-5	6.0	16	30	1.0P at 8	1.0P at 24	1.5P at 18	2670
	9	3.0	18	10	1.0P at 8	1.0P at 24	1.5P at 18	2080
	61-1	1.5	16	10	1.0P at 8	1.0P at 12	1.5P at 9	5930
	61-2	1.5	14	10	1.0P at 8	1.0P at 12	1.5P at 9	7170
10	4c	3.0	16	10	0.75F at 6	0.75F at 48	0.75F at 48	2000
	3d	3.0	16	10	0.75F at 6	1.0F at 24	1.0F at 24	3420
	10	3.0	18	10	0.75F at 6	1.0F at 24	1.0F at 24	2300

Legend: P = puddle weld; F = fillet weld; C = clinched seam connection.

along the seams and are welded at their ends to light longitudinal bent plates, termed fold line members in the sketch.

The design of the roof in the transverse direction does not differ from ordinary panel design for gravity loads. Only the normal component of the surface load need be considered. In the longitudinal direction, a girder is formed, with the upper and lower fold line members acting as compression and tension flanges, respectively, and the welded panels serving as the shear-carrying web. It may be evident that the membrane shear applied to the panels is of the same nature as that which was applied in the testing of horizontal diaphragms, although its intensity varies in this case from a maximum at the end frames to zero at midspan.

The decking contributes relatively little to longitudinal flexural strength. Therefore the longitudinal "flange" force may be approximately determined by dividing the in-plane moment by the slope depth of the plate. This force, divided by the member area, will give the uniformly distributed "flange" stress. At any "flange," the total stress is the algebraic sum of the stress produced by the in-plane bending of the two adjacent plates.

At the ends of the folded plate unit, a frame must be provided to receive the end reactions of the inclined plates. These in-plane end reactions may be considered to be uniformly distributed throughout the depth of each of the plates.

Several significant differences as compared with the behavior of concrete folded plates will be apparent. Transversely, the slab units are simple, not continuous, spans. As a consequence, beam strip moments and reactions are unchanged by small differences in deflection of the fold lines, and the corrective analysis for fold line deflections, which complicates the design calculations for concrete folded plates, is not required. In the longitudinal direction, for concrete roofs, the primary plate analysis may result in longitudinal stresses that differ on either side of a fold line. The resulting strain incompatibility causes shears along the fold line, acting equally and in the opposite direction on the two adjacent plates. These edge shears modify longitudinal stresses across the entire width of each plate. For the steel folded plate, the effect of such shear force is confined to the fold line member along that edge. The compatibility analysis associated with concrete folded plates is thus eliminated.

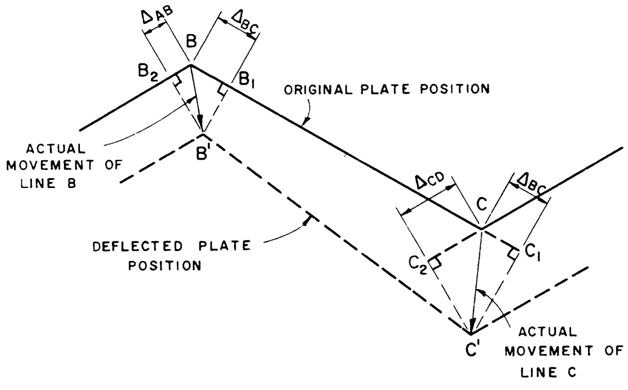


Fig. 8. Plate deflection diagram

Deflection of long span structures is always a concern. That of steel folded plates is easily predicted by a method which is similar to that used for determining truss deflection. The structure is temporarily separated at the fold lines, and the in-plane deflection of each plate is calculated separately. The fold lines are brought back into coincidence by means of a Williot displacement diagram, and the true movement of the fold line is determined graphically or analytically.

Figure 8 illustrates this procedure. The deflection Δ_{BC} is found as if that plate were independent of its neighbors; similarly, the deflections Δ_{AB} and Δ_{CD} are determined. To find the true position of the line **C**, plates **BC** and **CD** are rotated about their far ends until the points C_1 and C_2 coincide at **C′**. The location of **B′**, or any other fold line, is found in a similar manner. Movements being of a very small order of magnitude as compared with the slope width of the plates, it is satisfactory to consider that the rotational movement is perpendicular to the original position of the plate.

With regard to the calculation of the in-plane deflection, three contributing factors are considered:

(a) Flexural deflection
(b) Shear deflection
(c) Seam-slip deflection

The flexural deflection is calculated as for any beam. Consistent with the assumption made in flexural stress analysis, only the area of the fold line members need be considered in calculating moment of inertia. The plate

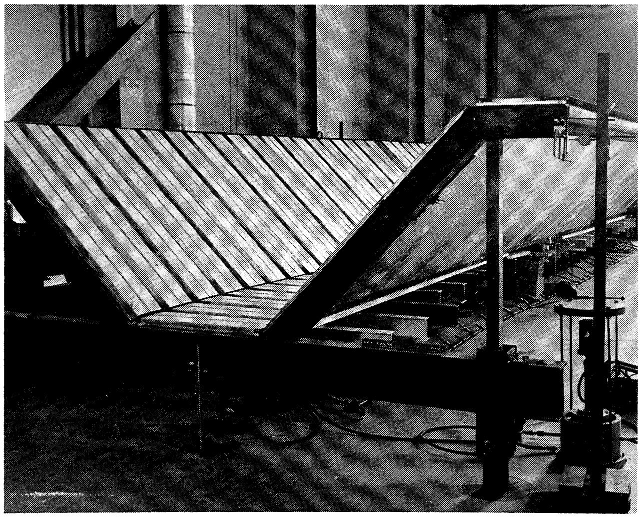


Fig. 9. Folded plate test

deflection due to shear is also found by conventional methods. The additional deflection due to relative movement between the individual panels consists of the summation between the support and mid-span of all individual seam-slip movements, which can be found by use of experimentally derived expressions.[1,3]

To demonstrate the validity of the above design proposals, a full scale prototype folded plate was built in the structural testing laboratory at Cornell (Fig. 9). The total span was 42.5 ft, and the width of the unit was 14 ft. A trapezoidal cross section was selected, using $1\frac{1}{2}$-in. deep, cellular steel panels of 0.06-in. thickness, and $3\frac{1}{2} \times \frac{1}{4} \times 3\frac{1}{2}$-in. bent plate fold line members. Typical diaphragm welding was used. It was convenient to test the structure inverted, with the loads applied by jacking upward against the two lower fold lines. The assembly was connected to rock-embedded floor anchors at each end frame.

The test structure performed in remarkably good agreement with the simple design theory proposed. The maximum load was 95 psf of horizontal projection, at which failure occurred in the puddle welds joining the plates to the end frames. Measured vertical and horizontal deflection of the fold line members almost exactly coincided with the predicted values. Flange stresses were about 20 percent lower than those predicted by simple theory, the difference undoubtedly being due to the longitudinal flexural contribution of the panel plate.

HYPERBOLIC PARABOLOIDS

A second shell form that is suitable for construction in steel is the hyperbolic paraboloid. The convenient property that this doubly curved surface contains two systems of straight line generators permits the use of standard steel panels oriented along either or both systems of generators. The individual panels, with high ratio of length to width and low torsional stiffness, can easily be warped to conform to the surface prior to welding the membrane.

Figure 10 shows a hyperbolic paraboloid surface, with points **B, O,** and **A** in one horizontal plane and point **C′** depressed by the amount *h* below that plane. Standard roof deck panels are used to provide the surface, bounded by steel structural members which need resist only

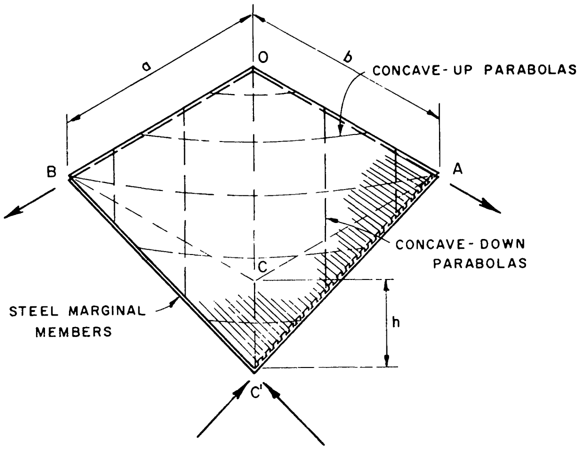


Fig. 10. Hyperbolic paraboloid

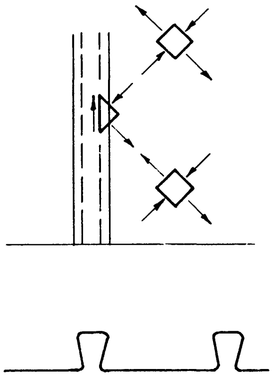


Fig. 11. Plan and elevation of shear membrane

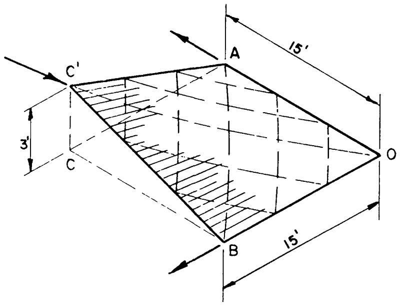


Fig. 12. Hyperbolic paraboloid test panel

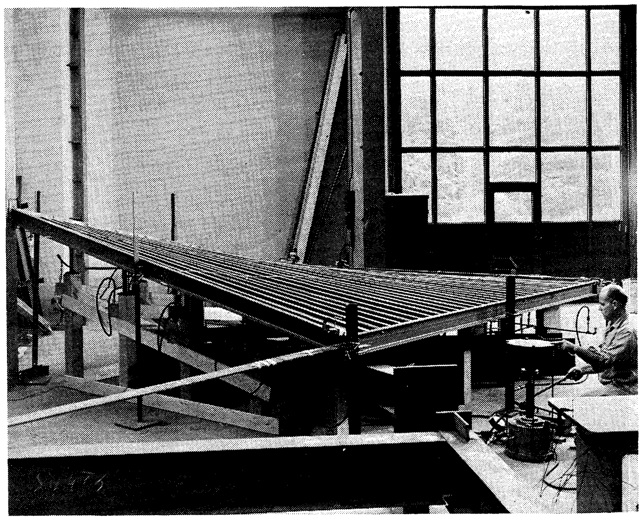


Fig. 13. Hyperbolic paraboloid test

direct tension or compression. A uniformly distributed downward load will require the external reactions shown for equilibrium.

Membrane analysis of this structural form[4] indicates the presence of principal tensile stress acting along the concave upward parabolic sections shown and equal principal compression in the perpendicular set of concave downward parabolas. If an open corrugated panel section is used, the question might well be raised as to how these tensile and compressive stresses are to be transmitted across the open ribs.

Figure 11 shows a plan and elevation of a portion of the membrane, with principal stresses acting on small elements of the steel sheet. A triangular element taken adjacent to the open rib illustrates that the components of principal tension and compression perpendicular to the rib cancel one another. The components parallel to the rib add, and the equilibrating shear acting on the element in the direction parallel to the rib is required. (Another way of looking at this situation is to observe that principal tension and equal perpendicular compression acting on any element constitute the equivalent of a state of pure shear acting on an element oriented at 45 degrees to the first. Consequently, all that the membrane needs to resist is pure shear applied in the direction parallel to the outside edges. This is precisely the type of loading which the tested horizontal shear diaphragms were found to resist successfully.)

Steel hyperbolic paraboloids may be constructed using a single layer of steel panels, oriented along either system of generators, or using a double layer with the layers spanning in perpendicular directions. The second alternative offers a substantial advantage in distributing load concentrations by bending resistance in two directions. While single-layer shells have proven capable of developing the necessary membrane state of stress (see below), rather severe distortion may result from nonuniform loading.

A prototype steel hyperbolic paraboloid was built in the laboratory at Cornell in late 1961.[5] The structure duplicated the quarter surface (Fig. 10) except that for testing convenience, it was inverted so that loads could be applied by jacking against the floor. The test structure (Figs. 12 and 13) measured 15 x 15 ft in plan, with a 3-ft rise. The panelling was a single layer of standard roof deck, 0.06-in. thick, with ribs 6 in. on centers and 1½ in. deep. The panels were welded to one another intermittently along the seams to provide the desired membrane shear resistance.

Six-inch channels were used for perimeter members. A 3 x ½-in. plate was welded at the mid-depth of the channels and was warped to conform to the local surface slope. Panels were then welded to the warped plate using standard puddle welds. (A second hyperbolic paraboloid was tested successfully using lighter tapered angles for perimeter members.)

The maximum load attained was 120 psf, corresponding to a membrane shear of 4,500 lb/ft. Yielding of the horizontal tie bars terminated the test. It is interesting to observe that the identical membrane system used in a horizontal test frame developed only 3,330 lb/ft membrane shear at failure. It may be that the curvature of the membrane restrained overall buckling of the surface, which would otherwise have led to a concentration of stress at the seam welds and a lower failure load. For the present time, it is apparent that experimental shear strengths of plane diaphragms can be used conservatively in the design of curved diaphragms with a strength reserve that may be a function of the principal curvatures.

As a check on the membrane state of stress in the shell, surface strain gages were mounted at four locations near the center of the surface. These were installed in pairs, mounted top and bottom of the steel sheet, so that an average value of strain at each location might be obtained which would cancel the effect of local distortions. Two pairs were oriented along the lines of principal compression, and two along lines of principal tension. Figure 14 shows the remarkably close agreement obtained between measured and computed values.

APPLICATIONS

A number of steel shell roofs have been built in the United States in the past few years. The first folded plate structure of this type was the Contra Costa County Library, built in California in 1961,[6] with a cross section

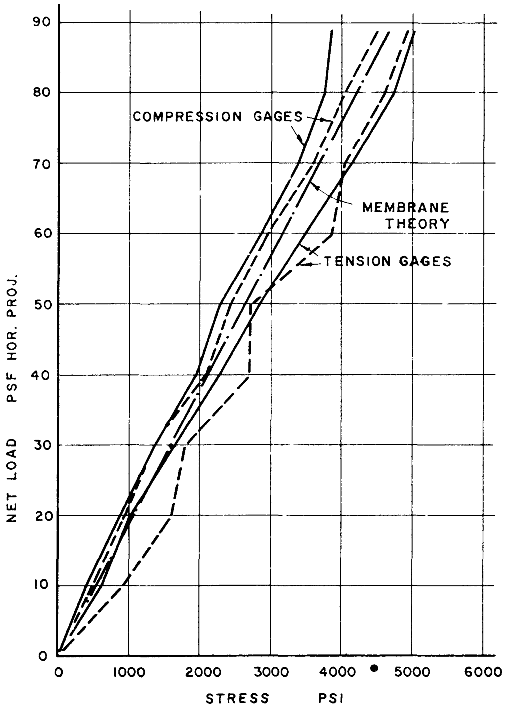


Fig. 14. Hyperbolic paraboloid membrane stress

nearly identical to that shown in Fig. 7. Some alternate sections are shown in Fig. 15. Figure 15a shows the simple sawtooth, while 15b is an unsymmetric variation of this. The trapezoidal shapes shown in Figs. 15c and 15d have been used, although it can be shown that the sawtooth is structurally more efficient and will require less welding. The main limitation in the selection of the cross section is that any two adjacent plates must not meet at too flat an angle; otherwise large distortions may occur.

In each case in Fig. 15, a vertical edge plate is shown. This normally will be required; otherwise the outside inclined plate must cantilever from the fold line at its interior edge. While short cantilevers can be used (see Fig. 9), ordinarily this is not advisable, and edge support is preferred. As an alternative to the use of a vertical edge plate, light longitudinal framing with intermittent columns can be provided, or a continuous wall may be used. In either case, only vertical loads need be carried. Any outward movement of the lower edge of the exterior plate must be accompanied by an in-plane component of deflection for that plate. Its considerable flexural resistance therefore resists the outward displacement.

Several recent examples of steel folded plate construction are shown in Figs. 16, 17, and 18. The Fort Crook School, Omaha, is shown under construction in Fig. 16. The roof spans 78 ft between supports, and was designed for a 30 psf live load. Figure 17 is an interior view of Northeast Junior College, Sterling, Colo. The undersurface of alternate panels has been perforated to improve acoustic properties. Figure 18 shows a long span folded plate roof, the Sullivan School, Sullivan, Mo.

An architect's sketch of what might be termed a pleated dome is shown in Fig. 19. Tapered radial folded plate elements converge to a column-supported central ring. A similar roof has been constructed for the Krueger Junior High School, Michigan City, Ind., in which no central supports were used. The folded plates spanned between a high level compression ring at the center of the building and a tension ring at the perimeter. The Michigan City dome was 80 ft in diameter, and used standard cellular steel panels.

The hyperbolic paraboloid has not yet found wide application, although several striking examples may be cited. Figure 20 shows the Johnson and Hardin Printing Plant in Cincinnati. The entire roof, measuring 107 x 133 ft, is supported by only four columns. The doubly-curved shell is formed of two layers of 1½-in. roof deck panels, mutually perpendicular, with the lower and upper panels of 18 gage and 20 gage steel respectively. A second doubly-curved shell, shown in Fig. 21, also used a double-layer steel deck membrane.[7]

An impressive fact about light gage steel shell structures is the high ratio of design live load to dead load. For the folded plate roof unit tested at Cornell, this ratio was 4.65 to 1. As a measure of the stiffness of

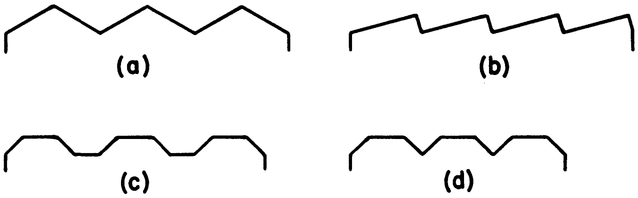
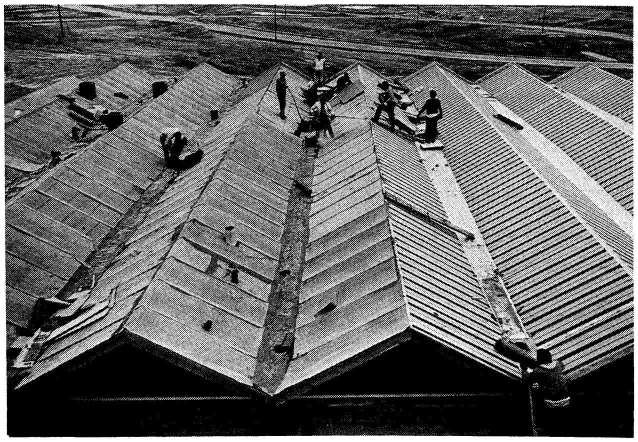


Fig. 15. Folded plate cross sections



Courtesy of Fenestra, Inc.

Fig. 16. Construction of Fort Crook School, Omaha

Courtesy of Fenestra, Inc.

Fig. 17. Northeast Jr. College, Sterling, Colorado

this very light structure, the midspan vertical deflection at 50 percent of the failure load was only 0.45 in. or 1/1,130 of the span. Data for the hyperbolic paraboloid are equally impressive. For the test shell, the ratio of design live load to dead load was 5.23 to 1. Full live load deflection of the outer corner of the 15 ft. square shell segment was less than 0.50 in.

Experience with the structural welding of light gage steel panels shows the desirability of special welder training and careful inspection. Welding using too much heat, or done too slowly, will burn the thin steel sheet around the weld, resulting in a defective connection. On the other hand, too little heat will produce improper fusion, as it is necessary to burn through one panel locally to weld to the one beneath. Recommendations as to welding rod type and size, current, and amperage have been carefully developed. If they are followed exactly, no difficulty should be experienced.

While the structural efficiency of steel shell roofs has been demonstrated, cost economy does not necessarily follow. Spectacular structures may be built almost regardless of expense because of architectural advantages or publicity factors, but, in the end, a new constructional form will achieve wide use only by reason of its basic low cost. What little data is now at hand relative to the cost of light gage steel shells indicates that such structures may prove competitive with more conventional construction. Unit costs for the pleated dome structure described, a highly unusual installation, were still within the range common for schools in the area. Prismatic folded plates, in particular, have been shown to be a highly economical means for achieving long spans, as well as architecturally appealing structures. As builders gain experience with construction techniques, including prefabrication of large components, there is every reason to expect that costs may be further reduced.

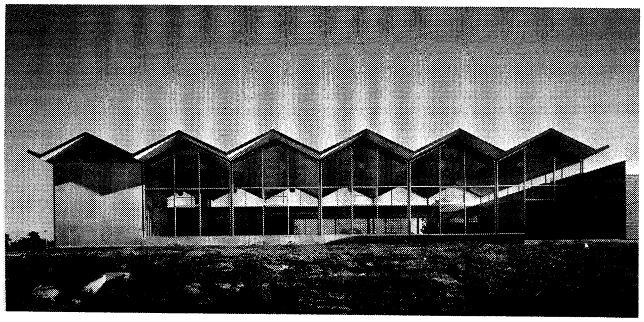


Fig. 18. Sullivan School, Sullivan, Missouri

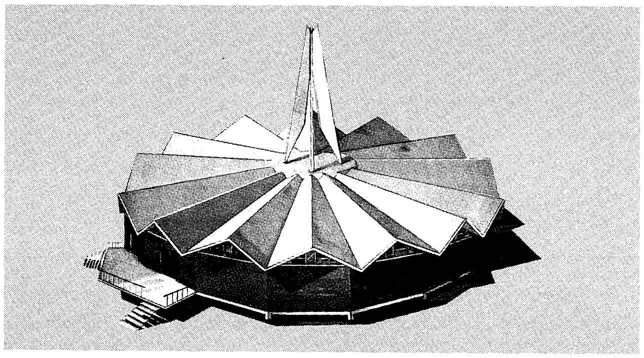
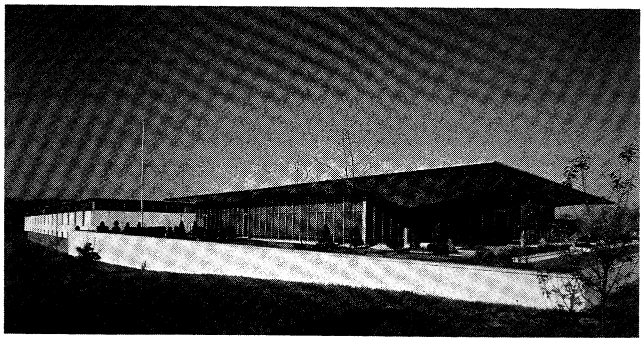


Fig. 19. Light gage steel pleated dome



Courtesy of Truman P. Young and Associates, Engineers

Fig. 20. Johnson and Hardin Printing Plant, Cincinnati

Courtesy of Truman P. Young and Associates, Engineers

Fig. 21. Frisch's Restaurant, Cincinnati

REFERENCES

1. *Nilson, A. H.* Shear Diaphragms of Light Gage Steel, *Journal of the Structural Division, Vol. 86 ST11, ASCE, Nov., 1960.*
2. *Luttrell, L. D.* Structural Performance of Light Gage Steel Diaphragms, *Cornell University Research Report for AISI, Aug., 1965.*
3. *Nilson, A. H.* Folded Plate Structures of Light Gage Steel, *Journal of the Structural Division, Vol. 87, ST7, ASCE, Oct., 1961.*
4. *Flügge, W.* Stresses in Shells, *Springer-Verlag, 1962.*
5. *Nilson, A. H.* Testing a Light Gage Steel Hyperbolic Paraboloid Shell, *Journal of the Structural Division, Vol. 88 ST5, Oct., 1962.*
6. *Shapiro, D.* Loads Test Steel Folded-Plate Roof, *Engineering News-Record, July 27, 1961.*
7. *Graham, H. T.* New Way to Build a Shell, *Architectural Record, March, 1962.*