Procedure for Design and Analysis of Hanger-type Connections

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The AISC *Manual* (Ref. 1) Part 4, gives a procedure for design of hanger-type connections. An examination of the procedure reveals some difficulties that can easily be removed by modifying the procedure. The difficulties in author's opinion are as follows:

1. The procedure is an iterative process. This can be time-consuming and requires selection of a preliminary trial flange thickness.
2. One can choose a flange thickness from a relatively wide range of values and find that all equations are satisfied. To obtain the *smallest* satisfactory flange thickness a large number of iterations should be performed.
3. In an analysis to obtain the load-carrying capacity of a given connection, an infinite number of load values within a certain range satisfy all the equations involved.

To demonstrate points 2 and 3 above, let us examine Ex. 4 given on page 4-11 of Ref. 2. By using the AISC Manual procedure and performing the calculations, one can observe that in this example any t_f value greater than 0.899 in. will satisfy the equations. Thus instead of a WT16.5 × 76 used in the example, a WT16.5 × 70.5 could be used to resist the applied load of 141.3 kips with a factor of safety greater than 2. In the same example, if one attempts to calculate the load-carrying capacity of the connection where a WT16.5 × 76 is used any value of load T less than 124.7/4 kips will satisfy all the equations. Items 2 and 3 above result from the fact that for values of α between zero and one, Eq. 5 given in the AISC Manual procedure is redundant and can be derived by combining Eqs. 3 and 4.

By using the same equations and assumptions used in the development of the Manual procedure, a modified procedure is proposed later in this paper that results in the following advantages:

1. The proposed procedure is a closed form (non-iterative) solution which eliminates the need for preliminary selection of t_f.
2. In a design problem, for a given load, only one t_f is calculated which is the smallest t_f to resist the load with a factor of safety of 2.
3. In an analysis problem, to calculate the load carrying capacity of a given connection, only one value of load is calculated which is the largest load that the connection can resist with a factor of safety of 2.

The technical merit of the assumptions or the concept used in the development of *Manual* procedure are not scrutinized here. It has been shown in Ref. 2 that the concept results in "better correlation between estimated connection strength and observed test results."[3] It should also be emphasized that use of AISC Manual procedure always results in a satisfactory but not necessarily the lightest design.

The objective of this paper is to propose a modified version of the AISC *Manual* procedure by rearranging the equations to achieve the above advantages.

In the original concept, as described in Ref. 3, the flange of the tee hanger is assumed to act as a beam supported at the edge of the bolt hole and at the tip of the flange as shown in Fig. 1. In addition it is assumed that a moment equal to the full plastic moment of the flange (half the plastic moment in allowable stress design) exists in the flange adjacent to the stem (Fig. 1). Based on these assumptions and static equilibrium of moments and reactions at segments *ab* and *bc* in Fig. 1, the following equations are derived. These equations relate applied load, geometry and yield-stress of the material.

$$\delta = 1 - d'/p \quad (1)$$

$$M = pt_f^2F_y/8 \quad (2)$$

$$\alpha = (Tb'/M - 1)/\delta \quad (3)$$

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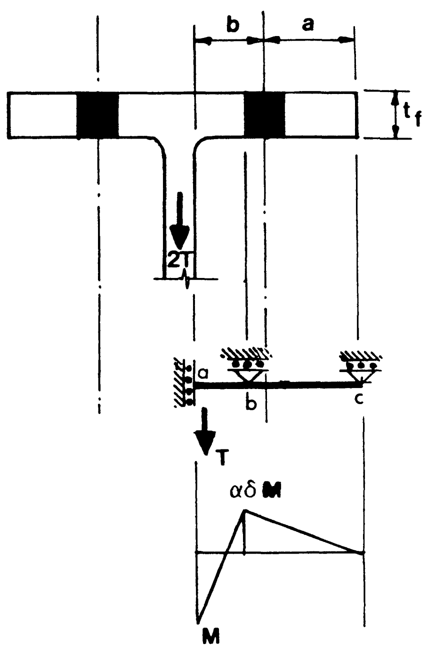

Fig. 1. Beam model of tee hanger flange

$$B_c = T\left[1 + \frac{\alpha\delta}{(1 + \alpha\delta)}(b'/a')\right] \quad (4)$$

$$t_f = \left[\frac{8B_c a' b'}{pF_y[a' + \delta\alpha(a' + b')]}\right]^{1/2} \quad (5)$$

$$Q = B_c - T \quad (6)$$

Two major failure modes (or limit states) associated with tee hangers are (1) beam mechanism failure of flange as a result of formation of plastic hinges at the edge of bolt hole and at a section along the face of the stem; and (2) tension failure of bolt as a result of bolt force reaching capacity of the bolt in tension. It is essential to realize that these two failure modes are independent and should be treated separately.

To consider flange failure one can use Eq. 3 with α equal to 1.0 which corresponds to the case where, in addition to the plastic hinge adjacent to the stem, a second plastic hinge with a moment equal to δM_p has also been developed at the edge of the bolt hole.

Therefore, the governing equation for failure of flange plate in bending is

$$1 = (Tb'/M - 1)/\delta \quad (7)$$

or in terms of t_f and T (and using a factor of safety of 2.0)

$$t_f = \left[\frac{8Tb'}{pF_y(1 + \delta)}\right]^{1/2} \quad (8)$$

$$T = \frac{(1 + \delta)}{8b'} pF_y t_f^2 \quad (9)$$

However, since bolts have a finite capacity, it is possible that bolt failure occurs before flange plate reaches a mechanism failure. For this case α will be less than 1.0 and the governing equations are Eq. 4 and 5. By eliminating $\delta\alpha$ between these two equations the following equation will result, which is the governing equation for bolt failure.

$$T = \frac{B_c a'}{a' + b'} + \frac{pF_y}{8(a' + b')} t_f^2 \quad (10)$$

Also

$$T \leqq B \qquad \text{and} \qquad B_c \leq B \quad (11)$$

Equations 9, 10 and 11 are plotted in Fig. 2 as a solid line. The curve is a limit state function separating satisfactory and unsatisfactory regions in $T - t_f$ space. Point B is the intersection of the two failure modes and corresponds to a balanced design where theoretically the plate and the bolt have exactly the same strength. Curve AB corresponds to a plate failure mode, whereas curve BCD represents a bolt failure mode. Curve BC corresponds to cases where $0 \leq \alpha \leqq 1$ and prying action exists, whereas line CD represents the cases for which $\alpha = 0$ and no prying action develops. Based on the above discussion, the following design and analysis procedures are proposed. Some steps in these procedures are the same as AISC *Manual* procedure.

ALLOWABLE STRESS DESIGN OF HANGER TYPE CONNECTION

Step 1
Determine required number and size of high strength bolts, using the allowable tension given in Table I-A of AISC Manual (Ref. 1) and p, the length of flange tributary to the support of one bolt.

Step 2
Compute the applied load T tributary per bolt by dividing total load by the number of bolts. Estimate the value of b, based on a given gage or the distance required for wrench clearance. Select value of distance a such that $a \leqq 1.25b$.

Step 3
Compute a', b', d' and δ from the following equations.

$$a' = a + d/2 \quad (12)$$

$$b' = b - d/2 \quad (13)$$

$$d' = d + 1/16 \quad (14)$$

$$\delta = 1 - d'/p \quad (15)$$

Step 4
Compute the balanced load T_0 from the following equation (Eq. 4 with $\alpha = 1$).

$$T_0 = \frac{B}{1 + \frac{\delta}{1 + \delta}(b'/a')} \qquad (16)$$

Step 5
If $T \leqq T_0$ then plate failure mode governs and the minimum required t_f should be calculated using the following equation (Eq. 3 with $\alpha = 1$).

$$t_f = \left[\frac{8Tb'}{pF_y(1 + \delta)}\right]^{1/2} \qquad (17)$$

Step 6
Check strength of web and the welds connecting web to flange (if any).

If $T \geqq T_0$ then bolt failure mode governs and minimum required t_f should be calculated from following equation.

$$t_f = \left[\frac{8[T(a' + b') - Ba']}{pF_y}\right]^{1/2} \qquad (18)$$

ANALYSIS OF HANGER-TYPE CONNECTION

To obtain allowable capacity of a given hanger-type connection, the following steps can be followed.

Step 1
Compute a', b', d', δ and M from the following equations.

$$a' = a + d/2$$

$$b' = b - d/2$$

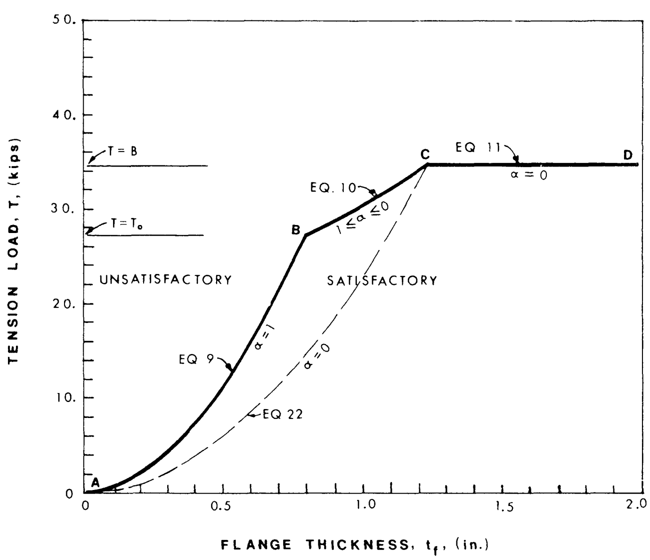

Fig. 2. $T - t_f$ curve for Example 4 in Ref. 2 (same as Example I in this paper)

$$d' = d + 1/16$$

$$\delta = 1 - d'/p$$

$$M = pF_y t_f^2/8$$

Step 2
The allowable tension per bolt T is the smallest value calculated from the following equations. The equations represent three segments of limit state curve in Fig. 2.

$$T_1 = \frac{(1 + \delta)M}{b'} \quad \text{(plate failure mode)} \qquad (19)$$

$$T_2 = \frac{Ba' + M}{a' + b'} \quad \text{(bolt failure mode with prying action)} \qquad (20)$$

$$T_3 = B \quad \text{(bolt failure mode without prying action)} \qquad (21)$$

FATIGUE

When considering fatigue, a maximum t_f can be calculated from the following equation which results in α being equal to 0.0 (no prying action) and $T = B_c$ (Eq. 3 with $\alpha = 0$).

$$t_f = \left[\frac{8Tb'}{pF_y}\right]^{1/2} \qquad (22)$$

Equation 22 is shown in Fig. 2 by dashed line.

Example 1

Repeat Example 4 in Ref. 2 and calculate required t_f using same dimensions as given in Ref. 2.

Given

$a' = 2.0$ in.

$b' = 1.183$ in.

$p = 6$ in.

$d = 1.0$ in.

$d' = 1.063$ in.

$\delta = 0.823$

$F_y = 36$ ksi

$B = 34.6$ kips (1-in. dia. A325 bolt)

$T = 114.3/4 = 28.6$ kips

Solution

$$T_0 = \frac{B}{1 + \frac{\delta}{1 + \delta}(b'/a')}$$

$$= \frac{34.6}{1 + \frac{0.823}{1.823}(1.183/2.0)} = 27.3 \text{ kips}$$

Since $T > T_0$, then bolt capacity governs and

$$\text{Req'd } t_f = \left[\frac{8T(a' + b') - Ba'}{pF_y}\right]^{1/2}$$

$$= \left[\frac{8[(28.6)(3.183) - 34.6(2.0)]}{6(36)}\right]^{1/2}$$

$$= 0.899 \text{ in.}$$

use WT16.5 × 70.5 ($t_f = .96$)

(bolt capacity governs)

Example 2.

Consider connection designed in p. 4-91 of AISC Manual Ref. 1, and calculate allowable load carrying capacity of the connection.

Given

$a' = 2.153$ in.

$b' = 1.417$ in.

$p = 4.5$ in.

$d = 0.75$ in.

$d' = 0.8125$ in.

$\delta = 0.819$ in.

$F_y = 36$ ksi

$B = 19.4$ kips

$t_f = 0.695$ in.

$M = pF_y t_f^2/8 = 9.78$ kip-in.

Solution

$$T_1 = \frac{(1 + \delta)M}{b'} = \frac{(1 + .819)(9.78)}{1.417} = 12.55 \text{ kips}$$

$$T_2 = \frac{Ba' + M}{a' + b'} = \frac{19.4(2.153) + 9.78}{3.57} = 14.4 \text{ kips}$$

$$T_3 = B = 19.4 \text{ kips}$$

Therefore

$T_{\text{allow}} = 12.55$ kips

(plate failure governs)

NOMENCLATURE

T	= applied tension per bolt (exclusive of initial tightening), kips
T_0	= balance tension load, kips
T_1, T_2, T_3	= allowable tension per bolt, kips
B	= allowable load on bolt, kips
B_c	= load per bolt including prying force, kips
M	= allowable bending moment tributary to tee-stub flange or angle leg by one bolt, kip-in.
F_y	= yield strength of the flange material, ksi
p	= length of flange, parallel to stem or leg, tributary to each bolt, in.
t_f	= required thickness of tee-stub flange or angle leg, in.
b	= distance from bolt center line (gage line) to the face of tee stem or angle leg, in.
a	= distance from bolt center line to edge of tee flange or angle leg but not more than $1.25b$, in.
d	= bolt nominal diameter, in.
d'	= width of bolt hole in flange parallel to tee stem or angle leg, in.
b'	= $b - d/2$, in.
a'	= $a + d/2$, in.
δ	= ratio of net area (at bolt line) and the gross area (at the face of the stem or angle leg)
Q	= prying force, kips
α	= ratio of flange moment at edge of bolt hole to δM
M_p	= plastic moment tributary to tee-stub flange or angle leg by one bolt, equal to $2M$, kip-in.

REFERENCES

1. *American Institute of Steel Construction, Inc.* Manual of Steel Construction *8th Ed., Chicago, Ill., 1980, p. 4-88.*
2. *American Institute of Steel Construction, Inc.* Engineering for Steel Construction *Chicago, Ill., 1984, p. 4-11.*
3. *Fisher, John W. and John H. A. Struik* Guide to Design Criteria for Bolted and Riveted Joints *John Wiley & Sons, Inc., New York, N.Y., 1974, p. 271.*