A Programmable Solution for Stepped Crane Columns

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The basic equations for the solution of a stepped column with a fixed base are easily derived, but the pinned top equations are the only forms found in generally available literature such as the AISC book on Industrial Buildings.[1] This paper presents the 10 basic equations required for a general solution of the stepped crane column and describes the use of these equations with the generally accepted Murray-Graham model of crane column action.[2]

The 10 equations are applicable to both bayonet and double-shaft crane columns and by setting I_2 equal to I_1 (see Fig. 1) may be applied to single-shaft columns with a bracketed crane girder. The equations also allow the designer to locate the crane thrust at the proper location instead of at the column step, a refinement which complicates needlessly the solution for manual calculation but which presents no problem to the computer.

The reader may use these equations, Anderson and Woodward's iterative solution for the effective length of stepped columns[3] and the procedures recommended in AISE Technical Report No. 13[4] to program a complete crane column solution. The author's program solves AISC Equations 1.6-1a & 1b or 1.6-2 using procedures and load combinations specified by the AISE, allowing a rapid selection of an economical combination of wide-flange sections.

The assumptions for the Murray-Graham stepped column model are:

Base: Fixed

Top: A. *No wind:* A pin is assumed midway between the top and bottom chords or midway between the top chord and knee brace, permitting rotation but no translation (see Fig. 1).

B. *With wind:* A slider is assumed at the bottom chord or at the knee brace, permitting translation but no rotation (see Fig. 2). M_0 is the moment required to maintain zero rotation at the top of the column. R_o is the force required to restore the deflection at the top of the column to a given fraction R of the deflection for R_o=0. For a bent with two identical columns, the fraction R is obviously 0.5, but when two or more bays share the lateral loads the fraction is not so obvious. If l_1, l_2 and l_3 (see Fig. 3) were the same for all columns:

$$R = \frac{I \text{ of column being designed}}{\Sigma I \text{ of all columns in bent}}$$

Since the I of an interior column is likely to be more than double that of an exterior column,

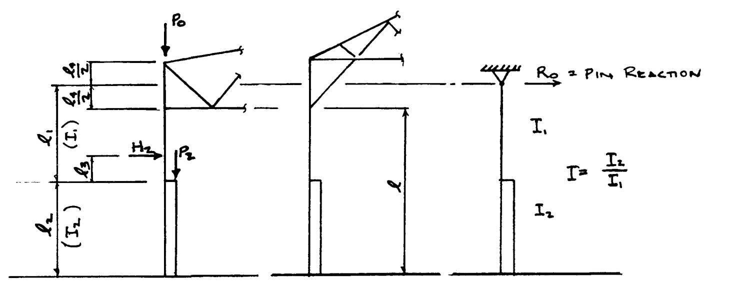

Fig. 1. Column model without wind

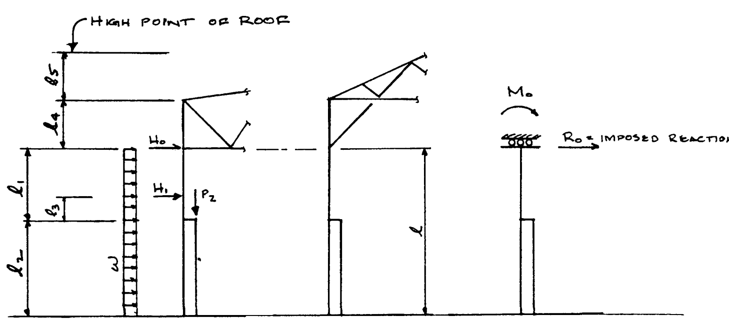

Fig. 2. Column model with wind

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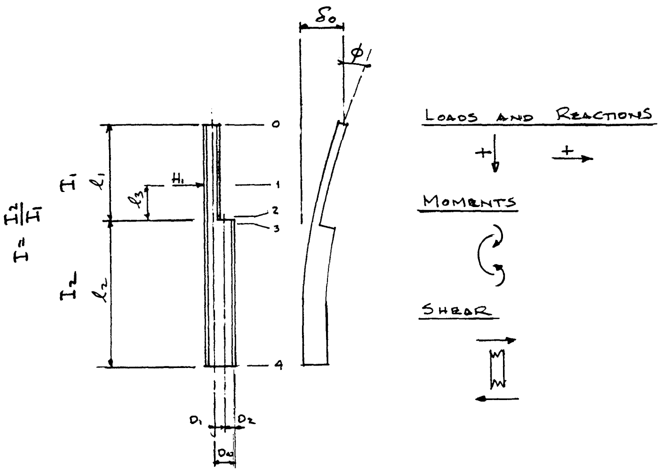


Fig. 3. Column geometry and sign convention

the use of R of 0.3 for two identical bays and 0.2 for more than two bays seems reasonable. Exact values of R may be determined after the initial member selection, but such over refinement is of dubious value in the opinion of the author.

The basic equations given below are for the deflection and rotation at the unrestrained top of the column for each of five loading conditions:

1. Horizontal load or reaction at top of column
2. Moment reaction at top of column
3. Crane thrust at any location on top shaft
4. Moment at step due to difference between roof and crane loads
5. Uniform wind load

The development of the 10 basic equations is routine by any classical method. Superposition of cantilever formulas as described in the Appendix was the method used by the author.

BASIC EQUATIONS

1. $EI_2\delta_{0H0} = H_0E_1 \qquad E_1 = \dfrac{I\ell_1^3}{3} + \ell_1^2\ell_2 + \ell_1\ell_2^2 + \dfrac{\ell_2^3}{3} \qquad (1)$

2. $EI_2\delta_{0H1} = H_1E_2 \qquad E_2 = \ell_2\left(\dfrac{\ell_2^2}{3} + \dfrac{\ell_2\ell_1}{2} + \dfrac{\ell_2\ell_3}{2} + \ell_1\ell_3\right) + I\ell_3^2\left(\dfrac{\ell_3}{3} + \dfrac{\ell_1 - \ell_3}{2}\right) \qquad (2)$

3. $EI_2\delta_{0W} = WE_3 \qquad E_3 = \dfrac{I\ell_1^4}{8} + \dfrac{\ell_1^3\ell_2}{2} + \dfrac{3\ell_1^2\ell_2^2}{4} + \dfrac{\ell_1\ell_2^3}{2} + \dfrac{\ell_2^4}{8} \qquad (3)$

4. $EI_2\delta_{0M2} = M_2E_4 \qquad E_4 = \ell_1\ell_2 + \dfrac{\ell_2^2}{2} \qquad (4)$

5. $EI_2\delta_{0M0} = M_OE_5 \qquad E_5 = \dfrac{I\ell_1^2}{2} + \ell_1\ell_2 + \dfrac{\ell_2^2}{2} = E_6 \qquad (5)$

6. $EI_2\phi_{0H0} = H_0E_6 \qquad E_6 = \dfrac{I\ell_1^2}{2} + \ell_1\ell_2 + \dfrac{\ell_2^2}{2} = E_5 \qquad (6)$

7. $EI_2\phi_{0H1} = H_1E_7 \qquad E_7 = \dfrac{\ell_2^2}{2} + \ell_2\ell_3 + \dfrac{I\ell_3^2}{2} \qquad (7)$

8. $EI_2\phi_{0W} = WE_8 \qquad E_8 = \dfrac{I\ell_1^3}{6} + \dfrac{\ell^2\ell_2}{2} + \dfrac{\ell_1\ell_2^2}{2} + \dfrac{\ell_2^3}{6} \qquad (8)$

9. $EI_2\phi_{0M2} = M_2E_9 \qquad E_9 = \ell_2 \qquad (9)$

10. $EI_2\phi_{0M0} = M_0E_0 \qquad E_0 = I\ell_1 + \ell_2 \qquad (10)$

PROGRAM OUTLINE

1. Enter column geometry (see Figs. 2 and 3)
   $\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, D_0$
2. Enter column loads (see Figs. 1 and 3)
   H_1, P_0, P_2, W
   To compute AISE load combinations, provide separate variables for:
   a. Live- and dead-load components of P_0
   b. Multiple and single crane values for P_2 and H_1
3. Enter wind deflection reduction ratio R.
4. Enter or read properties of trial sections
5. Enter or compute D_1, D_2, I_2 (see Fig. 3)
6. Compute $H_0 = W\ (\ell_4 + \ell_5)$ (see Fig. 2)
   $M_2 = P_2D_2 - P_0D_1$ (see Figs. 1 and 3)
7. A. No wind (see Figs. 1 and 3)
   a. Compute $\ell_1 = \ell + \dfrac{\ell_4}{2} - \ell_2$
   b. Compute E_1, E_2 and E_4 using Eqs. 1, 2 and 4.
   c. Set $EI_2\delta = 0$ and compute

$$R_0 = -\frac{H_1E_2 + M_2E_4}{E_1} \qquad (11)$$

   d. Compute loads and moments at Sects. 1, 2, 3, 4.
   e. Solve AISC Eqs. 1.6 − 1a & 1b or 1.6-2

   B. With wind (see Figs. 2 and 3)
   a. Compute $\ell_1 = \ell - \ell_2$
   b. Compute, E_1 through E_0 using Eqs. 1 to 10.
   c. Setting $EI_2\phi_0 = 0$

$$M_0 = -\left[\frac{(R_0 + H_0)E_6 + H_1E_7 + WE_8 + M_2E_9}{E_0}\right] \qquad (12)$$

$$EI_2\Delta_0 = \mathrm{H}_0E_1 + H_1E_2 + WE_3 + M_2E_4 + M_0E_5 \qquad (13)$$

d. Assuming $R_0 = 0$ solve Eq. 12 for M_0 and Eq. 13 for $EI_2\ \Delta_0$

e. Find R_0 required for final deflection of $REI_2\Delta_0$

$$R_0 = \frac{REI_2\Delta_0 - H_0E_1 - H_1E_2 - WE_3 - M_2E_4}{\left[E_1 - \dfrac{E_5E_6}{E_0}\right]} + \frac{\left[H_0E_6 + H_1E_7 + WE_8 + M_2E_9\right]\left[\dfrac{E_5}{E_0}\right]}{\left[E_1 - \dfrac{E_5E_6}{E_0}\right]} \qquad (14)$$

f. Solve Eq. 12 for final M_0

g. Compute loads and moments at critical sections 1, 2, 3 & 4 (see Fig. 3)

h. Compute the K-factors for top and bottom shafts in accordance with Ref. 3.

i. Compute AISC Eqs. 1.6-1a & 1b or 1.6-2 for selected load combinations (AISE Procedures, Ref. 4, recommended).

j. If trial sections are incorrect, return to Step 4; else print results.

NOMENCLATURE

Loads

H_0 = Concentrated horizontal load at top of column, kips
H_1 = Crane thrust, kips
M_2 = Moment at step, kip–in.
= (P_2) (crane eccentricity) − (P_0) (roof col. eccentricity)
P_0 = Roof load
P_2 = Crane load
W = Wind load, kips/in.

Reactions

M_0 = Restoring moment at top of column, kip-in.
R_0 = Restoring horizontal reaction at top of column, kips

Deflections

δ_0 = Deflection at top of column, in.
δ_{0H0} δ_{0H1}, δ_{0M2}, etc = deflection due to subscripted load, in.
$\Delta_0 = \Sigma\delta_0$ with $R_0 = 0$, in.
ϕ_0 = Rotation at top of column, rad
ϕ_{0H0}, ϕ_{0H1}, ϕ_{0M2} etc = rotation caused by subscripted load

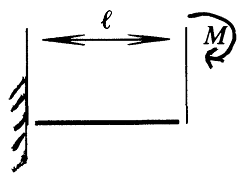
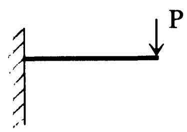
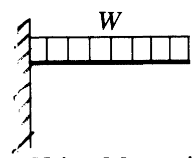
Properties

I_1 = Moment of inertia of top shaft, in^4
I_2 = Moment of inertia of bottom shaft, in^4
$I = \dfrac{I_2}{I_1}$

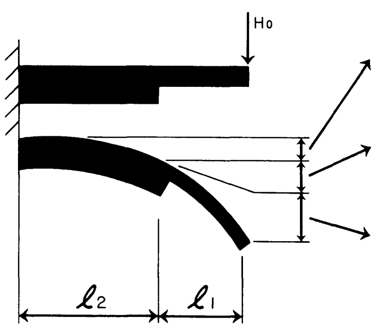
APPENDIX: DERIVATION OF EQUATIONS

While the basic equations can be derived by several methods, the superposition of cantilever beam deflections and rotations permits the equations to be written directly, without intermediate calculations. The method is described in Prof. J. P. Den Hartog's *Strength of Materials*[5] as "The Myosotis Method." I know of no other text where this useful alternate to area-moment methods is described.

The cantilever beam equations are:

	ϕ	δ
	$\dfrac{M\ell}{EI}$	$\dfrac{M\ell^2}{2EI}$
	$\dfrac{P\ell^2}{2EI}$	$\dfrac{P\ell^3}{3EI}$
	$\dfrac{W\ell^3}{6EI}$	$\dfrac{W\ell^4}{8EI}$

Using Myosotis to write eq. 1:



$$\left[\frac{(H_0\ell_1)\ell_2^2}{2EI_2} + \frac{H_0\ell_2^3}{3EI_2}\right]$$

$$\left[\frac{(H_0\ell_1)\ell_2}{EI_2} + \frac{H_0\ell_2^2}{2EI_2}\right]\ell_1$$

$$\left[\frac{H_0\ell_1^3}{3EI_1}\right]$$

Which when summed, cleared and I substituted for $\dfrac{I_2}{I_1}$ yields Eq. 1.

The general case of a fixed base column with unknown translational and rotational restraint at the top has four unknowns

(1) R_0 (2) M_0 (3) δ_0 (4) ϕ_o

And two conditions of equilibrium

(1) $\Sigma F_x = 0$ (2) $\Sigma M = 0$

The Murray-Graham wind model adds two boundary conditions

(1) $\Sigma\delta_0$ = a known amount (2) $\Sigma\phi_0 = 0$

Equations 1 to 10 are individual statements of equilibrium which may be superimposed as:

Eq. 15: $EI_2\delta_0 = (H_0 + R_0)E_1 + H_1E_2 + WE_3 + M_2E_4 + M_0E_5$

Eq. 16: $EI_2\phi_0 = (H_0 + R_0)E_6 + H_1E_7 + WE_8 + M_2E_9 + M_0E_0$

Equation 14 is the solution for R_0 of Eqs. 15 and 16 with $\phi = 0$ and $EI_2\delta_0 = REI_2\Delta_0$.

Equation 13 is not one of the simultaneous equations for solution, but is an intermediate step which forms the basis for the selection of the "known amount" of deflection, $REI_2\Delta_0$. □

REFERENCES

1. *Fisher, James M. and Donald R. Buettner* Light and Heavy Industrial Buildings *AISC, 1979, Chicago, Ill.*
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5. *Den Hartog, J. P.* Strength of Materials *McGraw-Hill, 1949, New York, N.Y.*