Deflections of Stepped Beams and Girders

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Stepped beams and girders, illustrated in Fig. 1, can be used in many structures. An appropriate application for stepped sections could be on the floor framing of high-rise buildings where the ceiling plenums are kept shallow to limit the floor to floor dimension. Stepped sections provide greater clearance under beams for the ducts and other mechanical systems which are often crowded into ceiling plenums. By proper selection, greater clearance may be achieved with the added benefit of reducing the weight of beams and girders.

However, stepped sections are rarely used. The reasons for their unpopularity are possibly due to the lack of familiarity with their deflection characteristics and the unavailability of deflection formulae for such beams and girders. Formulae for deflection and tables to assist in the selection of stepped sections are presented in this paper.

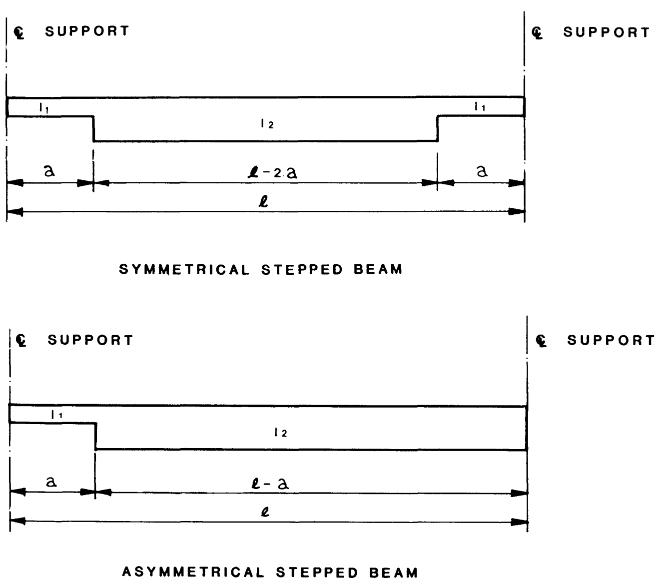

Figure 1

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THE UNIFORMLY LOADED SPAN

The simple beam parabolic moment curve for a uniformly loaded span is shown in Fig. 2. The ordinates are given as percentages of the maximum ordinate at the center of the span. At points a equal to $0.05L$, $0.10L$, $0.15L$ and $0.20L$, the ordinates are 19%, 36%, 51% and 64% respectively. Steps are most effective between $0.05L$ to $0.20L$ and are also the most economical. Table 1 shows the reduction in the depths of sections that are possible when the steps are placed at these points.

THE THEOREM OF EQUIVALENT MOMENTS

On any simple span the line loading can be uniform, triangular, parabolic or other pattern, and can be either partial or full. The line load may be replaced by a set

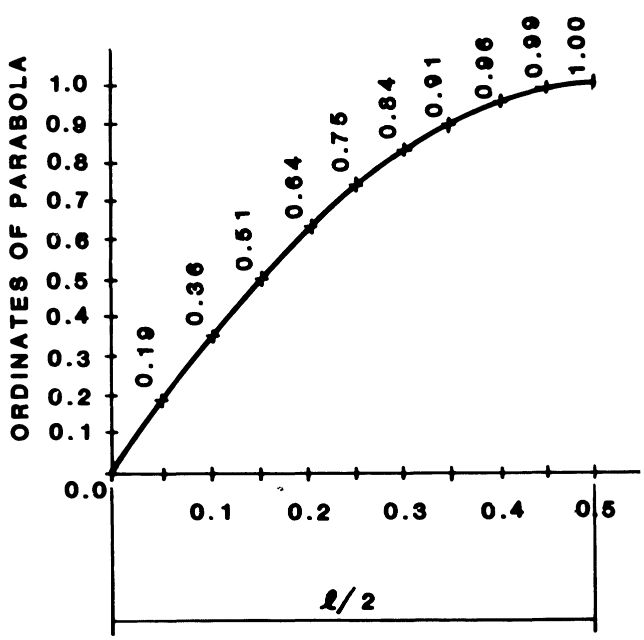

Figure 2

Table 1. Design Selection Table for Stepped Beam

"a" Segment % Maximum Moment		$0.20L$ 64	$0.15L$ 51	$0.10L$ 36	$0.05L$ 19
Main Section	S_x				
W36×300	1,110	W33×221 W36×210	W30×191	W24×162 W27×146 W33×130	W18×119 W21×101 W24×94 W27×84
W36×280	1,030	W30×211 W33×201 W36×194	W30×173	W24×146 W27×146 W30×132 W33×130	W14×120 W18×106 W21×93 W24×84
W36×260	953	W30×191 W36×182	W27×178 W30×173 W33×152 W36×150	W24×146 W30×124 W33×118	W14×120 W18×97 W21×93 W24×76
W36×245	895	W30×191 W36×170	W27×161 W30×173 W33×141	W21×147 W24×131 W30×116	W14×109 W16×100 W18×86 W21×83 W24×76
W36×230	837	W30×173 W36×160	W24×162 W27×161 W33×141 W36×135	W21×132 W24×131 W27×114 W30×108	W14×99 W16×89 W18×86 W21×83 W24×68
W33×221	757	W27×178 W33×152	W24×162 W27×146 W30×132 W33×130	W21×122 W24×117 W27×102 W30×99	W14×90 W16×89 W18×76 W21×68
W36×210	719	W27×161 W30×141	W24×146 W30×132 W30×118	W21×122 W24×104 W27×102 W30×99	W14×90 W16×77 W18×76 W21×68
W33×201	684	W27×161 W33×141	W24×146 W30×124 W33×118	W21×111 W24×104 W27×94	W14×90 W16×77 W18×71 W21×62
W36×194	664	W24×162 W27×161 W33×141 W36×135	W24×146 W27×146 W30×116	W21×111 W24×104 W27×94	W14×82 W16×77 W18×71 W21×62
W36×182	623	W24×162 W27×146 W33×130	W21×147 W24×131 W30×116	W18×119 W21×101 W24×94	W12×87 W14×82 W16×67 W18×65 W21×62
W36×170	580	W24×146 W30×132 W33×130	W21×132 W24×117 W27×114 W30×108	W18×106 W21×101 W24×94 W27×84	W12×79 W14×74 W16×67 W18×60 W21×57 W24×55
W36×160	542	W24×146 W30×124 W33×118	W21×122 W24×117 W27×114 W30×99	W14×120 W18×106 W21×93 W24×84	W12×79 W14×68 W18×60 W21×57 W24×55
W36×150	504	W21×147 W24×131 W30×116	W21×111 W24×104 W27×102 W30×99	W14×120 W18×97 W21×93 W24×76	W12×72 W14×68 W16×67 W18×55 W21×50
W33×141	448	W21×132 W24×117 W27×114 W30×108	W18×119 W21×101 W24×94	W14×99 W18×86 W21×83 W24×76	W12×65 W14×61 W18×50
W36×135	439	W21×122 W24×117 W27×114 W30×108	W18×119 W21×101 W24×94	W14×99 W16×89 W18×86 W21×83 W24×68	W12×65 W14×61 W16×50 W21×44
W33×130	406	W21×122 W24×104 W27×102 W30×99	W18×106 W21×101 W24×94 W27×84	W14×90 W16×89 W18×76 W21×73 W24×68	W12×58 W14×53 W16×50 W18×46 W21×44
W33×118	359	W18×119 W21×101 W27×94	W14×120 W18×97 W21×93 W24×84	W14×90 W16×77 W18×71 W21×62	W10×60 W12×53 W14×48 W16×45 W18×40
W30×116	329	W18×106 W21×101 W24×94 W27×84	W14×109 W16×100 W18×86 W21×83 W24×76	W12×87 W14×82 W16×67 W18×65 W21×62	W10×60 W12×50 W14×43 W16×40
W30×108	299	W14×120 W18×97 W21×93 W24×84	W14×99 W16×89 W18×86 W21×73 W24×68	W12×79 W14×74 W16×67 W18×60 W21×57 W24×55	W10×54 W12×45 W14×43 W16×36 W18×35
W30×99	269	W14×109 W16×100 W18×97 W21×83 W24×76	W14×90 W16×77 W18×76 W21×68	W12×72 W14×68 W16×67 W18×55 W21×50	W10×49 W12×40 W14×38 W16×36 W18×35
W27×94	243	W14×99 W16×89 W18×86 W21×73 W24×68	W14×82 W16×77 W18×71 W21×62	W12×65 W14×61 W16×57 W18×50	W10×45 W12×35 W14×34 W16×31
W27×84	213	W14×90 W16×77 W18×76 W21×68	W12×79 W14×74 W16×67 W18×60 W21×57 W24×55	W12×58 W14×53 W16×50 W18×46 W21×44	W10×39 W12×35 W14×30
W24×84	196	W14×82 W16×77 W18×71 W21×62	W12×72 W14×68 W16×67 W18×55	W12×53 W14×48 W16×45 W21×44	W10×39 W12×30 W14×30 W16×26
W24×76	176	W12×87 W14×74 W16×67 W18×65 W21×57 W24×55	W12×65 W14×61 W16×57 W18×50	W12×50 W14×43 W16×40	W10×33 W12×26

Table 1. (cont.)

"a" Segment		0.20*L*	0.15*L*	0.10*L*	0.05*L*
% Maximum Moment		64	51	36	19
W24×68	154	W12×72	W12×58	W10×49	W8×35
		W14×68	W14×53	W12×45	W10×30
		W16×67	W16×50	W14×38	W12×26
		W18×55	W18×46	W16×36	W14×22
			W21×44	W18×35	
W24×62	131	W12×65	W10×60	W10×45	W8×28
		W14×61	W12×53	W12×40	W10×26
		W16×57	W14×48	W14×34	W12×22
		W18×50	W16×45	W16×31	
		W21×44	W18×40		
W21×62	127	W12×65	W10×60	W10×45	W8×28
		W14×61	W12×50	W12×35	W10×26
		W16×50	W14×48	W14×34	W12×22
		W18×46	W16×40	W16×31	
		W21×44			
W24×55	114	W14×53	W10×54	W10×39	W8×28
		W16×45	W12×45	W12×35	W10×22
		W18×46	W14×43	W14×30	W12×19
		W21×44	W16×36		M14×18
			W18×35		
W18×55	98.3	W12×50	W10×45	W10×33	W8×21
		W14×43	W12×40	W12×30	W10×19
		W16×40	W14×34	W14×26	M14×18
W21×50	94.5	W10×54	W10×45	W10×33	W8×21
		W12×50	W12×40	W12×26	W10×19
		W14×43	W14×34		M14×18
		W16×40	W16×31		
W18×50	88.9	W10×54	W10×54	W8×35	W6×25
		W12×45	W12×35	W10×30	W8×21
		W14×43	W14×34	W12×26	W10×19
		W16×36	W16×31		W12×16
		W18×35			
W21×44	81.6	W10×49	W10×39	W8×35	W6×25
		W12×40	W12×35	W10×30	W8×18
		W14×38	W14×30	W12×26	W10×17
		W16×36		W14×22	W12×16
		W18×35			
W18×40	68.4	W10×45	W10×33	W8×28	W6×20
		W12×35	W12×30	W10×26	M6×20
		W14×34	W14×26	W12×22	W8×18
		W16×31			W10×15
					W12×14
W16×40	64.7	W10×39	W10×30	W8×28	W6×20
		W12×35	W12×26	W10×22	M6×20
		W14×30			W8×18
					W10×15
					M12×11.8
W18×35	57.6	W10×39	W8×35	W8×24	W6×20
		W12×30	W10×30	W10×22	M6×20
		W16×26	W12×26	W12×19	W8×15
			W14×22	M14×18	W10×12
					M12×11.8
W14×34	48.6	W8×35	W8×28	W8×21	M5×18.9
		W10×30	W10×26	W10×19	W6×15
		W12×26	W12×22	W12×16	W8×13
					M12×11.8

of concentrated loads such that the load intensity is in accordance with their spacings and where the outermost load at each end is distributed between the bearing point of the span and the first interior load. The moment at any load point of concentrated load may be expressed by the same moment formula for the line load. For the uniformly loaded span, see Fig. 3.

The theorem above allows moments to be calculated by using the moment equation for the simple beam uniformly loaded span for any concentrated load. For spans loaded with multiple concentrated loads, such as girders and trusses, moment calculations will be eased by application of the theorem.

By inspection of Fig. 3, one can see that with multiple concentrated loadings on a span, the moment curve will approach the uniformly loaded case. Should a concentrated load also be at the center of a span, the maximum moment will also be the same for both loadings. As the moment curves approach equality, so also will the deflection curves. Thus, the equivalent uniformly loaded span is the limiting condition for both moments and deflections of a span.

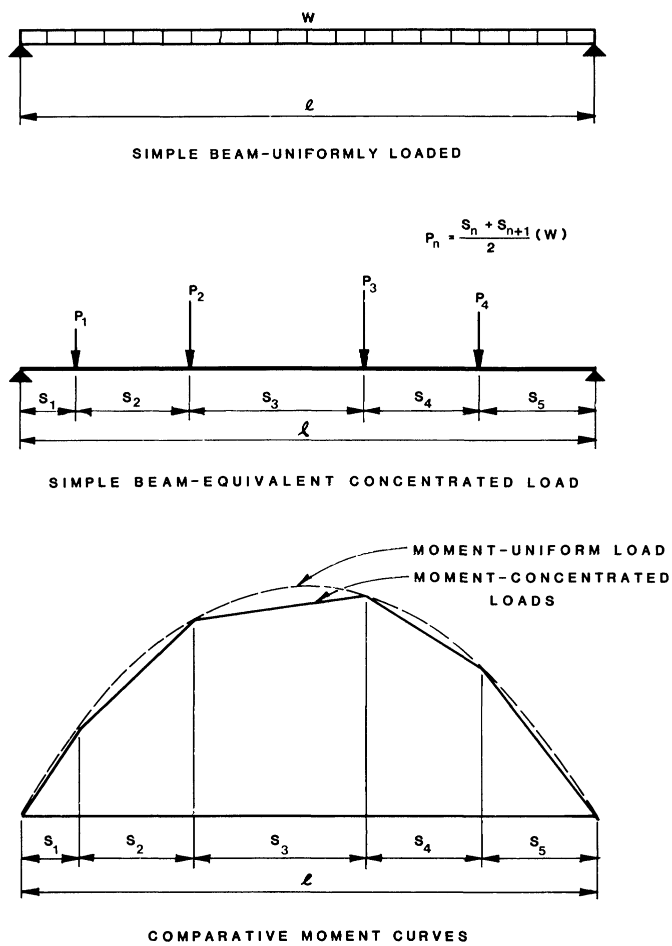

Figure 3

DEFLECTION OF UNIFORMLY LOADED STEPPED BEAMS AND GIRDERS

The deflection formulae presented below are for the deflection at the center of a span for a uniformly loaded, simple span with steps.

For symmetrical stepped beams as shown in Fig. 4:

$$\Delta_{\text{℄}} = \frac{WL^4}{EI_2}\left[\frac{5}{384} - \frac{c^3(4-3c)}{24}\right] + \frac{WL^4}{EI_1}\left[\frac{c^3(4-3c)}{24}\right] \tag{1}$$

For asymmetrical stepped beams as shown in Fig. 5:

$$\Delta_{\text{℄}} = \frac{WL^4}{48E}\left[\frac{5}{8I_2} - \frac{c^3(4-3c)}{I_2} + \frac{c^3(4-3c)}{I_1}\right] \tag{2}$$

The first term in the brackets of the formulae above can be seen to be the deflection constants for a regular prismatic beam. The additional terms account for the stepped segment. Where c is between 0.05 and 0.20, the influence of the step on the deflection is minor. Tables 2 and 3 compare the centerline deflections for stepped beams as illustrated in Figs. 6 and 7 with a standard prismatic beam. These tables show that the greatest increase in the deflection occurs when a equals $0.20L$. For the symmetrical beam, the steps increased the deflection by 0.13 in. or $L/2700$ of the span.

DEFLECTION OF POINT-LOADED STEPPED BEAMS AND GIRDERS

The formulae for the centerline deflection for a stepped beam with concentrated load or loads are derived by ap-

SYMMETRICAL STEPPED BEAM-
UNIFORMLY LOADED

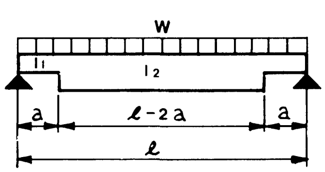

Figure 4

ASYMMETRICAL STEPPED BEAM-
UNIFORMLY LOADED

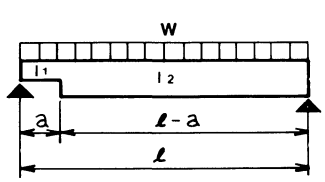

Figure 5

Table 2. Comparative Deflection at ℄—Symmetrical Stepped Beam

c	0.0	0.05	0.10	0.15	0.20
a (ft)	0.0	1.5	3.0	4.5	6.0
SECTION		W10×33			
SECTION			W10×60		
SECTION				W14×61	
SECTION	W24×76				W14×74
I_1 (in.^{4})	2,100 ($=I_2$)	170	341	640	796
$\Delta_{\text{℄}}$ (in.)	0.94	0.95	0.99	1.02	1.07

Table 3. Comparative Deflection at ℄—Asymmetrical Stepped Beam

c	0.0	0.05	0.10	0.15	0.20
a (ft)	0.0	1.5	3.0	4.5	6.0
SECTION		W10×33			
SECTION			W10×60		
SECTION				W14×61	
SECTION					W14×74
SECTION	W24×76				
I_1 (in.^{4})	2,100	170	341	640	796
$\Delta_{\text{℄}}$ (in.)	0.94	0.95	0.97	0.98	1.00

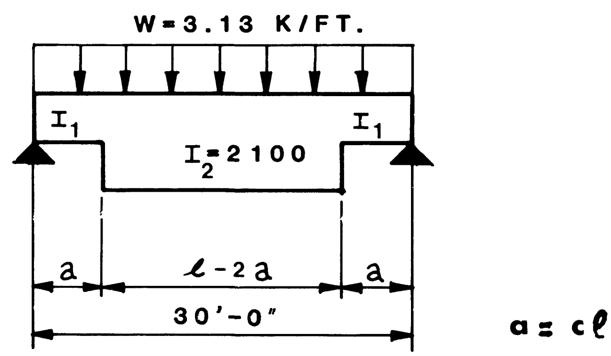

Figure 6

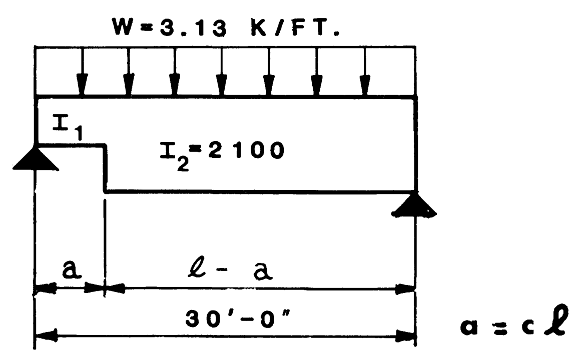

Figure 7

plying Maxwell's theorem of reciprocal deflection. This theorem states "the deflection at the point A caused by a load P at the point B is equal to the deflection at point B produced by moving the same load to the point A." Thus, for each concentrated load on a span, the sum of deflection for each load applied to the formulae with the applicable constants for that load will produce the centerline deflection for all concentrated loads on a span. The formulae are given below. For symmetrical stepped sections as shown in Fig. 8 where $x_a \leq a$:

$$\Delta_{\mathbb{C}} = \frac{P_a L^3}{E}\left[\frac{c^2 b}{4I_1} - \frac{b^3}{12\,I_1} + \frac{(1 - 4c^2)b}{16I_2}\right] \tag{3}$$

where $a \leq x \leq L/2$:

$$\Delta_{\mathbb{C}} = \frac{PL^3}{E}\left[\frac{c^3}{6I_1} - \frac{c^3}{6I_2} + \frac{b}{16I_2} - \frac{b^3}{12I_2}\right] \tag{4}$$

For asymmetrical stepped sections as shown in Fig. 9 where $x_a \leq a$:

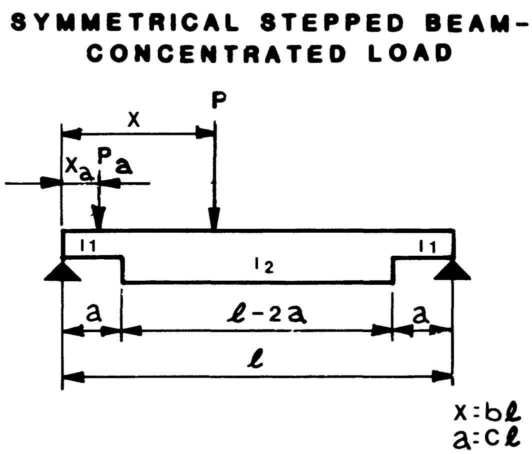

Figure 8

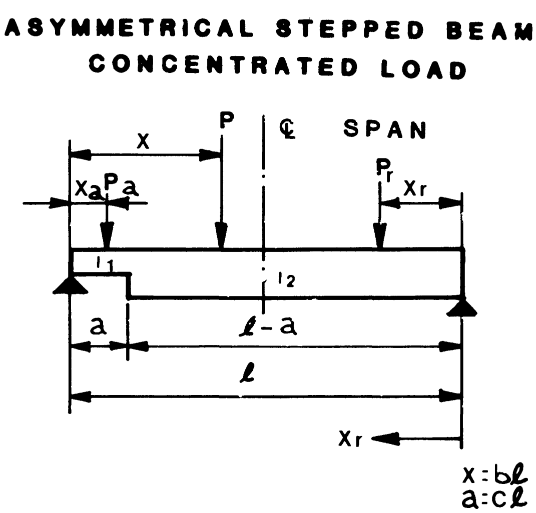

Figure 9

$$\Delta_{\mathbb{C}} = \frac{P_a L^3}{12EI_1}\left[c^2 b(3 - 2c) - b^3\right] + \frac{P_a L^3 b}{48EI_2}(3 - 12c + 8c^3) \tag{5}$$

where $a \leq x \leq L/2$:

$$\Delta_{\mathbb{C}} = \frac{Pa^3}{6EI_1}(1 - b) + \frac{PL^3 b}{48EI_2}(3 - 12c^2 + 8c^3) - \frac{PL^3}{12EI_2}(b + 2c)(b - c)^2 \tag{6}$$

where $x_r \leq L/2$ is measured from the support opposite the stepped segment:

$$\Delta_{\mathbb{C}} = \frac{P_r a^3 b}{6EI_1} + \frac{P_r L^3}{48EI_2}(3b - 8bc^3 - 4b^3) \tag{7}$$

If deflections are required at points other than at the centerline of a span for stepped sections, a deflection analysis by sections may be accomplished with available formulae. This solution involves segmenting the beam as illustrated in Fig. 10 and applying the deflection formulae to each section and combining the results.

An outline for calculating the deflection at points greater than a are listed below for Fig. 10.

1. The segment between the stepped section is assumed to be a simple beam with loads as shown in Fig. 10c. Determine the rotation θ_a due to the end moments and the load.
2. Determine the deflection due to the rotation θ_a which is θ_a multiplied by a.
3. Determine the deflection on the stepped segment a due to beam reaction V acting as a cantilever beam of span length a (Fig. 10d). Add this deflection to Step 2 above.
4. Determine the deflection on stepped segment a due to the uniform load on segment a (Fig. 10d) as a cantilever beam of span length a. Subtract this deflection from 2 and 3 above. The result is the deflection from the beam support to the end of segment a.
5. Calculate the deflection of the center segment (Fig. 10c) for both the uniform load and the end moment M_a on the center segment. Add this deflection to the deflection from 2, 3 and 4 above.

5a. Alternate Step 5
In lieu of span segment L-2a, use span length L and I_2 constant for length L. Substitute these factors into the simple beam, uniformly loaded deflection equation and calculate deflections at point a and beyond. Add the difference in deflection beyond point a to Steps 2, 3 and 4. The effect of end moment M_a is included by this method.

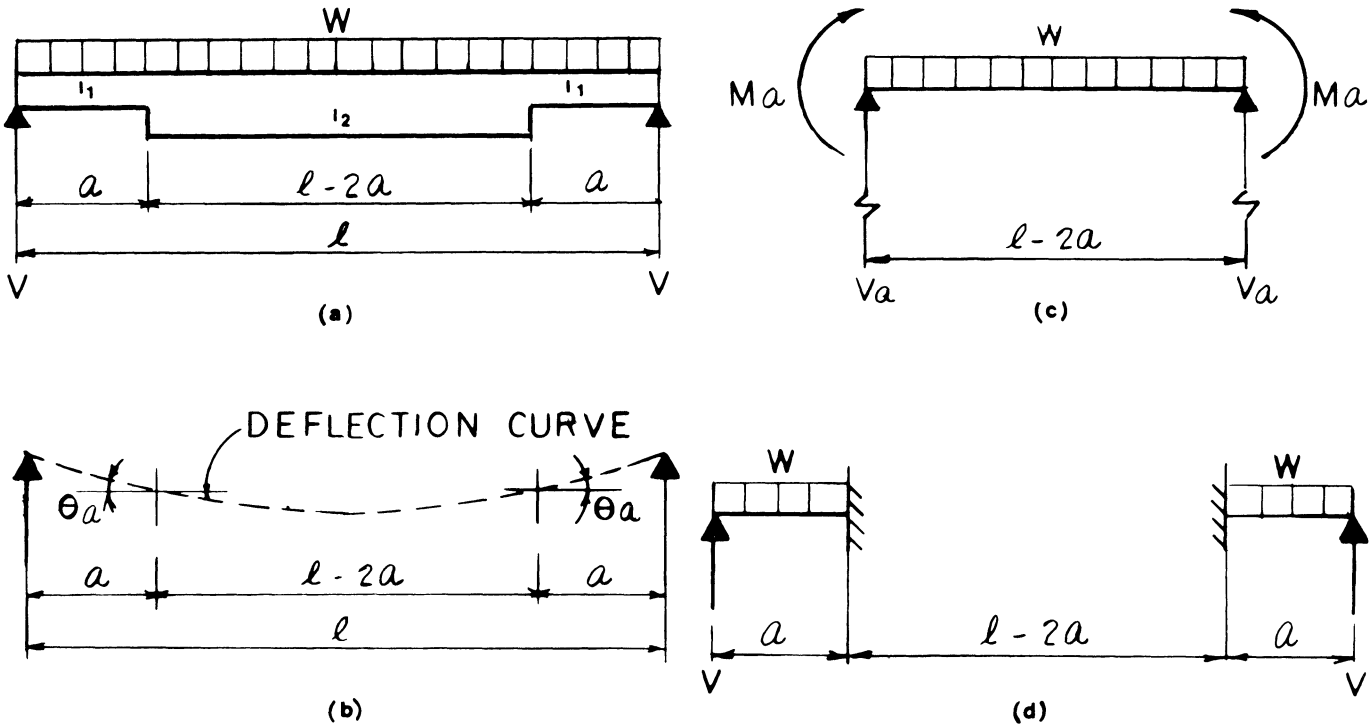

Figure 10

SUMMARY AND CONCLUSIONS

The deflection of a beam section is not significantly increased by the presence of a stepped segment whose length does not exceed 20% of the span length.

The benefits of the increased ceiling space at the stepped segment of a beam can be significant. It could result in reducing the cost of the structure by reducing the column lengths, the curtain wall and the mechanical systems that are usually installed in a building.

NOMENCLATURE

a = length of end segment of a stepped section
b = ratio of length from bearing of concentrated load where distance $x = bL$
c = ratio of length of stepped segment of beam where $a = cL$
x = distance of concentrated load from the bearing
x_r = distance of concentrated load from support in asymmetrical stepped beam where stepped segment occurs on opposite bearing side
E = Young's modulus of elasticity
I_1 = moment of inertia of stepped segment of a stepped beam
I_2 = moment of inertia of main segment of a stepped beam
S_x = section modulus of a beam
P = concentrated load on a span
P_n = concentrated load of a span at point n
S = spacing between concentrated loads on a span
S_n = spacing between concentrated loads on a span left of point n
S_{n+1} = spacing between concentrated loads on a span right of point n
S_1 = spacing between concentrated loads on a span between bearing and point 1
S_2 = spacing between concentrated loads on a span between S_1 and S_2
W = uniform load on a span
L = span length
K = kips or 1,000 pounds
M = moment
Δ = deflection
℄ = centerline of a beam span or beam support

REFERENCES

1. *American Institute of Steel Construction, Inc.* Manual of Steel Construction *8th Ed., Chicago, Ill., 1980.*
2. *Grinter, Linton E.* Vol. II, Theory of Modern Steel Structures *Macmillan Co., New York, N.Y. 1949.*
3. *Morris, I. E.* Handbook of Structural Design *Reinhold Publishing Corp., New York, N.Y., 1963.*