Combined Shear and Tension Stresses

SUBHASH C. GOEL

The current AISC Specification calls for allowable stress in tension, $F_t = 0.6F_y$, and that in shear, $F_v = 0.4F_y$, where F_y denotes the yield strength of the material. Checking the stresses in tension and shear separately against the corresponding allowable values in the base metal is generally considered adequate enough. This practice can be defended rationally in situations where maximum normal and shear stresses do not occur at the same point, such as in the design of bending members (except for plate girder webs where tension field action is utilized).

Quite frequently, however, situations are encountered where maximum normal and shear stresses occur simultaneously, such as in members subjected to combined bending and torsion or in connection design. Checking the section at the base of gusset plate for an inclined bracing member (Fig. 1) is a common example. Such situations are not covered in the current AISC Specification. Keeping the normal and shear stresses within their individual allowable limits in these cases may lead to grossly unconservative results. A rational design criterion for these situations should be based on combining the two stresses in some reasonable manner.

This paper examines the problem briefly from a theoretical viewpoint. A simple empirical equation is proposed for use in practical design. The proposed equation gives results which are in close agreement with those obtained by using Von Mises' yield criterion.

MAXIMUM PRINCIPAL STRESS CRITERION

The use of maximum principal stress theory can be justified in design situations where the failure mode is governed by brittle fracture, such as in fatigue loading. If this theory is used in design when yielding governs the failure mode and the maximum principal stress is kept at F_t (as is sometimes suggested), it could lead to unconservative results. In order to illustrate this point, let f_t and f_v denote the normal tension and shear stresses, respectively, at a point under consideration (Fig. 2). Equating the maximum principal stress to F_t gives,

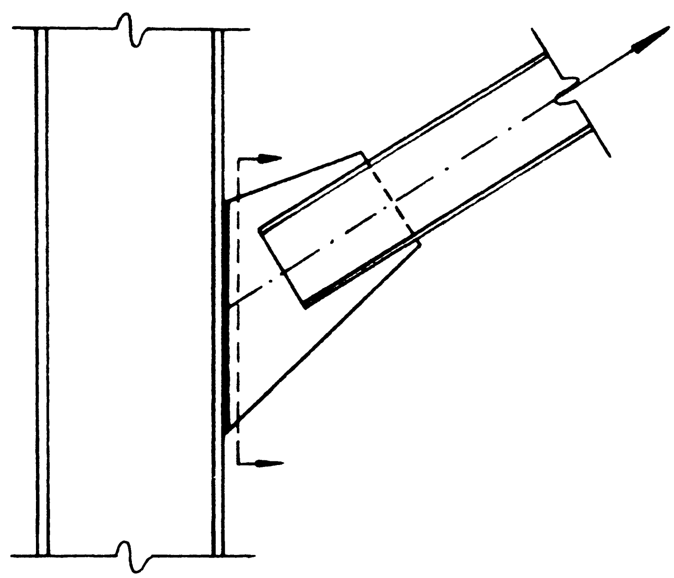

Fig. 1. Example

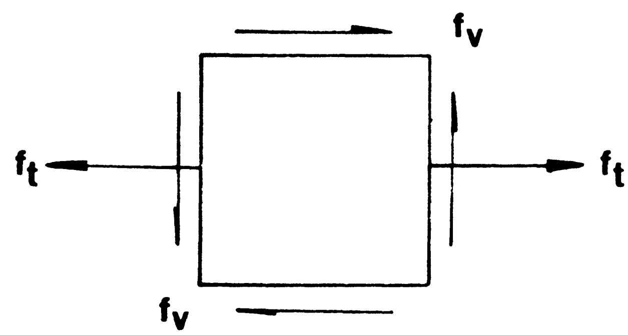

Fig. 2. Combined shear and tension stress

Subhash C. Goel is professor of Civil Engineering, the University of Michigan, Ann Arbor, Michigan.

$$\frac{f_t}{2} + \sqrt{\left(\frac{f_t}{2}\right)^2 + f_v^2} = F_t$$

or
$$\frac{1}{2}\left[\frac{f_t}{F_t} + \sqrt{\left(\frac{f_t}{F_t}\right)^2 + \frac{16}{9}\left(\frac{f_v}{F_v}\right)^2}\right] = 1 \qquad (1)$$

When Eq. 1 is plotted on $f_t/F_t - f_v/F_v$ axes, it can be seen from Fig. 3 that increasingly unconservative results are obtained as the tension stress becomes smaller. In the limit when $f_t = 0$, one gets $f_v/F_v = 1.5$. This would not be acceptable.

VON MISES' YIELD CRITERION

Von Mises' yield criterion is quite commonly used in design when yielding governs the strength, such as under normal load conditions. Application of this criterion to the combined stress condition under consideration herein (Fig. 2) leads to the following equation:

$$k^2(f_t^2 + 3f_v^2) = F_y^2 \qquad (2)$$

where k represents the factor of safety. Using $k = 1.67$, Eq. 2 can be transformed into the following:

$$\left(\frac{f_t}{F_t}\right)^2 + \frac{4}{3}\left(\frac{f_v}{F_v}\right)^2 = 1 \qquad (3)$$

A plot of Eq. 3 is shown in Fig. 3. It can be seen that the use of this equation gives somewhat conservative results for smaller values of tension stress. For $f_t = 0$, the equation would allow $f_v = 0.866F_v$, which may be considered a bit conservative.

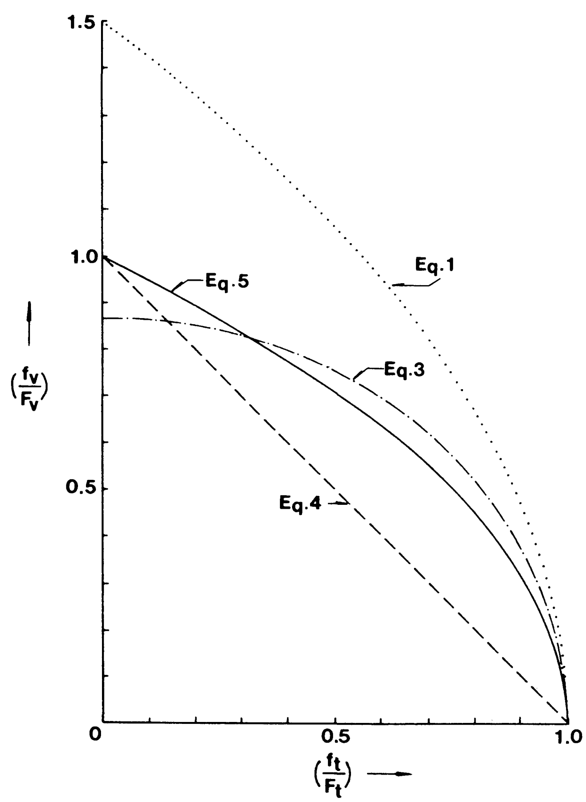

Fig. 3. Interaction curves

EMPIRICAL EQUATIONS

Equation 3, which is based on Von Mises' yield criterion, is simple enough for use in practical design. However, as noted above it gives somewhat conservative results in the region of small values of tension stress. It would be desirable to use an equation which incorporates factors of safety consistent with those in the current Specification over the entire range of application. As a first attempt linear interaction equation of the following form may be considered:

$$\frac{f_t}{F_t} + \frac{f_v}{F_v} = 1 \qquad (4)$$

A plot of this equation, shown by dashed line in Fig. 3, indicates that it may be overly conservative for the most part when compared with Eq. 3 which was derived from Von Mises' yield criterion.

Now consider the following empirical equation:

$$\frac{f_t}{F_t} + \left(\frac{f_v}{F_v}\right)^2 = 1 \qquad (5)$$

When this equation is plotted in Fig. 3 it is noticed that the results are quite close to those from Eq. 3 and the curve leads to $f_v/F_v = 1.0$ and $f_t/F_t = 1.0$ at the two ends. In addition, Eq. 5 is simple enough for practical design application.

CONCLUSION

An empirical equation of the form,

$$\frac{f_t}{F_t} + \left(\frac{f_v}{F_v}\right)^2 = 1$$

is proposed for design under combined direct tension and shear stresses. This equation has desired simplicity, and gives results which are close to those obtained by using the Von Mises' yield criterion and factors of safety consistent with the ones used in current AISC Specification.

It would be desirable to validate the above equation with some experimental results.