A Unified Approach for Stability Bracing Requirements

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Columns and beam flanges in compression often require intermediate lateral bracing to satisfactorily carry the required load. Usually this is a finite number of braces at a spacing S. Many engineers are familiar with the ideal stiffness $4\ P_{cr}/S$ needed to fully brace the compression member over the length S, and that the required stiffness is generally taken as twice the ideal stiffness, or $8\ P_{cr}/S$. If the compression member does not need to be *fully* braced at each support point, due to either the magnitude of loading required or the size of member used, considerably less bracing stiffness may be required than would be calculated by using $8\ P_{cr}/S$.

A means to evaluate an accurate value of required stiffness is important for examining structural members whose integrity is questionable because of limited bracing stiffness and strength, and for obtaining economical bracing design for members repeated many times in a structure. No simple technique other than continuous bracing expressions, exists for obtaining the less-than-full bracing integrity.

In this paper, the bracing stiffness required for the range from continuous point bracing to a single point brace is presented. The required brace strength is also summarized. The bracing expressions are given in the form most familiar for point bracing. The expressions provide an understanding of the relationship between bracing stiffness and buckling behavior of the compression member. Furthermore, an expression for tangent modulus of elasticity is proposed as a means to apply the bracing expressions for all levels of critical load less than the yield load.

BRACING STIFFNESS FOR CONTINUOUS BRACING SYSTEMS

From Timoshenko and Gere, *Theory of Elastic Stability*,[1]

$$P_{cr} = (\pi^2 EI/L^2)(m^2 + k_c L^4/m^2\pi^4 EI) \qquad (1)$$

where: L = the overall member length
k_c = the continuous bracing stiffness
m = the number of ½ sine waves in the buckled shape

Timoshenko shows $k_c L^4/\pi^4 EI = m^2(m+1)^2$ at buckling.

$$\therefore \quad P_{cr} = (\pi^2 EI/L^2)[m^2 + (m+1)^2]$$

Since $m^2 + (m+1)^2 = 2m^2 + 2m + 1 = 2m(m+1) + 1$,

$$P_{cr} = (\pi^2 EI/L^2)[2m(m+1) + 1]$$
$$= (\pi^2 EI/L^2)[2L^2/\pi^2\sqrt{k_c/EI} + 1]$$
$$= 2EI\sqrt{k_c/EI} + \pi^2 EI/L^2$$

Thus, $P_{cr} = 2\sqrt{k_c EI} + P_E$

$\therefore$ the ideal continuous bracing stiffness

$$k_c = (P_{cr} - P_E)^2/4EI \qquad (2)$$

Neglecting P_E for $P_{cr} >> P_E$, $P_{cr} = 2\sqrt{k_c EI}$ or $k_c = P_{cr}^2/4EI$.

For the case of inelastic action replace E with E_t. Using $P_{cr} = \pi^2 E_t I/L^2_e$

$$L_e = \pi\sqrt{E_t I/P_{cr}} \qquad (3)$$

where L_e = the effective length of the buckled column.

$$\therefore \ k_c = (\pi^2 P_{cr}/4)(P_{cr}/\pi^2 E_t I)$$

$= \pi^2 P_{cr}/4L_e^2$. Thus, the ideal continuous bracing stiffness is:

$$k_c \simeq 2.5\ P_{cr}/L_e^2 \qquad (4)$$

BRACING STIFFNESS WITH A FINITE NUMBER OF SUPPORTS

For finite braces at a spacing equal to S, where S is small relative to L_e,

$$K_i = k_c S = (2.5 P_{cr}/L_e^2)S \qquad (5)$$

See Fig. 1. Based on Winter's work[2] when S equals L_e,

$$K_i = 4P_{cr}/L_e \qquad (6)$$

Equation 6 is often used when S is less than L_e, with S being substituted for L_e. This is not correct. The results

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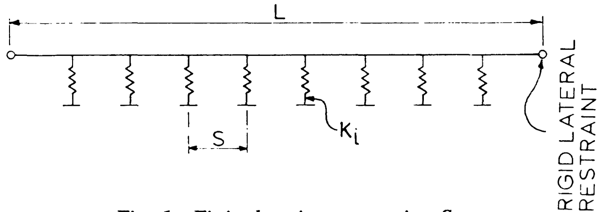

Fig. 1. Finite bracing at spacing S

from such calculations can grossly overestimate the bracing stiffness required. One must remember that the length L_e in Eq. 6 defines the required buckling length of the member. Using a S which is less than L_e in Eq. 6 presumes a shorter mode shape, which requires more stiffness than is necessary.

For a small S, relative to L_e, Eq. 5 provides an accurate solution for bracing stiffness. However, as S increases relative to L_e, using Eq. 5 results in a substantial error.

Solutions are presented in Ref. 3 for critical buckling loads (P_{cr}) for cases when brace stiffness is less than the stiffness required to force the column to buckle in its highest mode, i.e., the number of one-half waves equal to the number of braces plus one. Solutions are presented for one, two, three and four intermediate brace points, as well as for columns continuously braced.

Shown in Fig. 2 are the solutions from Ref. 3 for cases with three and four intermediate braces replotted using an absissa of K_iS/P_E. It can be seen from Fig. 2 that the bracing expression, Eq. 5, provides a solution for P_{cr} with good accuracy to approximately three-quarters of their maximum P_{cr}/P_E value. Based on bracing Eqs. 5 and 6, bracing stiffness over the entire range of S less than or equal to L_e can be conservatively calculated from the expression

$$K_1 = [2.5 + 1.5(S/L_e)^4][P_{cr}S/L_e^2] \qquad (7)$$

This equation provides a transition between the continuous bracing equation, which has a numerical coefficient of approximately 2.5, and the solution for finite full bracing, which has a maximum numerical coefficient of 4.

The accuracy of Eq. 7 is illustrated in Fig. 2, where it has been plotted for the cases of four and three intermediate brace points. Also shown in Fig. 2 is the expression $4P_{cr}/S$.

When the number of intermediate braced points is less than three, it has been found that Eq. 7 becomes overly conservative as S approaches L_e.

BRACING STIFFNESS FOR LESS THAN THREE INTERMEDIATE SUPPORTS

Provided in Ref. 2 for a single intermediate brace, Fig. 3, is the relationship:

$$P_{cr} = P_E + (3/16)(K_iL) \text{ for } P_E < P_{cr} \leq \pi^2 E_t I/S^2$$

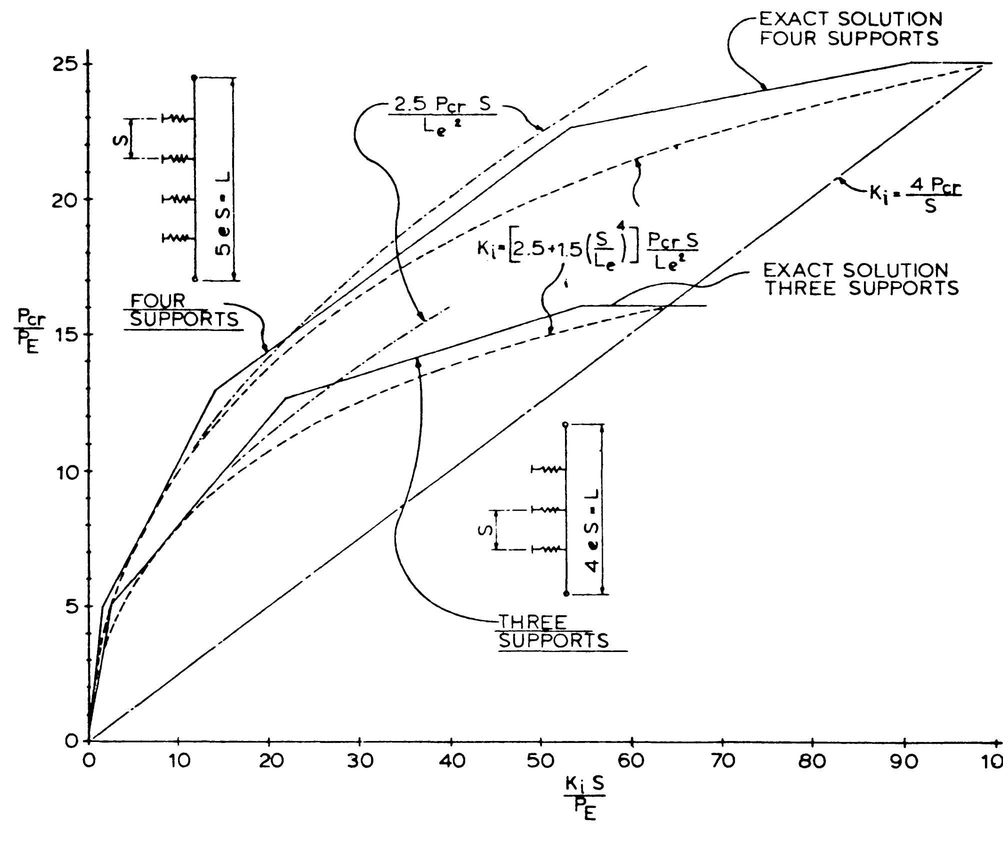

Fig. 2. Solution for three or more intermediate supports

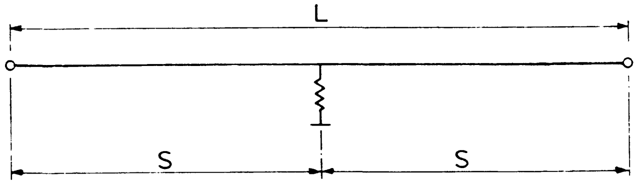

Fig. 3. One intermediate brace

If $2S$ is substituted for L, one obtains

$$K_i = (8/3)(P_{cr} - P_E)/S \qquad (8)$$

where,

P_E = the buckling load with no intermediate support
$= \pi^2 E_t I/L^2$

This relationship is plotted in Fig. 4.

Also plotted in Fig. 4 for the one intermediate support condition is the commonly used expression $K_i = 2P_{cr}/S$ (see Ref. 1). It is recommended the designer requiring more accuracy use Eq. 8.

For the situation where two intermediate supports exist (Fig. 5) the "exact" solution from Ref. 2 is again replotted in Fig. 4 in terms of K_iS/P_E along with the commonly used equation $K_i = 3P_{cr}/S$ (see Ref. 2). One can see from Fig. 4 the discrepancy between the exact solution and the commonly used expression for K_i when full P_{cr} is not necessary.

The empirical equation:

$$K_i = 3P_{cr}S/L_e^2 \qquad (9)$$

is suggested for more accurate solutions, as shown in Fig. 4.

EVALUATION OF THE TANGENT MODULUS OF ELASTICITY

The tangent modulus E_t equals E as long as the member remains elastic. Using the CRC expression to represent the inelastic region,

$$P_{cr} = P_y[1 - .5(L_e/rC_c)^2] \qquad (10)$$
$$\text{for } P_{cr} > .5\ P_y.$$

Solving for L_e in Eq. 10,

$$L_e = rC_c\ \sqrt{2(1 - P_{cr}/P_y)} \qquad (11)$$

with $rC_c = \sqrt{I/A}\ \sqrt{2\pi^2 E/F_y}$

$= \pi\sqrt{2EI/P_y}$ by definition,

the equivalent length $L_e = \pi\sqrt{(4EI/P_y)(1 - P_{cr}/P_y)}$

$= \pi\sqrt{E_t I/P_{cr}}$ from Eq. 3

From this

$$E_t = (4EP_{cr}/P_y)(1 - P_{cr}/P_y) \qquad (12)$$

when $P_{cr}/P_y > .5$ or $L_e/r < C_c$.

Using E_t in Eq. 3, the procedure for determining the ideal stiffness K_i can be used for all $P_{cr} < P_y$.

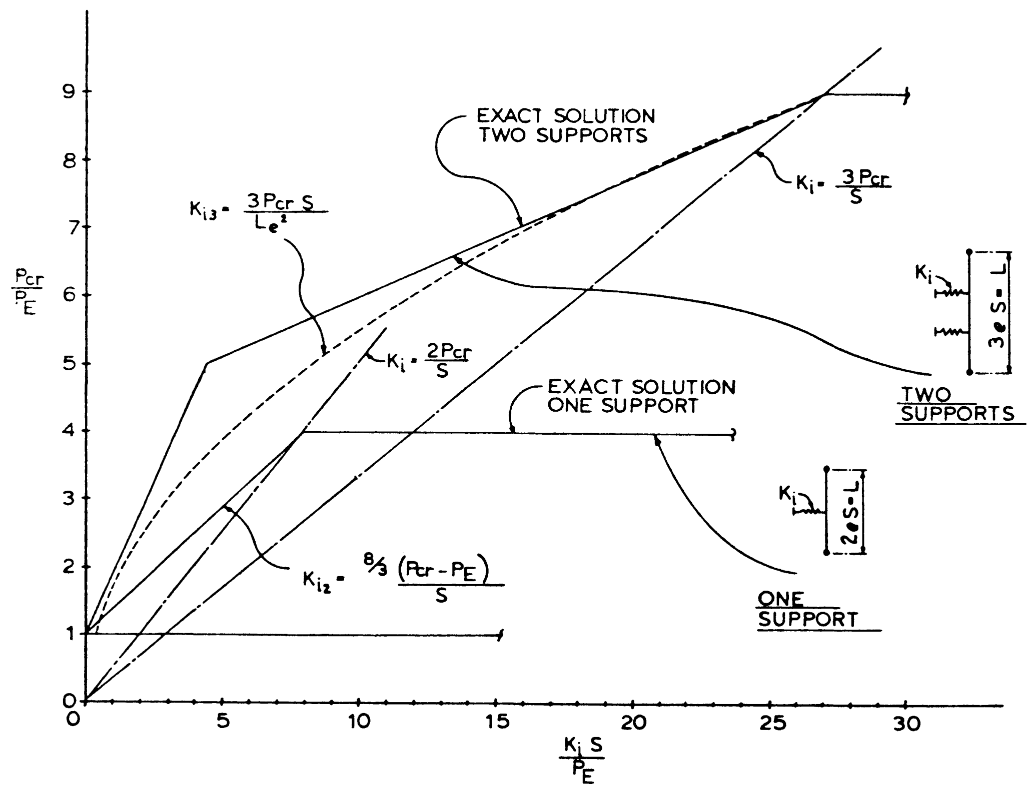

Fig. 4. Solution for one and two intermediate supports

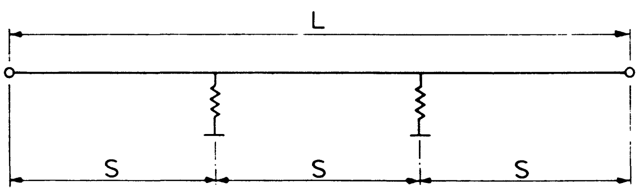

Fig. 5. Two intermediate braces

REQUIRED BRACE FORCE AND STIFFNESS

The brace force required:

$$F_{req} = K_{act} d \tag{13}$$

where d = the maximum lateral displacement immediately preceding failure and K_{act} = the furnished stiffness,[2] the F_{req} for an initially imperfect column is $K_i(d_o + d)$, where d_o is the initial out-of-straightness.

$$\therefore \quad K_{act} = K_i(d_o/d + 1) \tag{14}$$

$$\therefore \quad d = [K_i/(K_{\text{actual}} - K_{\text{i}})]d_o \tag{15}$$

It is generally accepted that the required stiffness K_{req} must be at least twice K_i so that $d = d_o$; d_o is usually set at $S/500$, but can be $S/250$ or any value that appears reasonable for the particular situation. Thus, for design purposes, the following equations are recommended:

For a single intermediate support,

$$K_{req} = 5.33(P_{cr} - P_E)/S; \quad P_E < P_{cr} \leq \pi^2 E_t I/S^2 \tag{16}$$

For two intermediate supports,

$$K_{req} = 6\ P_{cr} S/L_e^2; \quad P_E < P_{cr} \leq \pi^2 E_t I/S^2 \tag{17}$$

For three or more intermediate supports,

$$K_{req} = [5 + 3(S/L_e)^4] P_{cr} S/L_e^2; \quad P_{cr} > P_E \tag{18}$$

The required brace force,

$$F_{req} = d_o K_i/(1 - K_i/K_{act}); \quad K_{act} \geq K_{req} \tag{19}$$

The following three example problems illustrate the application of the proposed bracing expressions:

Example 1

This example is presented to illustrate the wide range of required stiffnesses which can be obtained. Assume a section with $r_y = 1.2$ in., $A = 6$ in.^{2}, $I_y = 8.64$ in.^{4} and $F_y = 50$ ksi. The column is to carry a P_{cr} of 100 kips and is to be braced with intermediate point braces as shown in Fig. 1 with a bracing spacing $S = 128.4$ in. Determine the bracing stiffness required.

Method 1: Using the expression
$K_{req} = 8P_{cr}/S = 8(100)/128.4 = 6.23$ kips/in.

Method 2: Using Eq. 6 with a factor of safety = 2.0,
$K_{req} = 8P_{cr}/L_e$.

From Eq. 3 $L_e = \pi\sqrt{E_t I/P_{cr}}$

$= \pi\sqrt{29{,}000(8.64)/100} = 157.3$ in.

Note that $E_t = E$ since $P_{cr} < .5\ P_y = .5(50)(6)$
$= 150$ kips,
or $L_e/r = 131.1 > C_c = 107$.
$\therefore \quad K_{req} = 8(100)/157.3 = 5.09$ kips/in.

Method 3: Using Eq. 18, the required stiffness need not be more than:
$K_{req} = [5 + 3(128.4/157.3)^4](100)(128.4)/(157.3)^2$
$= 3.29$ kips/in.

Example 2

This example illustrates the determination of brace spacing to achieve a desired column capacity with a specific column. A column (F_y = 36 ksi) is to be laterally braced about its weak axis so the column can carry a P_{cr} of 200 kips. Assume the column is braced sufficiently about its strong axis and more than two intermediate braces are required. The furnished braces are assumed to have a stiffness of 5.00 kips/in.

$I_y = 37.1$ in.^{4} and $A = 9.13$

Solution:
Find L_e.
From Eq. 3, $L_e = \pi\sqrt{E_t I/P_{cr}}$
$P_y = (A)(F_y) = 9.13 \times 36 = 328.68$ kips
$P_{cr}/P_y = 200/328.68 = 0.608$.
Since $P_{cr}/P_y > .5$,
From Eq. 12, $E_t = (4EP_{cr}/P_y)(1 - P_{cr}/P_y) = 27{,}647$ ksi
$\therefore \quad L_e = 224.98$ in., say 225 in.
Try a bracing spacing of 225 in., i.e., $S = 225$ in.
From Eq. 18,
$K_{req} = [5 + 3(225/225)^4]200(225)/(225)^2 = 7.11$ kips/in.
Since only 5 kips/in. is available from each brace, the braces must be spaced at a closer interval.
Try S = 205 in.
$K_{req} = [5 + 3(205/225)^4]200(205)/(225)^2$
$= 5.72$ kips/in. > 5.00 **n.g.**
Try 180 in., $K_{req} = 4.43$ kips/in. < 5.00 **o.k.**
Use braces at 180 in. o.c.

From Eq. 19,
$F_{req} = (d_o)2.215/(1 - 2.215/5.00)$
Note: $K_i = K_{req}/2 = 2.215$
$= (d_o)\ 3.977$
Assuming $d_o = S/500 = 180/500 = 0.36$ in.,
$F_{req} = 3.977\ (.36) = 1.43$ kips

Example 3

This example illustrates evaluation of column capacity for a given column and bracing situation. Determine the critical buckling load for the W12×40 column shown in Fig. 6. Assume the column sections shown are pinned at their ends and sidesway is prevented at their ends. Also, assume A36 steel is used.
For the W12×40: $r_y = 1.93$, $r_x = 5.13$, $A = 11.8$
For the W12×26: $I_x = 204$, $S_x = 33.4$
$C_c = 126.1$

Solution:

1. Determine the stiffness provided (K_{act}) by the strong axis of the W12×26 bracing the weak axis of the W12×40. Assume the two intermediate supports on the W12×40 exert forces in the same direction. The minimum stiffness of the supports can be found from the deflection equation for a simple beam loaded with two equal symmetric loads.

$$\Delta = (Px/6EI)(31a - 3a^2 - x^2)$$
$$\Delta = (Pa/6EI)(31a - 4a^2)$$
$$\Delta/P = [(8)/6EI][3(24)(8) - 4(8)^2]\ 1{,}728$$
$$= 4{,}423{,}680/6EI$$
$$= 4{,}423{,}680/[6(29{,}000)(204)]$$
$$= .125 \text{ kip/in.}$$

Therefore, the stiffness $K_{act} = 1/.125 = 8.0 =$ kips/in. The axial stiffness of the W8×31ties can be neglected. The connections of the ties to the columns are assumed to be rigid.

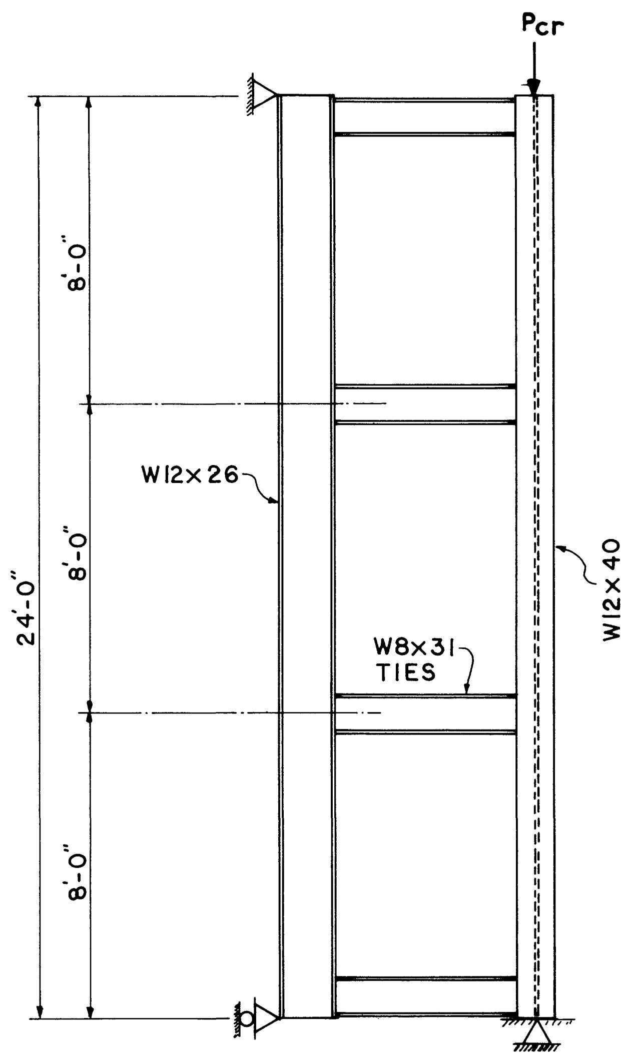

Fig. 6. Example 3

If fully braced, $L_e/r = 8(12)/1.93 = 49.7$ and $F_a = 18.38$ ksi

$P_{allow} = 18.38(11.8) = 216.9$kips

and $P_{cr} \simeq 216.9(1.7) = 369$kips

and $K_{req} = 6P_{cr}/S = 6(369)/(8)(12) = 23$ kips/in.

∴ the W12×40 is only partially braced.

2. From Eq. 17, for two intermediate braces,

$K_{req} = 6P_{cr}S/L_e^2$

For $P_E < P_{cr} < \pi^2 E_t I/S^2$.

From Eq. 11, $L_e = rC_c\sqrt{2(1 - P_{cr}/P_y)}$

P_{cr} can be found by substituting L_e from Eq. 11 into Eq. 17, and substituting K_{act} for K_{req}.

The result is,

$P_{cr} = P_y/[3SP_y/(K_{act}\ r^2 C_c^2) + 1]$

Thus, $P_{cr} = P_y/(3)(96)(424.8)/[(8.0)(1.93)^2(126.1)^2] + 1$

$= .795\ P_y$

$= 337.6$kips

and $L_e = (1.93)(126.1)\sqrt{2(1 - .795)}$

$= 155.8$ in. $= 12.98$ ft

Since $P_{cr} > .5\ P_y$, an acceptable solution has been found. Had P_{cr} been less than half of P_y,

$L_e^2 = \pi^2 EI/P_{cr}$ (Euler buckling) and substitution into Eq. 17 would yield

$P_{cr} = \sqrt{\pi^2 EIK_{act}/6S}.$

3. Determine the allowable buckling load of the W12×40.

$l/r_x = 24(12)/5.13 = 56.14$

$L_e/r_y = 155.8/1.93 = 80.75$ controls

$F_a = 15.27$, thus the allowable load is

$15.27(11.8) = 180.2$ kips

The brace force required from Eq. 13 is $K_{act}d$. If $d = d_o$ with d_o considered to be .375 in., $F_{req} = 8.0(.375) = 3.0$ kips
The stress in the W12×26 would be
3.0 kips(8 ft)(12)/33.4 = 8.62 ksi

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