Effective Width Criteria for Composite Beams

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A key factor in the analysis and design of composite flexural members is the effective width of the slab, i.e., that part of the slab that can be relied upon to act compositely with the steel beam. The current design specification for steel building structures in the United States³ gives detailed criteria, as do others.^{2,5} However, these have mostly been developed on the basis of analyses and tests of composite beams without steel deck. It is the purpose of this paper to evaluate the performance of some of the current effective width criteria, and to recommend practical improvements for use in future specifications.

SCOPE

Some of the current design criteria that are used to determine the effective width of the slab of composite beams will be evaluated. The results include an examination of the deflection characteristics of composite beams with formed steel deck, including a comparison of theoretical and experimental data. Since only service load deflections have any practical, design-oriented value, the study has been limited to that range of response. For all practical purposes, composite members behave elastically under these conditions.

The investigation also examines the influence of the various parts of the steel deck and makes recommendations for design usage. Complete data for all of the analyses described in the paper are given in Ref. 10.

EVALUATION OF EXISTING CRITERIA

Current Specifications

The primary advantage of a composite beam comes from the utilization of the concrete slab and the steel flange as the compression flange of the bending member. Since the slab must be present in the floor system anyway, it is obviously economical to make use of its substantial compressive load capacity.

The current design rules for composite bending members utilize the concept of effective width to determine the portion of the slab that acts compositely with the steel member. The advantage of this approach is that, if the effective width is found, the analysis and design of the composite beam can be performed by simple bending theory.

Reidar Bjorhovde is Professor of Civil Engineering and Engineering Mechanics, University of Arizona, Tucson, Arizona. The AISC Specification³ requires that the effective width, b_e , be set equal to the smallest of the following equations:

$$b_e = L/4 \tag{1}$$

$$b_e = 2b' + b_f \tag{2}$$

$$b_e = 16t_c + b_f \tag{3}$$

where

L = beam span

b' = one-half the clear distance between beams

 b_f = beam flange width

 t_c = thickness of the concrete slab

Similarly, for highway bridge design, the effective width criteria of the AASHTO specification² are identical to those given by the AISC Specification, except that Eq. 3 is replaced by $b_e = 12t_c$.

The use of one-fourth of the beam span as one limitation of the effective width can be developed from the theory of elasticity as that which replaces the actual width.⁹ Thus, elementary flexural theory gives the correct value of the uniform maximum bending stress when it is applied to the transformed beam cross section.

The provision relating the effective slab width to the steel beam flange width, plus some multiple of the slab thickness, appears to have been taken from earlier specifications. These requirements incorporated a measure intended to protect against crippling or buckling in steel compression members.⁸

The requirements for the design of composite beams in the proposed AISC Load and Resistance Factor Design (LRFD) Specification^{4,7} have simplified the criteria for b_e to: (a) one-eighth of the beam span, center-to-center of supports; (b) one-half the distance to the adjacent beam(s); or (c) the distance to the edge of the slab.

Adekola¹ proposed a formula for an effective-width factor based on deflection considerations and the use of the transformed cross section. He also demonstrated the influence of flexible shear connectors by studying the relationship between partial interaction and shear lag effects. Using the *deflection factor*, defined as the ratio of the midspan deflection of any composite beam in a floor system with several beams when there is full interaction, and the central composite beam deflection of the same system, the stiffness characteristics of the members were evaluated. It was demonstrated that the effective width increases with an increasing degree of interaction, because relatively more of

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the slab participates in increasing the stiffness of the equivalent T-beam as interaction increases. This is an important aspect of composite beam behavior which has not been recognized previously. It is possible to use it to develop improved effective width criteria for composite beams with less than full interaction.

Comparison of Existing Criteria and Experimental Data

The calculations of the effective width in this study are based on a set of composite beam test data that were provided by Fritz Engineering Laboratory at Lehigh University.⁶ All of the tests were for individual beams with a concrete slab on a formed steel deck. The results of these computations are based on deflection considerations; they have been correlated with the test results.

Some of the results of the theoretical and experimental evaluation are shown in Figs. 1 through 4. These figures compare elastic load-deflection relationships and the test results for the composite beam, using the effective width criteria of various codes.^{1,3,4,5} The theoretical deflections have been calculated on the basis of elementary bending theory, using an effective moment of inertia for the composite beam as given by the AISC Specification.³ This in-

corporates a measure of the effect of the degree of interaction.

Among other things, it is seen that for a considerable range of deflections, the criteria for the effective width taken as

- (a) the total width of the slab
- (b) the AISC Specification criteria
- (c) the AISC Specification criteria, ignoring the thickness effect

overestimate the stiffness of composite beams with formed steel deck. It was found that these criteria produce a consistent difference in deflection with respect to the experimental data for all of the beams that were analyzed.

Adekola's approach¹ gives good estimates of the effective width, but is not consistent for the different variables that apply to composite beams with formed steel deck. For example, his method gives poor results when the composite beam span is large and has a low degree of shear connection.

The British design code⁵ largely underestimates the stiffness of composite beams with formed steel deck. However, for beams with a small slab width or with a high degree of



Fig. 1. Applied load vs. midspan deflection of a composite test beam (beam W16×45, $V'_h/V_h = 0.255)^6$



Fig. 2. Applied load vs. midspan deflection of a composite test beam (beam $W16 \times 40$, $V'_h/V_h = 0.392)^{\circ}$

shear connection, this approach gives good results.

The composite beam deflections that were obtained using the effective-width requirements of the AISC LRFD criteria^{4,7} are very close to the test results in some cases. However, the criteria give conservative results for beams with a large slab width and a low degree of shear connection. On the other hand, the LRFD approach of using the total slab width is easy to apply for a designer.

The results of the preceding evaluations show that it is difficult to arrive at a unique relationship that incorporates all of the different variables that affect the design of composite beams with formed steel deck. In order to reduce the discrepancy between theory and tests, it is therefore suggested that more than one relationship may be necessary to encompass the maximum number of the variables that influence the design of composite beams with formed steel deck.

FORMULATION OF EFFECTIVE WIDTH CRITERIA

It has been established^{4,5,6,7,10} that the deflection of a composite beam using formed steel deck is mostly influenced by the beam span, the slab width and the horizontal shear ratio. Other parameters, such as the rib geometry, the stud shear connector dimensions and the concrete strength (for the same weight of concrete) do not appear to be as important. The differences in deflection for composite beams having the same beam span or slab width are therefore mostly controlled by the degree of shear connection.

To develop improved effective width criteria, linear regression analyses of the theoretical load-deformation data obtained using the various effective width requirements were performed. The composite beam parameters whose effects were analyzed in this way were (a) the steel beam span, (b) the slab width and (c) the degree of shear connection.

The study was performed for a range of composite beam stiffnesses. Comparing these results with the test data, the relationship between the parameters that influence the behavior of the composite beams was obtained. Based on the degree of partial shear connection, correction factors for the beam span and the slab width were chosen on a least squares basis, so the regression analysis produced the smallest error for each effective width criterion. This process was repeated until the variable with the smallest adjusted regression sum of squares gave an error no larger than 5% (in some cases the actual error was less than 1%).



Fig. 3. Applied load vs. midspan deflection of a composite test beam (beam W16×45, $V'_h/V_h = 0.675)^6$



Fig. 4. Applied load vs. midspan deflection of a composite test beam (beam W14×30, $V'_h/V_h = 0.836)^6$

Considerable differences were found between beams with low and high degrees of shear connection; different correction factors are therefore proposed for the composite beam span, based on the degree of interaction. Consequently, a Class 1 factor should be used when the degree of interaction is less than 0.5, and a Class 2 factor should be used when the degree of interaction is greater than or equal to 0.5.

The results of the regression analysis indicate that the effective width that is defined on the basis of the beam span should be determined as follows (see Fig. 5):

Class 1:
$$b_e = \frac{L}{12}$$
 (4a)

Class 2:
$$b_e = \frac{L}{8}$$
 (4b)

The regression analysis that was performed with the slab width as the major parameter indicates that the effective width should be given as (see Fig. 3):

$$b_e = b' + b_f \approx \frac{b_o}{2} \tag{5}$$

The results of the above approach were compared with the experimental data and good agreement was found.

To improve the design criteria given in the AISC Specification,³ it is recommended that the designer evaluate the effective width according to an estimated or preferred degree of shear connection. It has been shown that when the effective width is determined as suggested in this paper, more accurate results will be obtained. The proposed criteria are also easy to incorporate into a practical design procedure.

APPLICATION OF RECOMMENDED CRITERIA

The following design example illustrates the practical usage of the recommended effective width criteria.



Fig. 5. Recommended criteria for the effective width of composite beams with formed steel deck

Given:

- Beam span: L = 20 ft
- Beam spacing: $b_o = 6$ ft
- Steel grade: ASTM A36, $F_y = 36$ ksi
- Allowable bending stress (assuming that the shape that will be used is compact): $F_b = 0.66F_y = 23.8$ ksi Concrete strength: $f'_c = 3.71$ ksi (laboratory test value)
- The concrete is semi-lightweight ($w_c = 117.9 \text{ pcf}$), giving a modular ratio of $n = E_s/E_c = 13.4$ on the basis of a measured E_c .⁶

Uniform live load: $w_L = 3.5$ kips/ft

A 4-in. concrete slab is used, along with a 1¹/₂-in. composite metal deck with the ribs running perpendicular to the beam.

Solution:

1. Bending moments

Dead load:
4 in. S.L.W. slab; deck; mesh = 0.0423 kips/ft²
Steel (assumed) =
$$\frac{0.0075 \text{ kips/ft}^2}{0.0498 \text{ kips/ft}^2}$$

 $w_D = 0.0498 \times 6 = 0.300 \text{ kip/ft}$
 $M_D = \frac{w_D L^2}{8} = \frac{0.3(20)^2}{8} = 15 \text{ kip-ft}$

Live load:

$$M_L = \frac{w_L L^2}{8} = \frac{3.5(20)^2}{8} = 175$$
 kip-ft

2. Required section moduli (100% interaction) For bare steel beam:

$$(S_s)_{req} = \frac{M_D}{F_b} = \frac{15(12)}{23.8} = 7.56 \text{ in.}$$

For composite (transformed) cross section:

$$(S_{tr})_{req} = \frac{M_D + M_L}{F_b} = \frac{(15 + 175)(12)}{23.8} = 95.8 \text{ in.}^3$$

3. Determine effective width for desired degree of shear connection

Choose a value of V'_h/V_h larger than or equal to 0.5, to be able to utilize a Class 2 factor for the effective width (maximum). Try $V'_h/V_h = 0.675$.

Then

$$b_e = \frac{L}{8} = \frac{20.12}{8} = 30$$
 in. (governs)

or

$$b_e = \frac{b_o}{2} = \frac{6.12}{2} = 36$$
 in.

4. Select beam shape and determine cross-sectional properties

Try using a W16×45, where $A_s = 13.3$ in.², d = 16.13 in., $I_s = 586$ in.⁴, $S_s = 72.7$ in.³

Calculate the composite (transformed) cross section properties for 100% interaction:

$$y_{tr} = 11.27$$
 in.
 $A_{tr} = 13.30 + 5.60 = 18.90$ in.²
 $I_{tr} = 1050$ in.⁴ (for deflection computations)

The transformed moment of inertia is used for deflection calculations, and hence the actual *n*-value has been used to find y_{tr} , A_{tr} and I_{tr} . For stress computations, however, the AISC Specification, Sect. 1.11.2.2, recommends that the transformed properties be determined on the basis of the *n*-value for normal weight concrete of the same strength. Here, for $f'_c = 3.71$ ksi, it is found that n = 8.35. This gives

$$y_{tr} = 12.42$$
 in.
 $I_{tr} = 1217$ in.⁴
 $S_{tr} = 98.0$ in.³ (for stress computations)

5. Calculate effective cross-sectional properties for partial interaction

Effective moment of inertia (Formula 1.11-6 of AISC Specification):

$$I_{eff} = I_s + \sqrt{\frac{V'_h}{V_h}} (I_{tr} - I_s)$$

= 586 + \sqrt{0.675} (1050 - 586) = 967 in.⁴

Effective section modulus:

$$S_{eff} = S_s + \sqrt{\frac{V'_h}{V_h}} (S_{tr} - S_s)$$

= 72.7 + \sqrt{0.675} (98.0 - 72.7) = 93.5 in.³

It is found that $I_{eff} = 0.921I_{rr}$ and $S_{eff} = 0.954S_{tr}$. The losses in bending stiffness and strength that are prompted by having less than 100% interaction are therefore minimal in this case.

6. Shoring check (AISC Specification Eq. 1.11-2)

$$S_{tr} = \left(1.35 + 0.35 \frac{M_L}{M_D}\right) S_s \ge (S_{tr})_{act}$$
$$= \left(1.35 + 0.35 \frac{175}{15}\right) 72.7 = 395 > 93.5 \text{ in.}^3 \quad \mathbf{o.k}$$

7. Check of concrete and steel stresses

Steel stresses:

Dead load:

$$f_b = \frac{M_D}{S_s} = \frac{15.12}{72.7} = 2.48 \text{ ksi} << F_b$$
 o.k.

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Total load:

$$f_b = \frac{M_D + M_L}{S_{eff}} = \frac{(15 + 175)12}{93.5} = 24.4 \text{ ksi} \approx F_b$$

Based on the effective section modulus, the steel is overstressed by approximately 2.5% under total load. This is deemed acceptable.

Concrete stress:

The concrete stress is limited to $0.45 f'_c$, which for this case is 1.67 ksi. The actual transformed cross sectional properties are to be used:

$$S_c = \frac{I_{eff}}{y_c} = \frac{967}{20.13 - 11.27} = 109.1 \text{ in.}^3$$

Concrete stress due to M_L :

$$f_c = \frac{M_L}{n S_c} = \frac{175.12}{13.4(109.1)}$$

= 1.44 ksi < 1.67 ksi **o.k.**

8. Check of deflection

Under normal composite design circumstances, the dead and live load deflections would be determined independently and compared with suitable performance criteria. In this case, however, only the live load deflection is needed for comparison with the measurements that were made for the test beam of identical proportions⁶ (see Fig. 3). It is noted that the actual stiffness properties of the beam must be used.

Live load deflection:

$$\Delta_L = \frac{M_L L^2}{160I_{eff}} = \frac{175(20)^2}{160(967)} = 0.452 \text{ in.}$$

This constitutes a deflection-to-span ratio of approximately 1/530, and compares to the test deflection of 0.52 in. as giving a slightly stiffer beam (the computed deflection is 13% less than the test value). In addition to being more accurate, the method is also a significant improvement over other design approaches in that the results are consistent.

SUMMARY AND CONCLUSIONS

Several criteria are currently used to determine the effective width of composite beams. However, tests and other data for composite beams with formed steel deck suggest that improvements can be made to arrive at a more accurate and economical representation. Deflection limits are not well defined. Whereas most designers will limit the maximum live load deflection to L/360 of the beam span, it must be understood this is an empirical expression intended primarily to prevent extensive cracking in plastered surfaces. An accurate evaluation depends on many factors, such as the geometry and the material properties of the slab and the steel beam. The following conclusions can be stated:

- 1. The deflection of composite beams with formed steel deck is influenced most significantly by the beam span, the slab width and the degree of shear connection.
- 2. The procedure used in the current AISC Specification for determining the effective width has been found to give excessive beam stiffness properties. As a result, beams designed on this basis tend to underestimate actual deflections.
- 3. The AISC Load and Resistance Factor Design (LRFD) criteria for the effective width were found quite realistic. They are inconsistent only in cases of composite beams with a large slab width and a low degree of shear connection.
- 4. Adekola's method and the British specification were found to underestimate the composite beam stiffness. However, of all of the experiments analyzed, Adekola's approach appeared to give the most consistent results. On the other hand, it is too complex for practical usage.
- 5. Results of the proposed formulation for the effective width are in good agreement with actual test data. However, it must be remembered all the results presented in this study were based on linearly elastic behavior only. On the other hand, the service load characteristics of most composite beams closely resemble elastic response; the recommendations therefore are believed to be realistic.
- 6. Use of the suggested approach gives the designer a better way of accounting for the effects of partial shear connection on the strength and behavior of a composite beam with formed steel deck.

NOMENCLATURE

- A_s = cross-sectional area of steel beam
- A_{tr} = transformed area of composite beam cross section
- b = total slab width
- b' = one-half the clear distance to the adjacent beam
- b_e = effective width
- b_f = steel beam flange width
- b_o = center-to-center distance between adjacent beams
- d =steel beam depth
- E_c = modulus of elasticity for concrete
- E_s = modulus of elasticity of steel
- F_b = allowable bending stress in steel beam
- F_v = yield stress of steel
- f_b = bending stress in steel
- f_c = bending stress in concrete
- f'_c = compressive failure stress of concrete

- I_{eff} = effective moment of inertia of the composite cross section with partial interaction
- I_s = moment of inertia of the steel beam
- I_{tr} = moment of inertia of the transformed composite cross section with full interaction
- L = span length
- M_D = dead load moment
- M_L = live load moment
- $n = \text{modular ratio} = E_s/E_c$
- S_c = section modulus of composite beam, referred to the top of the concrete slab
- S_{eff} = effective section modulus of the composite beam with partial interaction, referred to the bottom flange of the steel beam
- S_s = section modulus of bare steel beam
- S_{tr} = section modulus of transformed composite cross section with full interaction, referred to bottom flange
- t_c = slab thickness
- V_h = total horizontal shear to be resisted by connectors under full composite action
- V'_h = total horizontal shear to be resisted by connectors providing partial composite action
- w_c = unit weight of concrete
- w_D = uniformly distributed dead load
- w_L = uniformly distributed live load
- y_c = distance from centroid of transformed composite cross section to the top of the concrete slab
- y_{tr} = as defined for y_c , but measured to the bottom fiber of the steel beam
- Δ_L = live load deflection

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