Stepped Columns: A Simplified Design Method

CARLO A. CASTIGLIONI

Summary

A simple method is offered for the design of stepped columns which presents, with respect to the classical effective length method, some advantages (mainly swiftness and precision), when designing members in compression and bending.

The method is based on a simplified model with two degrees of freedom. It is possible to obtain the ultimate interaction domains for stepped members, taking into account the effects of both geometrical and mechanical imperfections and of the loading path.

Some of these domains are presented, and compared with available numerical results.

The problem of how to determine the ultimate load-carrying capacity of stepped-steel columns has been extensively—even if not exhaustively—treated in literature. Only limited research has, however, been carried out on the behavior of these structural elements when taking into account both the non-linearity of the constitutive law of the material and the geometrical non-linearity.

Most of the preceding studies[1-6] dealt with the problem of determining the elastic critical load of axially compressed members, with various conditions of end restraints and loading. The only attempt, to the author's knowledge, to determine the load-carrying capacity in the elasto-plastic range for a stepped column is a work of Barnes and Mangelsdorf.[7] That paper, however, considers only axial effects and disregards compression and bending, which is the most frequently occurring stress state for these members.

It may be concluded that the only aspect to be investigated so far is related to the elastic behavior of stepped columns, and when determining the ultimate load carrying capacity of such elements, reference is usually made to the effective length parameter.

Carlo A. Castiglioni is Assistant Professor, Structural Engineering Department, Politecnico di Milano, Milano, Italy.

Design practice[8-11] reflects the theoretical state of the research. The tendency is to design stepped members carrying out separate checks for the two shafts, by using the effective length method and the axial-thrust bending-moment interaction formulas which are valid for members with uniform cross section.

With reference to AISE Recommendations,[8] such formulas can be written as:

$$P/P_{c,n} + C_m M/[M_p\,(1-P/P_E)] \leq 1 \qquad (1)$$

where

- P is the total axial thrust in the shaft (upper or lower)
- M is the maximum first order bending moment
- C_m is a reduction coefficient ≤ 1 which is a function of the bending moment's distribution
- M_p is the fully plastic bending moment of the profile
- $P_{c,n}$ and P_E are respectively the ultimate and the critical elastic loads, calculated on the base of the effective slenderness ratio of the shaft under consideration.

Several general and specific critical considerations may be developed about this kind of approach; in particular it should be noted:

- the effective length is derived from the critical multiplier of the axial loads acting on the column, and is linked to a prefixed value of the ratio of these loads. The effective length is therefore different for different load combinations. The methods based on calculating the effective length of structural members lose (at least in part) their advantage of being easy and quick to apply when a number of different load combinations must be taken into account.
- design methods based on the concept of effective length do not work well for the interaction between column segments. This requires a series of separate checks.

On the basis of the preceding critical considerations, this author, following an approach developed by previous international research on the behavior and stability of members with uniform cross section and axial load, has performed a numerical study.[12,13] This study follows step-by-step the response of a stepped member (affected by both geometrical and mechanical imperfections) during a number of different loading paths, up to the attainment of the collapse situation.

Ultimate interaction domains, for the elements considered in the study were numerically obtained in terms of the two vertical loads P_1 and P_2 (respectively applied on the upper and lower shaft) and compared with those deductible on the basis of design methods based on the effective length and Formula 1. It was pointed out that:

- The shapes of the interaction domains obtained numerically are very similar to those obtained using the effective length concept.
- For simple compression members, there is a close agreement between numerical results and those obtained by the effective length method, which enables a fundamentally correct evaluation of the ultimate load-carrying capacity for stepped columns.
- For members in compression and bending:
  a. With a method based on the concept of effective length (which is implicitly linked with the concept of instability of equilibrium as a bifurcation problem), it is possible to understand correctly which situation is associated with the collapse of the structural element. It is not possible to appreciate the effect of geometrical imperfections on the behavior of the member or on the shape of its ultimate interaction domain. (The author pointed out[13] this effect is relevant and different in the two shafts).
  b. The method based on effective length tends to always be on the safe side when the collapse situation is reached in the lower shaft (the situation of greatest practical interest), while it tends to be on the unsafe side when the collapse occurs in the upper shaft.
- The safety factor assumed, using a method based on the effective length concept, is not homogeneous and is a function of the vertical load ratio.

The knowledge of the ultimate interaction domains has the advantage of allowing the safety margin associated with the various load combinations (which can occur during the life of the structure) to be appreciated in global terms in design checks. If reference is made to these ultimate domains, methods based on axial-thrust bending-moment interaction formulas, such as (1), are decidedly complex from the computational point of view, since for every load combination they require:

- the calculation of the effective length
- the solution of the interaction formula with regard to the axial load

Furthermore, to obtain a better precision in the solution, the reduction coefficient C_m should also be defined for the different values of the ratio between the applied loads.

A simple approach was proposed[12,13] based on the use of an interaction formula directly written in terms of the applied vertical loads.

$$\left(\frac{P_1}{P_{1C,M}}\right)^{\beta} + \left(\frac{P_2}{P_{2C,M}}\right)^{\beta} = 1 \tag{2}$$

In Formula 2, $P_{1C,M}$ and $P_{2C,M}$ are the maximum values of the loads P_1 and P_2 sustainable by the column in the presence of a single vertical load; $P_{1C,M}$ and $P_{2C,M}$ implicitly take into account the possible transverse loads acting on the column, and can be defined making reference to the two situations shown in Figs. 1b and 1c.

The use of such formulas requires the definition of the value of exponent β and the availability of a sufficiently simple method for determining the loads $P_{1C,M}$ and $P_{2C,M}$; it has the implicit advantage over using Formula 1 (i.e. determining the coefficient C_m) only for calculating $P_{1C,M}$ and $P_{2C,M}$, i.e., when one of the two vertical loads is absent.

In the case of members with uniform cross section, it was shown in a preceding paper[12] that it is possible, with an acceptable degree of approximation, to adopt $\beta = 1.0$ for elements subjected to centric vertical loads and $\beta = 0.9$ for elements subjected to eccentric vertical loads. In the same

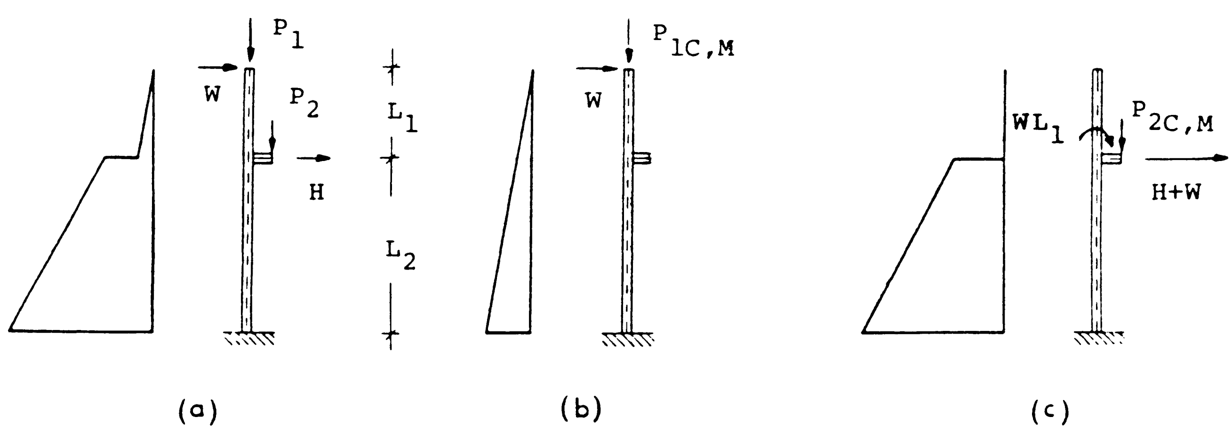

Fig. 1. Loading conditions associated with calculation of $P_{1C,M}$ and $P_{2C,M}$

paper however, it was pointed out the obvious influence of coefficient C_m on the intersection of the domain with the load axis (i.e., on the values of $P_{1C,M}$ and $P_{2C,M}$).

It has been tried, but it has not yet been possible to extend the same approach to columns with variable cross section, because coefficient β has a very wide range of variation, and it is influenced by too many parameters. Some research is still going on, trying to determine the values of β to be used in Formula 2, in the case of stepped elements.

From preceding studies,[12,13] it has been noted that the collapse of a stepped member is mainly associated with two different and non-correlated situations: the collapse of the upper shaft or the collapse of the lower shaft. In both cases the collapse situation is reached in the most stressed section of the shaft. The collapse situation of these elements seems to be caused more by local buckling in a well defined area of one of the shafts (the most stressed cross section), rather than by global instability of the whole member.

It is possible to predict where in the shaft, but is not possible to know a-priori in which one of the two shafts the collapse will occur, this last fact depends on the loading conditions.

Starting from these considerations, this paper simulates the behavior of stepped columns with a simple model with two degrees of freedom. The deformability of the element is concentrated in the two most stressed cross sections, and the interaction between the two shafts is disregarded.

THE MODEL

The Equilibrium Equations

If the column is considered as simply cantilevered at its lower edge (a simplifying and conservative scheme when dealing with mill building columns, because the rotational restraint effect of the roof structure is ignored), the most stressed section of each shaft is its lower section. The ultimate load carrying capacity of the stepped element can be determined using the simple model shown in Fig. 2. (It is assumed the presence of adequate bracings preventing the out-of-plane buckling of the column.)

The model consists of two rigid bars and two cells in which the deformability has been concentrated. The upper shaft has a length L_1, a cross sectional area A_1 and a moment of inertia (with respect to the center of gravity) I_1. The lower shaft has a length L_2, a cross sectional area A_2 and a moment of inertia (with respect to the center of gravity) I_2. The two shafts are connected together taking into account an eccentricity e_{12} between them.

Two vertical loads P_1 and P_2 are applied with an eccentricity e_1 and e_2 respectively at the top of each shaft, together with two horizontal forces F_1 and F_2. In addition, a horizontal force H, proportional by a constant coefficient ξ to the vertical load P_2 may be present at the top of the lower shaft: $H = \xi P_2$.

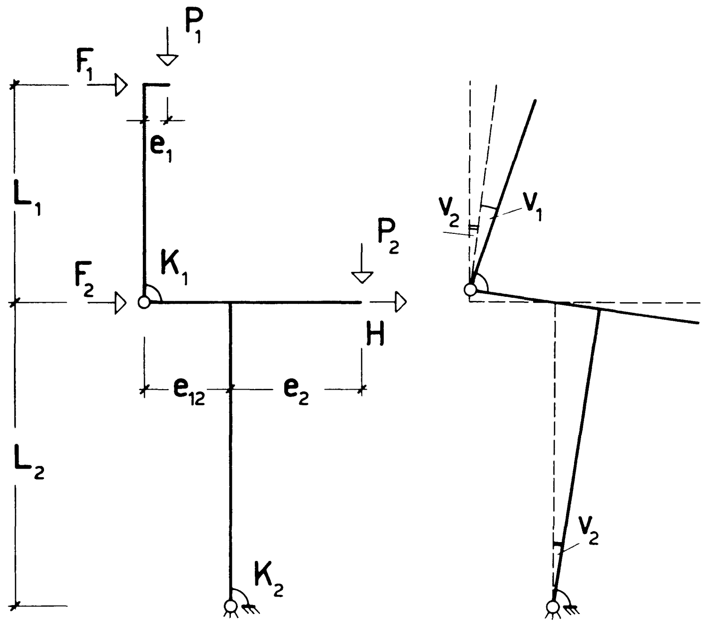

Fig. 2. The model

The two degrees of freedom of the model may be identified with the relative rotation v_1 between the upper and the lower shaft, and with the absolute rotation v_2 of the lower shaft with respect to the vertical axis. Initial geometrical imperfections $f_{01} = v_{01} L_1$ and $f_{02} = v_{02} L_2$ have been assumed at the top of the upper and of the lower shaft respectively, v_{01} and v_{02} being the initial values of v_1 and v_2 respectively.

The equilibrium conditions for the model in a displaced configuration characterized by two rotations v_1 and v_2 can be derived by equating in each cell the internal bending moments to the external ones due to the applied loads.

Two equations can be written:

$$P_1 e_1 + P_1 L_1 (v_1 + v_2) + F_1 L_1 = K_1 (v_1 - v_{01}) \qquad (3)$$

$$P_2 e_2 + P_2 L_2 v_2 + F_2 L_2 + H L_2 + F_1 (L_1 + L_2) + P_1 (e_1 + e_{12}) + P_1 [(L_1 + L_2) v_2 + L_1 v_1] = K_2 (v_2 - v_{02}) \qquad (4)$$

When the external loads, the initial out-of-straightness and the bending stiffnesses K_1 and K_2 of the two shafts are known, Formulas 3 and 4 form a system of linear equations in which the unknowns are the two rotations v_1 and v_2, i.e., the parameters which define the equilibrium configuration of the model. The collapse situation may be reached either in the upper or in the lower shaft. In the first case, rotation v_1 is equal to the ultimate limit rotation $v_{1\,lim}$ and $v_2 < v_{2\,lim}$, while in the second case rotation $v_1 < v_{1\,lim}$ and v_2 is equal to $v_{2\,lim}$.

Equivalence Between Model and Real Column

The parameters which govern behavior of the model must be defined so there is complete equivalence between the model and the simulated real element. Equating the Euler elastic critical load and the ultimate limit bending moment for each step imposes that the discrete model and the continuous real member have the same global elastic deformability, and they locally reach their ultimate strength under the same bending stresses.

So, for each step, two equations may be written from which the two unknown parameters (the bending stiffness K and the ultimate limit rotation v_{lim}) can be determined. In each shaft of the model, the Euler critical load can be defined respectively as:

$$P_{cr1} = K_1/L_1 \text{ and } P_{cr2} = K_2/L_2$$

while the ultimate limit bending moment can be defined respectively as:

$$M_{PL1} = K_1(v_{1\,lim} - v_{01}) \quad \text{and} \quad M_{PL2} = K_2(v_{2\,lim} - v_{02})$$

For the real column, the Euler critical loads of the two shafts are respectively:

$$P_{cr1} = \pi^2 EI_1 / 4L_1^2 \quad \text{and} \quad P_{cr2} = \pi^2 EI_2/4L_2^2$$

where E is the Young modulus.

The ultimate limit bending moment is not a constant in a cross section of a member which is subjected to variable axial loads, but is different for different values of the axial load.

For the cross section, a linear interaction domain can be assumed (on the safe side) of the kind:

$$\frac{M}{M_u} + \frac{N}{N_u} = 1$$

where

M_u = maximum bending moment sustainable by the cross section in absence of axial load at the plastic adaptation limit state (i.e., $M_u = \psi f_y S$, where the coefficient ψ amplifying the section modulus S, is called the plastic adaptation coefficient, and $1 \leq \psi \leq \alpha$, where α is the shape factor of the cross section[14] and f_y the yield stress of the material)

N_u = maximum axial load sustainable by the cross section, in absence of bending moment (i.e. $N_u = f_y A$).

When the value of the axial load in the shaft is known, then it is possible to define:

$$M_{PL1} = M_{u1}\left(1 - \frac{P_1}{P_{1u}}\right) \text{ and}$$

$$M_{PL2} = M_{u2}\left(1 - \frac{P_1 + P_2}{P_{2u}}\right)$$

where

$$M_{u1} = \psi_1 f_y S_1$$

$$M_{u2} = \psi_2 f_y S_2$$

$$P_{1u} = f_y A_1$$

$$P_{2u} = f_y A_2.$$

By equating the corresponding expressions, the four unknown parameters are determined:

$$v_{1\,lim} = \left(1 - \frac{P_1}{P_{1u}}\right)\frac{M_{u1}}{K_l} + v_{01} \tag{5a}$$

$$K_1 = \pi^2 EI_1/4L_1 \tag{5b}$$

$$v_{2\,lim} = \left(1 - \frac{P_1 + P_2}{P_{2u}}\right)\frac{M_{u2}}{K_2} + v_{02} \tag{6a}$$

$$K_2 = \pi^2 EI_2/4L_2 \tag{6b}$$

Note that posing the equivalence of the Euler elastic critical loads separately in the various steps does not imply that the same equivalence exists between the whole model and the real structure. The operating way was forced because the critical load of the model depends on the ratio of the bending stiffnesses K_1 and K_2 of the two steps, which are a priori unknown.

However, as the deformability of the column was concentrated in the two cross sections at the bottom of each step, the approximation introduced here turns out to have no influence on the final results.

The Ultimate Interaction Domains

It is possible to reduce Formulas 3 and 4 to two expressions respectively of the kind $v_1 = v_1(v_2)$ and $v_2 = v_2(v_1)$, by solving Formula 3 with respect to v_1 and Formula 4 with respect to v_2.

Substituting Formula 3, solved with respect to v_1, into Formula 4 the following expression for v_2 is obtained:

$$v_2 = \frac{1}{K_2 - P_1(L_1 + L_2) - P_2L_2 - \dfrac{P_1^2 L_1^2}{K_1 - P_1L_1}}\Bigg[K_2 v_{02} + P_1(e_1 + e_{12}) + F_1(L_1 + L_2) + F_2L_2 + \xi P_2L_2 + P_2e_2 + (P_1L_1)\frac{P_1e_1 + F_1L_1 + K_1v_{01}}{K_1 - P_1L_1}\Bigg] \tag{7}$$

When the geometrical characteristics of the column are known, this expression gives a value of v_2 as a function of the external loads. The collapse situation is reached in the lower shaft when the loading condition is such that $v_2 \geq v_{2\,lim}$ (Formula 6a). Equating v_2 to $v_{2\,lim}$ and varying the values of the vertical loads, Formula 7 describes a curve in the plane $P_1 \div P_2$. This curve defines an admissible region: all the points contained in the area bounded by the coordinate axis and by the curve represent admissible loading conditions for the lower shaft. The points on the curve represent combinations of loads which cause the limit situation to be reached in the lower shaft. The points external to this admissible area represent load combinations which cannot be sustained by the column, and cause the collapse of the lower shaft.

Analogously substituting Formula 4 solved with respect to v_2 into Formula 3, an expression is reached:

$$v_1 = \frac{1}{K_1 - P_1L_1 - \dfrac{P_1{}^2L_1{}^2}{K_2 - P_2L_2 - P_1(L_1 + L_2)}} \times \Bigg[P_1e_1 + F_1L_1 + K_1\,v_{01} + P_1L_1\frac{K_2v_{02} + P_1\,(e_1 + e_{12}) + F_1\,(L_1 + L_2)}{K_2 - P_2L_2 - P_1\,(L_1 + L_2)} + \frac{F_2L_2 + \xi P_2L_2 + P_2e_2}{K_2 - P_2L_2 - P_1\,(L_1 + L_2)}\Bigg] \qquad (8)$$

which, when the geometrical characteristics of the column are known, defines the value of v_1 as a function of the external loads. The collapse situation in the upper shaft is reached when the loading condition is such that $v_1 \geq v_{1\,lim}$ (Formula 5a).

Equating v_1 to $v_{1\,lim}$ and varying the values of the vertical loads, Formula 8 also describes a curve in the plane $P_1 \div P_2$: all the points contained in the region bounded by the coordinate axis and the curve represent admissible loading conditions for the upper shaft. The column cannot sustain combinations of loads represented by points external to the admissible region, without collapse of the upper shaft.

If the two shafts have different cross-sectional properties, then the two curves represented by Formulas 7 and 8 intersect each other. The ultimate interaction domain for the column is the intersection of the two admissible regions for the two shafts, and the boundary of the domain is the envelope of the two curves.

If the columns have a constant cross section, the two curves do not intersect, and the region bounded by Formula 7 is completely contained into that bounded by Formula 8, this turns out to be the ultimate interaction domain of the element.

Use of the Model

Even if it is possible to evaluate in a substantially correct way the global behavior of the column, the real stiffness of the stepped member cannot be correctly evaluated using the model as it is, because of rough simplifying assumptions on which the model is based.

Preceding studies[12,13] have shown that by using the effective length concept it is possible to evaluate with good precision (at least from an engineering point of view) the maximum values P_{1uc} and P_{2uc} of the centric vertical loads sustainable by the real column (at the top of the whole column and at the top of the lower shaft respectively) in the absence of other loads (both vertical and horizontal).

In fact, to obtain the values of P_{1uc} and P_{2uc} for the real column, it is enough to enter with the values of the effective length (calculated separately for the upper and for the lower shaft[4,6]) on the stability curves for the upper and lower shaft respectively.

Let P^*_{1uc} and P^*_{2uc} be the corresponding maximum values of the centric vertical loads sustainable by the model. It is possible to reduce the approximation introduced with the initial assumptions, normalizing the domains obtained using the model over the values P^*_{1uc} and P^*_{2uc}, i.e. reducing the ultimate interaction domains in a non-dimensional form, in the plane $P_1/P^*_{1uc} \div P_2/P^*_{2uc}$.

These domains, because in a non-dimensional form, cannot be used by the designer for practical applications, but must be dimensionalized using the two values P_{1uc} and P_{2uc} calculated, for the real column, in a fast and easy way, as already explained. The domains are now ready to be used by the designer.

By following this, it is also possible, although indirectly, to include into the model the effect of residual stresses on the ultimate value of the load carrying capacity of the member.

A short interactive computer program has been set up, which solves Formulas 7 and 8 for the different combinations of loads considered. Once the statical and geometrical properties of a stepped member are entered as input data, the computer automatically furnishes as output the ultimate interaction domains in the non-dimensional form, in the plane $P_1/P^*_{1uc} \div P_2/P^*_{2uc}$.

Comparison with Numerical Results

Some comparisons were done between the domains obtained with a numerical simulation method,[16] and those obtained using the simplified model presented in this paper. In Figs. 3 to 8 the domains are shown in a non-dimensional form, in the plane $P_1/f_yA_1 \div P_2/f_yA_2$.

Figures 3 to 6 show the domains relative to a prismatic member, a W8×31 shape, subjected to step-wise axial loads.

Figures 7 and 8 show the domains relative to a stepped column, with a W8×108 shape used as lower shaft and a W8×31 shape used as upper shaft.

It is possible to see in the various loading conditions taken into account, that there is good agreement between the model and the numerical simulation.

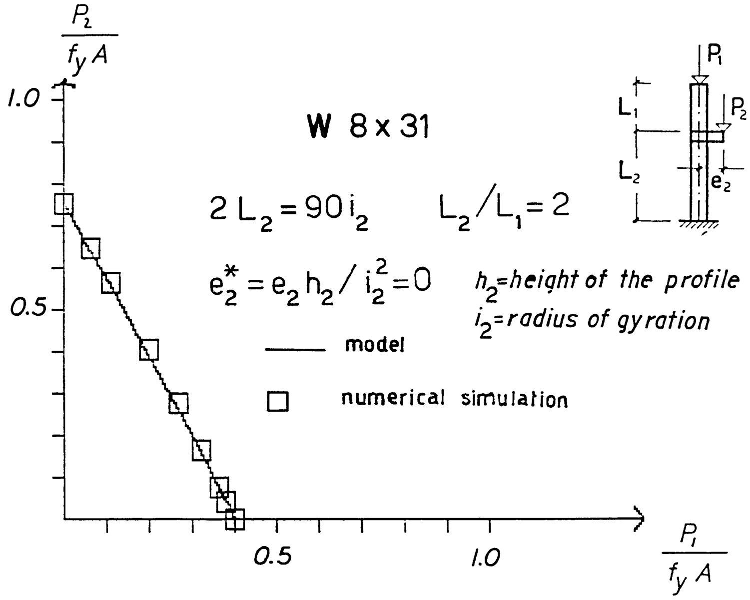

Fig. 3. Comparison between domains obtained with the simplified model and numerical simulation, in the case of a prismatic member with step-wise axial loads, in simple compression.

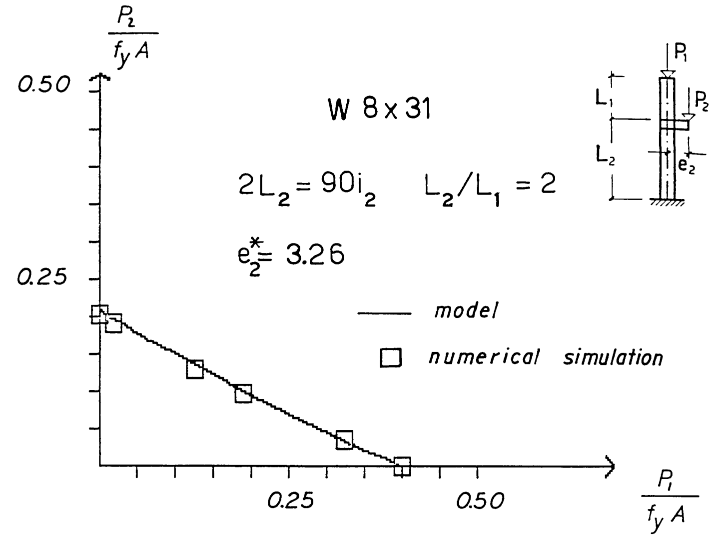

Fig. 4. Comparison between domains obtained with the simplified model and numerical simulation in the case of a prismatic member with step-wise axial loads, in compression and bending.

CONCLUSION

In this paper, a simple method is presented to determine the ultimate interaction domains for stepped columns. The method requires use of the effective length concept only for calculating the ultimate values of the centric axial loads applied at the top of the lower shaft (P_{2uc}) and of the whole column (P_{1uc}). These values are then used to render in a dimensional form the ultimate interaction domains determined in a non-dimensional form using a simple model with two degrees of freedom.

Using this model, it is possible to take into account the effect of both mechanical and geometrical imperfections and of the loading path, on the shape of the ultimate interaction domains for stepped structural members. It is possible to obtain the ultimate domains avoiding all the difficulties connected with the use of methods based on the effective length concept and axial-thrust bending-moment interaction formulas (such as 1), which require long calculations when dealing with members in compression and bending.

The method presented in this paper also represents an overcoming of that proposed by the author,[12,13] based on Formula 2, whose results are heavily influenced by the values adopted for the coefficient C_m of Formula 1, when calculating the values of $P_{1C,M}$ and $P_{2C,M}$ used in Formula 2.

The method was checked in a number of cases, showing a good agreement with the numerical results obtainable.[16] However, before any use or application of the method in standard design practice, more extensive research and checks (both numerical and experimental) are required.

REFERENCES

1. *Timoshenko, S.P. and J.M. Gere* Theory of Elastic Stability *McGraw-Hill, 1961, New York, N.Y. (par. 2.14, pp. 113).*
2. *Harvey, J.W.* Buckling Loads for Stepped Columns *ASCE Proceedings, ASCE Journal of the Structural Division, Vol. 90, No. ST2, April 1964 (pp. 201).*
3. *Dalal, S.T.* Some Non-Conventional Cases of Column Design *AISC Engineering Journal, January 1969, Chicago Ill. (p. 28).*
4. *Anderson, J.P. and J.H. Woodward* Calculation of Effective Lengths and Effective Slenderness Ratios of Stepped Columns *AISC Engineering Journal, October 1972, Chicago, Ill. (p. 157).*
5. *Fisher, J.M.* One Engineer's Opinion *AISC Engineering Journal, 1st Qtr., 1980, Chicago, Ill. (pp. 14).*
6. *Agrawal, K.M. and A.P. Stafiej* Calculation of Effective Lengths of Stepped Columns *AISC Engineering Journal, 1st Qtr., 1980, Chicago, Ill. (pp. 96).*
7. *Barnes, W.D. and C.P. Mangelsdorf* Allowable Axial Stresses in Segmented Columns *AISC Engineering Journal, 1st Qtr., 1979, (pp. 11).*
8. *Association of Iron and Steel Engineers* Guide for the Design and Construction of Mill-Buildings *AISE Technical Report No. 13, August 1979.*
9. *Murray, J.J. and T.C. Graham* The Design of Mill-Buildings *Iron and Steel Engineer, February 1957 (pp. 159).*
10. *Huang, H.C.* The Design of Mill Building Columns *Iron and Steel Engineer, March 1968 (pp. 97).*

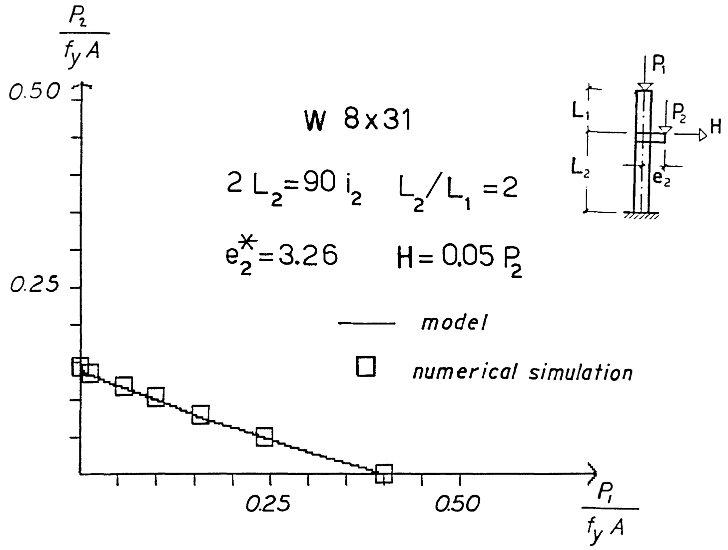

Fig. 5. Comparison between domains obtained with the simplified model and numerical simulation, in the case of a prismatic member with step-wise axial loads, in compression and bending, in presence of a transversal force H, simulating effect of crane sway.

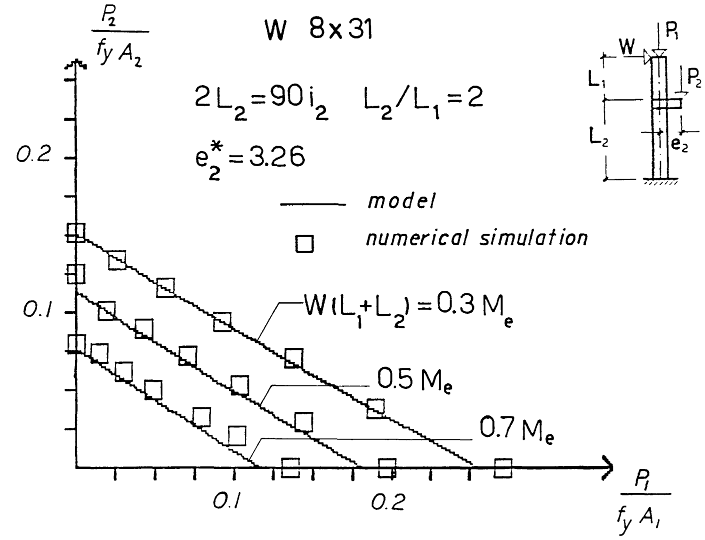

Fig. 6. Comparison between domains obtained with the simplified model and numerical simulation, in case of a prismatic member with step-wise axial loads, in compression and bending, in presence of a transversal force W, simulating effect of wind. Three values of horizontal force W generate at the base of column a bending moment respectively equal to .3, .5, .7 of elastic limit bending moment M_e.

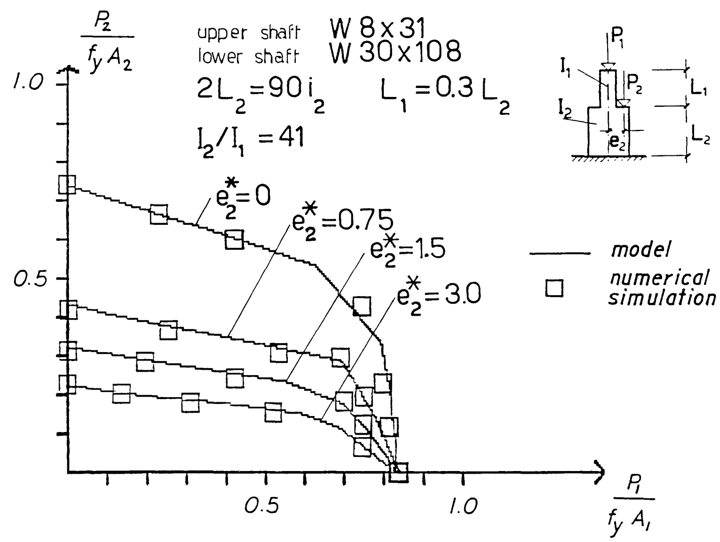

Fig. 7. Comparison between domains obtained with the simplified model and numerical simulation, in the case of a stepped column, for different values of eccentricity of load P_2 applied at top of lower shaft.

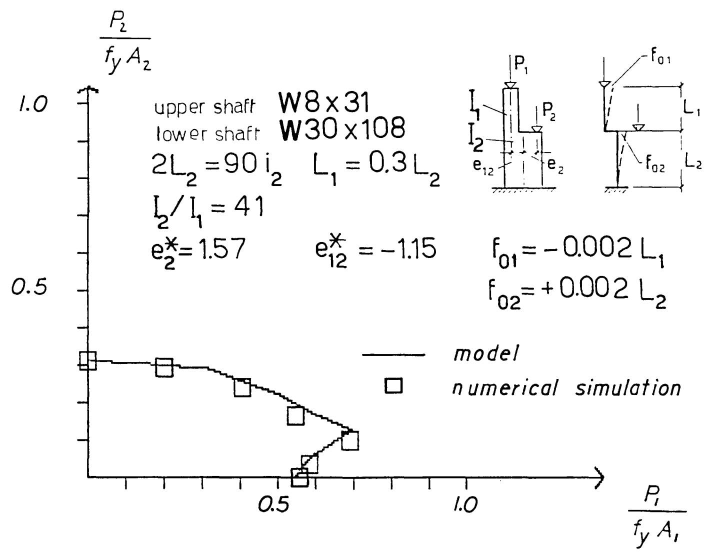

Fig. 8. Comparison between domains obtained with the simplified model and numerical simulation, in case of a stepped column in compression and bending, taking into account effect of shape of initial geometrical imperfections.

11. *Huang, H.C.* Determination of Slenderness Ratios for Design of Heavy Mill-Building Columns *Iron and Steel Engineer, November 1968 (pp. 123).*
12. *Castiglioni, C.A., F. Genna, and R. Zandonini* Ultimate Strength Analysis of Steel Members Subjected to Stepwise Axial and Transversal Loads *Costruzioni Metalliche, No. 4, 1982 (pp. 161).*
13. *Castiglioni, C.A. and R. Zandonini* On the Ultimate Strength of Light Mill Building Columns *SSRC, Proceedings of the 3rd International Colloquium on Stability of Metal Structures, Toronto, Canada, May 1983.*
14. *Ballio, G. and F.M. Mazzolani* Theory and Design of Steel Structures *Chapman and Hall, 1983, London, New York.*

15. *Ballio, G., V. Petrini, and C. Urbano* Loading Effects in Beam Columns *Meccanica, Vol. 9, December 1974.*
16. *Castiglioni, C.A.* A Numerical Method for the Simulation of the Static and Dynamic Behavior of Steel Structures Subjected to Buckling *Dept. of Structural Engrg., Politecnico di Milano, Tech. Report No. 1/82.*

NOMENCLATURE

A	= area of cross section
A_1, A_2	= values of A respectively for upper and lower shaft
C_m	= reduction coefficient to be introduced in Formula 1
e_1	= eccentricity of load P_1 with respect to axis of upper shaft
e_2	= eccentricity of load P_2 with respect to axis of lower shaft
e_{12}	= eccentricity between axis of upper shaft and that of lower one
E	= Young's modulus of material
f_y	= yield stress of material
f_{01}, f_{02}	= initial geometrical imperfections (horizontal displacement at top of upper and lower shaft respectively)
F_1, F_2	= horizontal forces applied respectively at top of upper and of lower shaft
H	= horizontal force applied at top of lower shaft, proportional to vertical load P_2
K	= bending stiffness
K_1, K_2	= values of K respectively for upper and lower shaft
I_1, I_2	= moment of inertia of cross section respectively for upper and lower shaft
L_1, L_2	= length respectively of upper and lower shaft
M	= first order bending moment
M_p	= fully plastic bending moment
M_{PL1}, M_{PL2}	= ultimate limit bending moment respectively for upper and lower shaft
M_u	= maximum bending moment sustainable by cross section in absence of axial load, at plastic adaptation limit state
M_{u1}, M_{u2}	= values of M_u respectively for upper and lower shaft
N_u	= maximum axial load sustainable by cross section in absence of bending moment
P	= axial load
$P_{c,n}$	= ultimate axial load
P_{cr1}, P_{cr2}	= Euler elastic critical load respectively of upper and lower shaft, calculated as if shaft were completely disconnected from the other, and simply cantilevered at its base
P_E	= Euler elastic critical load of shaft, calculated on base of effective length
P_1	= axial load applied at top of whole column
P_2	= axial load applied at top of lower shaft
$P_{1C,M}$, $P_{2C,M}$	= maximum values of P_1 and P_2 respectively, sustainable by column in presence of single vertical load and of possible transversal actions.
P_{1u} P_{2u}	= values of N_u respectively for upper and lower shaft
P_{1uc}, P_{2uc}	= maximum values of centric vertical loads P_1 and P_2 respectively, sustainable by column in absence of other loads (both vertical and horizontal).
P^*_{1uc}, P^*_{2uc}	= *analogous to* P_{1uc} and P_{2uc}, related to the model
P_{1UC}, P_{2UC}	= *values of* P_{1uc} and P_{2uc} obtained with a numerical simulation method
S	= section modulus
S_1, S_2	= values of S respectively for upper and lower shaft
v_1	= relative rotation between the upper and lower shaft
v_2	= absolute rotation of lower shaft with respect to vertical axis
v_{01}, v_{02}	= initial geometrical imperfections (initial values of v_1 and v_2)
v_{lim}	= ultimate limit rotation
v_{1lim}, v_{2lim}	= ultimate limit values respectively of v_1 and v_2
W	= horizontal force (analogous to F_1) simulating wind load
α	= shape factor for cross section
β	= numerical coefficient to be used in Formula 2
Ψ	= plastic adaptation coefficient
Ψ_1, Ψ_2	= values of Ψ for upper and lower shaft respectively
ξ	= numerical coefficient (ratio between H and P_2 : $\xi P_2 = H$)