Design and Optimization of Plate Girders and Weld-fabricated Beams for Building Construction

WALTER H. FLEISCHER

Currently, design of plate girders and weld-fabricated beams necessitates an iteration of trial designs to establish the web-buckling range used in analysis.

The material presented here illustrates an expeditious mathematical and nomographic design method for bisymmetric plate girders and weld-fabricated beams, as well as material-optimization for any given moment-shear combination. The method shown is based on the web-buckling theory (no tension-field action), and on the assumption of flange configurations that do not necessitate bending-stress reductions in the compression flange. Design examples are given numerically and shown on nomographs.

CROSS-SECTIONAL PROPERTIES

In general terms, a girder's moment of inertia can be expressed as follows (see Fig. 1):

$$I = d^2 \frac{A_w}{12} + 2\left(\frac{d-h}{2}\right)^2 \frac{A_f}{12} + 2\left(\frac{d+h}{4}\right)^2 A_f \quad (1)$$

Developed:

$$I = \frac{d^2}{12} \times [A\,(1 + \phi + \phi^2) - d\,t\,\phi\,(1 + \phi)] \quad (2)$$

Hence:

$$S = \frac{d}{6} \times [A\,(1 + \phi + \phi^2) - d\,t\,\phi\,(1 + \phi)] \quad (3)$$

Walter H. Fleischer is Senior Structural Consultant with Bethlehem Steel Corporation, Bethlehem, Pennsylvania

Re-arranging:

$$A = \frac{1}{1 + \phi + \phi^2} \times \left[\frac{6\,S}{d} + d\,t\,\phi\,(1 + \phi)\right] \quad (4)$$

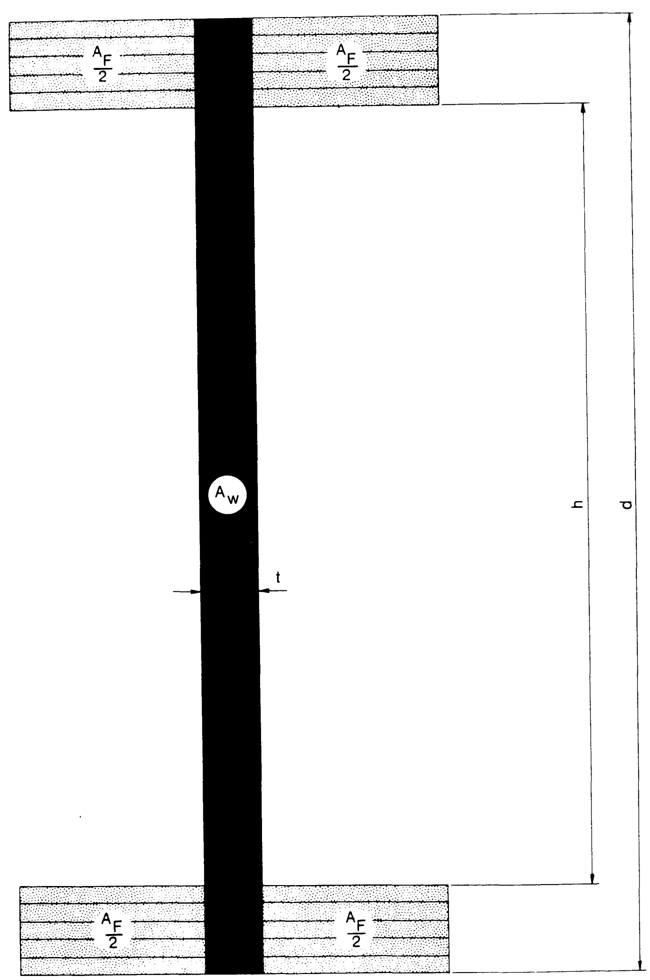


Figure 1

Substituting: $A_f = \dfrac{A - d\,t}{2}$

$$A_f = \frac{1}{1 + \phi + \phi^2} \times \left(\frac{3\,S}{d} - \frac{d\,t}{2}\right) \qquad (5)$$

Very close results can be obtained by the simplifying approximations $(1 + \phi + \phi^2) = 3\,\phi$ and $(1 + \phi) = 2\,\phi^{1/2}$, which will reduce the cross-sectional property equations to:

$$S = d\,\phi \left(\frac{A}{2} - \frac{d\,t\,\phi^{1/2}}{3}\right) \qquad (3a)$$

$$A = \frac{2\,S}{d\,\phi} + \frac{2\,d\,t\,\phi^{1/2}}{3} \qquad (4a)$$

$$A_f = \frac{S}{d\,\phi} - \frac{d\,t}{6\,\phi} \qquad (5a)$$

FLANGE DESIGN NOTES

To avoid reductions of allowable stresses in the compression flange, the following AISC Specification[1] requirements have to be met:

1. $\dfrac{b}{t_f}$ is to be smaller than $\dfrac{190}{F_y^{1/2}}$ (Sect. 1.9.1)
2. $\dfrac{h}{t}$ is to be smaller than $\dfrac{760}{(0.6\,F_y)^{1/2}}$ (Sect. 1.10.6)

To qualify as a fully compact compression flange:

3. $\dfrac{b}{t_f}$ is to be smaller than $\dfrac{130}{F_y^{1/2}}$ (Sect. 1.5.1.4.1-(2))

In all cases, a laterally supported compression flange is assumed.

SPECIFICATION REQUIREMENTS

Plate girder design is governed by the following AISC Specification[1] requirements:

$$f_v = \frac{V}{d\,t} \quad \text{(Sect. 1.5.1.2.1)} \qquad (6)$$

$$F_v = \frac{F_y}{2.89}(C_v) \leqq 0.40\,F_y \quad \text{(Eq. 1.10-1)} \qquad (7)$$

where $C_v = \dfrac{45{,}000\,k\,t^2}{F_y\,h^2}$ when C_v is less than 0.8 (7a)

$$= \frac{60\,t}{h}\left(\frac{10\,k}{F_y}\right)^{1/2}$$

when C_v is more than 0.8* (7b)

in which

$k = 4.00 + \dfrac{5.34\,h^2}{a^2}$ when $\dfrac{a}{h}$ is less than 1.0

$= 5.34 + \dfrac{4.00\,h^2}{a^2}$ when $\dfrac{a}{h}$ is more than 1.0

$= 5.34$ when $\dfrac{a}{h}$ is more than 3.0**

in which a is the clear distance between stiffeners, in.

$$\frac{h}{t} \leqq \frac{14{,}000}{[F_y(F_y + 16.5)]^{1/2}} \quad \text{(Sect. 1.10.2)} \qquad (8)$$

$$\frac{h}{t} \leqq 260 \quad \text{(Sect. 1.10.5.3)} \qquad (9)$$

$$\frac{h}{t} \leqq \frac{760}{(0.6\,F_y)^{1/2}} \quad \text{for no reduction in flange stress (Sect. 1.10.6)} \qquad (10)$$

$$\frac{h}{t} \leqq \frac{640}{F_y^{1/2}} \quad \text{for compact web***} \qquad (11)$$

(Sect. 1.5.1.4.1-(4))

Note:

Eq. 7a equals Eq. 7b at $\dfrac{h}{t} = 75 \times \left(\dfrac{10\,k}{F_y}\right)^{1/2}$ (12)

Eq. 7 equals 0.4 F_y at $\dfrac{h}{t} = \dfrac{150}{2.89} \times \left(\dfrac{10\,k}{F_y}\right)^{1/2}$ (13)

Eq. 10 governs over Eq. 8 for $F_y < 187$
Eq. 10 governs over Eq. 9 for $F_y > 14$

WEB DESIGN—GENERAL CONDITION

Plate girder webs fall into one of the following five buckling ranges:

1. $F_v = 0.4 \times F_y$ (for convenience, this will be called ``plastic buckling'')
2. C_v is more than 0.8 (compact web, inelastic buckling)
3. C_v is less than 0.8 and Eq. 11 applies (compact web, elastic buckling)
4. C_v is less than 0.8 and Eq. 10 applies (non-compact web, elastic buckling)

**The exact value of $60 \times 10^{1/2}$ is used in lieu of the approximate value of 190 used in the AISC Specification.*[1]

***This represents "Accepted Engineering Practice."*

****The buckling parameter $\frac{h}{t}$ is used in lieu of the ratio $\frac{d}{t}$ used in the AISC Specification.*[1]

5. C_v is less than 0.8 and Eq. 10 is exceeded (reduction in compression flange allowable bending stress—not considered herein)

The applicable range is found by the following algebraic manipulations:

1. Combine Eq. 6 = 0.4 F_y with h/t to be equal to or smaller than Eq. 13

$$\frac{V}{d\,t} = 0.4\,F_y \quad \text{and} \quad \frac{h}{t} \leqq \frac{150}{2.89} \times \left(\frac{10\,k}{F_y}\right)^{1/2}$$

2. Combine Eq. 6 = Eq. 7 using Eq. 7b with h/t to be equal to or smaller than Eq. 12

$$\frac{V}{d\,t} = \frac{F_y}{2.89} \times \frac{60\,t}{h} \times \left(\frac{10\,k}{F_y}\right)^{1/2} \quad \text{and}$$

$$\frac{h}{t} \leqq 75 \times \left(\frac{10\,k}{F_y}\right)^{1/2}$$

3. Combine Eq. 6 = Eq. 7 using Eq. 7a with Eq. 11

$$\frac{V}{d\,t} = \frac{F_y}{2.89} \times \frac{45{,}000\,k\,t^2}{F_y\,h^2} \quad \text{and} \quad \frac{h}{t} \leqq \frac{640}{F_y^{1/2}}$$

4. Combine Eq. 6 = Eq. 7 using Eq. 7a with Eq. 10

$$\frac{V}{d\,t} = \frac{F_y}{2.89} \times \frac{45{,}000\,k\,t^2}{F_y\,h^2} \quad \text{and}$$

$$\frac{h}{t} \leqq \frac{760}{(0.6\,F_y)^{1/2}}$$

5. Not considered in this paper

The results of these operations will be:

$$1.\ V \geqq \frac{d^2\,F_y^{3/2}\,\phi}{948.2} \times \left(\frac{5.34}{k}\right)^{1/2} \quad \text{(plastic-compact)} \qquad (14a)$$

$$2.\ V \geqq \frac{d^2\,F_y^{3/2}\,\phi}{1{,}980} \times \left(\frac{5.34}{k}\right)^{1/2} \quad \text{(inelastic-compact)} \qquad (14b)$$

$$3.\ V \geqq \frac{d^2\,F_y^{3/2}\,\phi}{3{,}153} \times \left(\frac{k}{5.34}\right) \quad \text{(elastic-compact)} \qquad (14c)$$

$$4.\ V \geqq \frac{d^2\,F_y^{3/2}\,\phi}{11{,}360} \times \left(\frac{k}{5.34}\right) \quad \text{(elastic-non-compact)} \qquad (14d)$$

$$5.\ V < \frac{d^2\,F_y^{3/2}\,\phi}{11{,}360} \times \left(\frac{k}{5.34}\right) \quad \text{(not considered)} \qquad (14e)$$

WEB DESIGN

For plastic buckling:
Take Eq. 6 $\leqq$ 0.4 F_y and solve for t:

$$\frac{V}{d\,t} \leqq 0.4\,F_y \quad \text{therefore} \qquad (15a)$$

$$t \geqq 2.5 \times \frac{V}{d\,F_y}$$

For inelastic buckling:
Take Eq. 6 to be equal to or smaller than Eq. 7 using Eq. 7b and solve for t:

$$\frac{V}{d\,t} \leqq \frac{F_y}{2.89} \times \frac{60\,t}{h} \times \left(\frac{10\,k}{F_y}\right)^{1/2} \quad \text{therefore} \qquad (15b)$$

$$t \geqq 0.1234 \times \frac{V^{1/2}\,\phi^{1/2}}{k^{1/4}\,F_y^{\,1/4}}$$

For elastic buckling:
Take Eq. 6 to be equal to or smaller than Eq. 7 using Eq. 7a and solve for t:

$$\frac{V}{d\,t} \leqq \frac{F_y}{2.89} \times \frac{45{,}000\,k\,t^2}{F_y\,h^2} \quad \text{therefore} \qquad (15c)$$

$$t \geqq 0.0400 \times \frac{V^{1/3}\,d^{1/3}\,\phi^{2/3}}{k^{1/3}}$$

In general: $$t \geqq m\,V^{(1-n)/2} d^n \qquad (16)$$

with values for m and n from Table 1.

GENERAL DESIGN PROCEDURE

Given: S, V, d, F_y, k, ϕ

1. Compare V with Eqs. 14a – 14e to establish the range
2. Obtain t_{min} from Eqs. 15a – 15c according to the range
3. Choose $t \geqq t_{min}$
4. Obtain $A_{f\,min}$ from either Eq. 5 or Eq. 5a
5. Choose $A_f > A_{f\,min}$ observing the "Flange Design Notes"
6. Obtain net weight of girder from $w = (2\,A_f + d\,t) \times 3.4$, plf

Example 1:

S = 1,600 in.^{3}	V = 300 kips
d = 60 in.	F_y = 36 ksi
k = 5.34	ϕ = 0.96

1. 14a: $V \geqq 787$? No

   14b: $V \geqq 377$? No

   14c: $V \geqq 237$?

   Yes; therefore: compact web – elastic buckling
2. $t_{min} = 0.584$ in.
3. Take $t = 0.625$ in.
4. $A_{f\,min} = 21.3$ in.^{2}
5. Take $A_f = 17.125 \times 1.25 = 21.4$ in.^{2}
   check for compactness:

$$\left(\frac{17.125 + 0.625}{1.25} = 14.2\right) < \left(\frac{130}{36^{1/2}} = 21.7\right) \textbf{ o.k.}$$

6. $w = (2 \times 21.4 + 60 \times 0.625) \times 3.4 = 273$ plf

MATERIAL OPTIMIZATION

For the cross-sectional dimensions encountered in practice, the ratio $h/d = \phi$ has very little influence on optimization results and can be treated as a constant. Introducing Eq. 16 into Eq. 4 will result in a general formulation of the girder cross-sectional area:

$$A = \frac{1}{1 + \phi + \phi^2} \times \left[\frac{6\,S}{d} + m\,\phi\,(1 + \phi)\,V^{(1-n)/2}\,d^{(1+n)}\right] \tag{17}$$

in which S, V and ϕ are given; m and n are obtainable from Table 1 for the given values of F_y, k and ϕ; and d is unknown.

To obtain the minimum area, the first derivative $\frac{\partial A}{\partial d}$ has to equal zero and the second derivative $\frac{\partial^2 A}{\partial d^2}$ has to be positive:

$$\frac{\partial A}{\partial d} = \frac{1}{1 + \phi + \phi^2} \times \left[-\frac{6\,S}{d^2} + (1 + n)\,m\,\phi\,(1 + \phi) \times V^{(1-n)/2}\,d^n\right] = 0 \tag{18}$$

$$\frac{\partial^2 A}{\partial d^2} = \frac{1}{1 + \phi + \phi^2} \times \left[\frac{12\,S}{d^3} + n\,(1 + n)\,m\,\phi\,(1 + \phi) \times V^{(1-n)/2}\,d^{(n-1)}\right] > 0$$

Solving Eq. 18 for d will result in the most economical girder depth:

$$d^* = \left[\frac{6\,S}{(1 + n)\,m\,\phi\,(1 + \phi)\,V^{(1-n)/2}}\right]^{1/(2+n)} \tag{19}$$

Eq. 19 can now be used to obtain other girder parameters. Introducing Eq. 19 into Eq. 16 will indicate the most economical web thickness:

$$t^* = m^{2/(2+n)}\,V^{(1-n)/(2+n)}\left[\frac{6\,S}{(1 + n)\,\phi\,(1 + \phi)}\right]^{n/(2+n)} \tag{20}$$

Introducing Eq. 19 into Eq. 17 will show the material-optimized cross-sectional area:

$$A^* = \frac{2 + n}{1 + \phi + \phi^2} \times \left(\frac{6\,S}{1 + n}\right)^{(1+n)/(2+n)} \times V^{(1-n)/[2 \times (2+n)]} \times [m\,\phi\,(1 + \phi)]^{1/(2+n)} \tag{21}$$

Eq. 19 divided by Eq. 20 results in the optimized d/t ratio:

$$\frac{d^*}{t^*} = \left(\frac{1}{m}\right)^{3/(2+n)} \times \left[\frac{6\,S}{(1 + n)\,\phi\,(1 + \phi)\,V^{3/2}}\right]^{(1-n)/(2+n)} \tag{22}$$

Eq. 19 times Eq. 21 results in:

$$A^*\,d^* = \frac{6\,S}{1 + \phi + \phi^2} \times \frac{2 + n}{1 + n} \tag{23}$$

Eq. 19 squared times Eq. 20 will yield:

$$d^{*2}\,t^* = \frac{6\,S}{(1 + n)\,\phi\,(1 + \phi)} \tag{24}$$

Eq. 19 times Eq. 20 divided by Eq. 21 shows the most economical web area – total area ratio:

$$\frac{d^*\,t^*}{A^*} = \frac{1 + \phi + \phi^2}{(2 + n)\,\phi\,(1 + \phi)} \tag{25}$$

Note that Eqs. 23-25 do not contain web shear V; in addition, Eq. 25 is independent from the section modulus S.

The most economical depth given by Eq. 19 is not very sensitive to optimization, as will be shown in the following:

By multiplying Eq. 17 with the girder depth d and dividing by Eq. 23, the following expression ensues:

$$\frac{A\,d}{A^*\,d^*} = \frac{1+n}{2+n} \times \left[1 + \frac{m\,\phi\,(1+\phi)\,V^{(1-n)/2}\,d^{(2+n)}}{6\,S}\right] \qquad (26)$$

Re-arranging:

$$\frac{A}{A^*} = \frac{1+n}{2+n} \times \left[\frac{d^*}{d} + \frac{d^{*(2+n)}}{d^{*(1+n)}} \times \frac{m\,\phi\,(1+\phi)\,V^{(1-n)/2}\,d^{(1+n)}}{6\,S}\right] \qquad (27)$$

Raising Eq. 19 to the $(2+n)$ power and introducing it into Eq. 27 will give the ratio of the cross-sectional areas of the non-optimized girder to the optimized girder:

$$\frac{A}{A^*} = \frac{1}{.2+n} \times \left[\frac{d^*}{d}(1+n) + \left(\frac{d}{d^*}\right)^{(1+n)}\right] \qquad (28)$$

Fig. 2 shows the weight increases if the chosen girder depth differs from the optimized depth. It demonstrates that even substantial changes in girder depths do not affect girder economy appreciably.

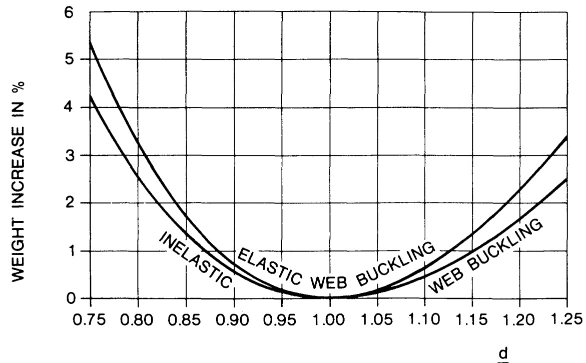


Figure 2

By introducing the values for n and m from Table 1 into Eqs. 19-28 and using the previously recommended approximations $(1 + \phi + \phi^2) = 3\,\phi$ and $(1 + \phi) = 2\,\phi^{1/2}$, the working equations given in Table 2 are obtained.

Table 1

Web buckling	n	m
Plastic	-1	$\frac{2.5}{F_y}$
Inelastic	0	$0.1234 \times \left(\frac{\phi^2}{k\,F_y}\right)^{1/4}$
Elastic	$\frac{1}{3}$	$0.0400 \times \left(\frac{\phi^2}{k}\right)^{1/3}$

Table 2

Dimension or property	Inelastic Web Buckling	Elastic Web Buckling	Equation
d^*	$4.930\left(\frac{S^2\,k^{1/2}\,F_y^{1/2}}{V\,\phi^4}\right)^{1/4}$	$5.621\left(\frac{S^3\,k}{V\,\phi^{13/2}}\right)^{1/7}$	(19)
t^*	$0.1234\left(\frac{V^2\,\phi^2}{k\,F_y}\right)^{1/4}$	$0.0712\left(\frac{S\,V^2\,\phi^{5/2}}{k^2}\right)^{1/7}$	(20)
A^*	$0.8113\left(\frac{S^2\,V}{k^{1/2}\,F_y^{1/2}}\right)^{1/4}$	$0.6226\left(\frac{S^4\,V}{k\,\phi^{1/2}}\right)^{1/7}$	(21)
$\frac{d^*}{t^*}$	$39.95\left(\frac{S^2\,k^{3/2}\,F_y^{3/2}}{V^3\,\phi^6}\right)^{1/4}$	$78.95\left(\frac{S^2\,k^3}{V^3\,\phi^9}\right)^{1/7}$	(22)
$A^*\,d^*$	$\frac{4\,S}{\phi}$	$\frac{7\,S}{2\,\phi}$	(23)
$d^{*2}\,t^*$	$\frac{3\,S}{\phi^{3/2}}$	$\frac{9\,S}{4\,\phi^{3/2}}$	(24)
$\frac{d^*\,t^*}{A^*}$	$\frac{3}{4\,\phi^{1/2}}$	$\frac{9}{14\,\phi^{1/2}}$	(25)
$\frac{A}{A^*}$	$\frac{1}{2}\left[\frac{d}{d^*} + \frac{d^*}{d}\right]$	$\frac{1}{7}\left[3\left(\frac{d}{d^*}\right)^{4/3} + 4\,\frac{d^*}{d}\right]$	(28)

WEB DESIGN—OPTIMIZED CONDITION

The following combinations of equations are used to determine the limits of the buckling ranges, paralleling those for the General Condition:

2a. Combine Eq. 13 and Eq. 22 times ϕ to obtain the lower limit for inelastic web buckling (compact):

$$\frac{h}{t} = 39.95 \frac{S^{1/2}\, k^{3/8}\, F_y^{\,3/8}}{V^{3/4}\, \phi^{1/2}} \quad \text{and}$$

$$\frac{h}{t} \geqq \frac{150}{2.89} \times \left(\frac{10\, k}{F_y}\right)^{1/2}$$

2b. Combine Eq. 12 and Eq. 22 times ϕ to obtain the upper limit for inelastic web buckling (compact):

$$\frac{h}{t} = 39.95 \frac{S^{1/2}\, k^{3/8}\, F_y^{\,3/8}}{V^{3/4}\, \phi^{1/2}} \quad \text{and}$$

$$\frac{h}{t} \leqq 75 \times \left(\frac{10\, k}{F_y}\right)^{1/2}$$

3a. Combine Eq. 12 and Eq. 22 times ϕ to obtain the lower limit for elastic web buckling (compact):

$$\frac{h}{t} = 78.95 \frac{S^{2/7}\, k^{3/7}}{V^{3/7}\, \phi^{2/7}} \quad \text{and}$$

$$\frac{h}{t} \geqq 75 \left(\frac{10\, k}{F_y}\right)^{1/2}$$

3b. Combine Eq. 11 and Eq. 22 times ϕ to obtain the upper limit for elastic web buckling (compact):

$$\frac{h}{t} = 78.95 \frac{S^{2/7}\, k^{3/7}}{V^{3/7}\, \phi^{2/7}} \quad \text{and}$$

$$\frac{h}{t} \leqq \left(\frac{640}{F_y^{\,1/2}}\right)$$

4. Combine Eq. 10 and Eq. 22 times ϕ to obtain the upper limit for elastic web buckling (non-compact, $F_b = 0.6\, F_y$):

$$\frac{h}{t} = 78.95 \frac{S^{2/7}\, k^{3/7}}{V^{3/7}\, \phi^{2/7}} \quad \text{and}$$

$$\frac{h}{t} \leqq \frac{760}{(0.6\, F_y)^{1/2}}$$

The results will be:

$$\text{2a.}\quad \frac{S^2\, F_y^{\,7/2}}{V^3\, \phi^2} \geqq 658 \left(\frac{k}{5.34}\right)^{1/2} \qquad (29a)$$

$$\text{2b.}\quad \frac{S^2\, F_y^{\,7/2}}{V^3\, \phi^2} \leqq 2{,}870 \left(\frac{k}{5.34}\right)^{1/2} \qquad (29b)$$

(inelastic-compact)

$$\text{3a.}\quad \frac{S^2\, F_y^{\,7/2}}{V^3\, \phi^2} \geqq 5{,}100 \left(\frac{k}{5.34}\right)^{1/2} \qquad (29c)$$

$$\text{3b.}\quad \frac{S^2\, F_y^{\,7/2}}{V^3\, \phi^2} \leqq 15{,}100 \left(\frac{5.34}{k}\right)^{3} \qquad (29d)$$

(elastic-compact)

$$\text{4.}\quad \frac{S^2\, F_y^{\,7/2}}{V^3\, \phi^2} \leqq 301{,}000 \left(\frac{5.34}{k}\right)^{3} \quad \text{(elastic-non-compact)} \qquad (29e)$$

No optimization is possible below the limit of Eq. 29a and between the limits of Eqs. 29b and 29c.

OPTIMIZED DESIGN PROCEDURE

The required result of this procedure is to obtain a girder weight which is a minimum for given values of S, V, F_y, k and ϕ. This procedure is carried out as follows.

1. Compare $\dfrac{S^2\, F_y^{\,7/2}}{V^3\, \phi^2}$ with Eqs. 29a-29e to establish the range

If elastic web buckling governs, proceed as follows:

2. Obtain t^* from (Eq. 20 − elastic) in Table 2
3. Choose $t \approx t^*$
4. Obtain d_{max} from Eq. 15c :

$$d_{max} = 83{,}150 \frac{t^3}{V\, \phi^2} \times \left(\frac{k}{5.34}\right)$$

5. Choose $d \leqq d_{max}$
6. Obtain $A_{f\,min}$ from either Eq. 5 or Eq. 5a
7. Choose $A_f \geqq A_{f\,min}$ observing the Flange Design Notes
8. Obtain net weight of girder from $w = (2\, A_f + d\, t) \times 3.4$, plf

Example 2:

$S = 1{,}600$ in.^{3} $\quad V = 300$ kips $\quad F_y = 36$ ksi
$k = 5.34$ $\quad \phi = 0.98$

1. $\dfrac{S^2\, F_y^{\,7/2}}{V^3\, \phi^2} = 27{,}600$ (elastic − non-compact)
2. $t^* = 0.641$ in.
3. Take $t = 0.625$ in.
4. $d_{max} = 70.5$ in.
5. Take $d = 70$ in.
6. $A_{f\,min} = 15.9$ in.^{2}
7. Take $A_f = 16 \times 1 = 16.0$ in.^{2}
   check for flange stress reduction:

$$\left(\frac{16 + 0.625}{1} = 16.6\right) < (190/36^{1/2} = 31.7)$$

$$(68/0.625 = 109) < [760/(0.6 \times 36)^{1/2} = 164] \quad \textbf{o.k.}$$

8. $w = (2 \times 16 + 70 \times 0.625) \times 3.4 = 258$ plf

If inelastic web buckling governs, proceed as follows:

2. Obtain t^* from Eq. 20 – inelastic in Table 2
3. Choose $t \geqq t^*$
4. Obtain d^* from Eq. 24 – inelastic in Table 2:

$$d^* = \left(\frac{3\,S}{t\,\phi^{3/2}}\right)^{1/2}$$

5. Choose $d \approx d^*$ but $\frac{2.5\,V}{t\,F_y} < d < \frac{3.61\,V}{t\,F_y}$ from Eq. 15a, and Eq. 15b introduced into Eq. 14b, respectively
6. Obtain $A_{f\,min}$ from either Eq. 5 or Eq. 5a
7. Choose $A_f \geqq A_{f\,min}$ observing the Flange Design Notes
8. Obtain net weight of girder from $w = (2\,A_f + d\,t) \times 3.4$, plf

Example 3:
$S = 300$ in.^{3} $V = 290$ kips $F_y = 36$ ksi
$k = 5.34$ $\phi = 0.97$

1. $\frac{S^2\,F_y^{7/2}}{V^3\,\phi^2} = 1{,}100$ (inelastic – compact)
2. $t^* = 0.556$ in.
3. Take $t = 0.5625$ in. (9/16 in.)
4. $d^* = 41$ in.
5. Take $d = 40$ in. $(36 < 40 < 52)$
6. $A_{f\,min} = 3.86$ in.^{2}
7. Take $A_f = 6.9375 \times 0.5625 = 3.90$ in.^{2}
   check for compactness:
   $[(6.9375 + 0.5625)/0.5625 = 13.3] <$
   $[130/(36)^{1/2} = 21.7]$ **o.k.**
8. $w = (2 \times 3.9 + 40 \times 0.5625) \times 3.4 = 103$ plf

NOMOGRAPHIC SOLUTIONS

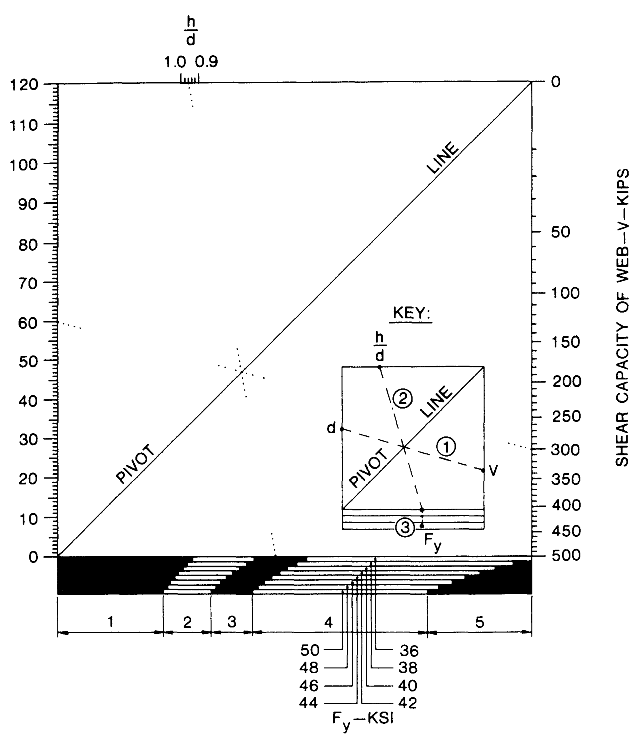
(For $k = 5.34$, i.e., unstiffened webs)

The following nomographs will expedite design because the required parameters can be obtained with minimal calculations. As before, the examples will demonstrate the general and material-optimized conditions.

General Condition: (girder depth d is given)
1. Find the range from Graph 1
2. Find t_{min} from General Condition graph for corresponding range
3. Continue at Step 3 of General Design Procedure

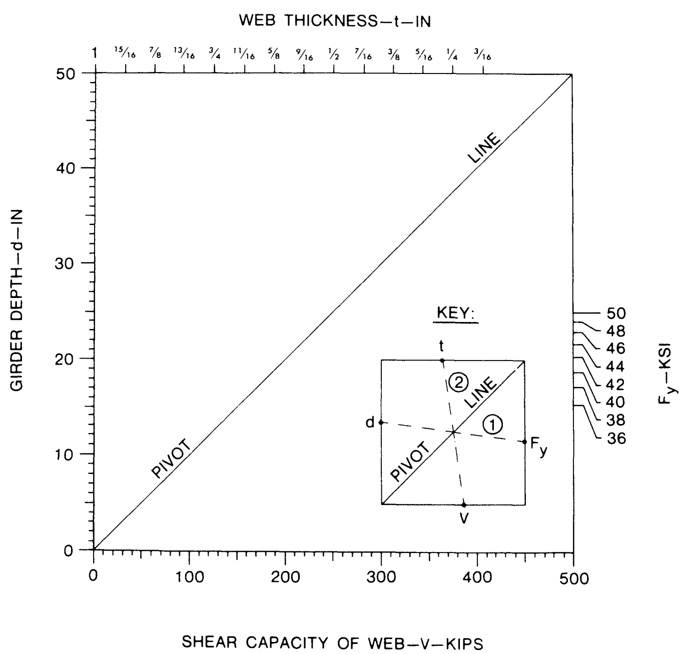
Optimized Condition (elastic):
1. Find the range from Graph 5
2. Find $t \approx$ (t^* from Graph 7)
3. Find d_{max} = (d from Graph 4)
4. Continue at Step 5 of Optimized Design Procedure – elastic

Graph 1



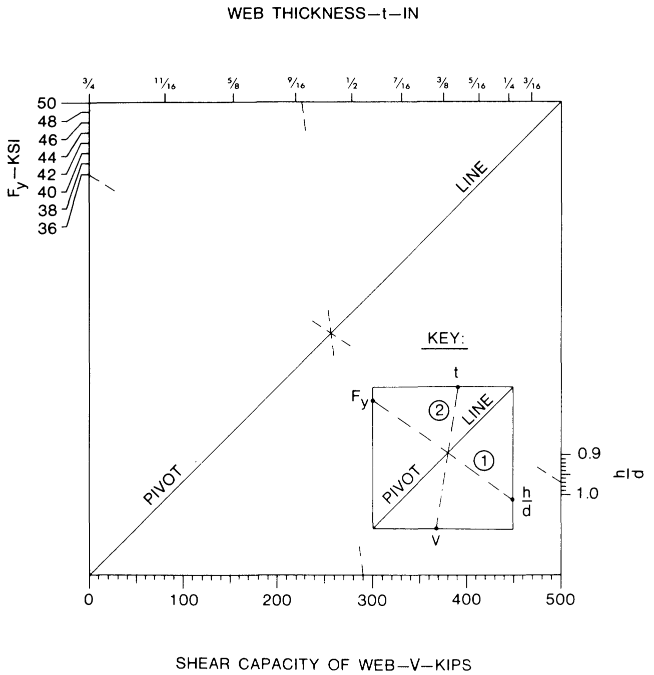
GENERAL CONDITION
RANGE 1: $F_v = 0.4\,F_y$
RANGE 2: $C_v \geqslant 0.8$ (COMPACT WEB–INELASTIC BUCKLING)
RANGE 3: $C_v < 0.8$ (COMPACT WEB–ELASTIC BUCKLING)
RANGE 4: $C_v < 0.8$ (NONCOMPACT WEB–ELASTIC BUCKLING)
RANGE 5: $C_v < 0.8$ (REDUCTION IN ALLOW. COMPR. FLG. STRESS)

Graph 2



GENERAL CONDITION
RANGE 1: $F_v = 0.4\,F_y$

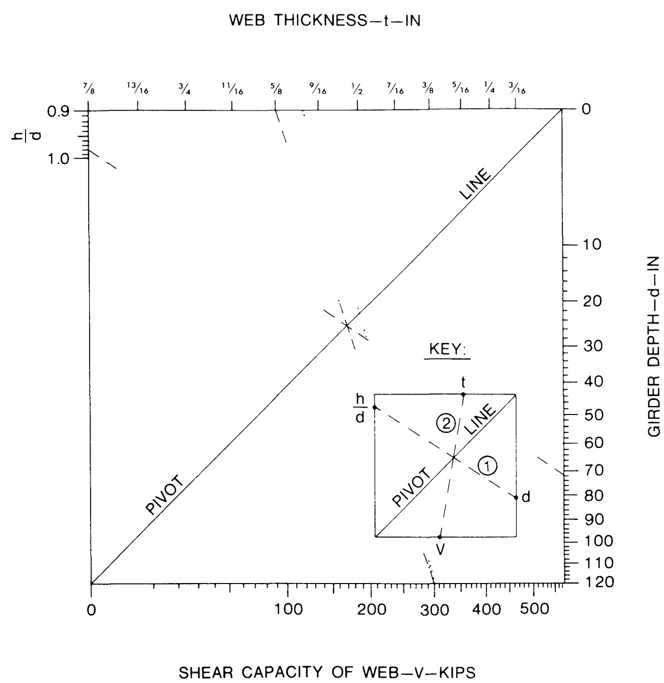
Graph 3



GENERAL CONDITION

RANGE 2: $C_V \geqslant 0.8$ (COMPACT WEB—INELASTIC BUCKLING)

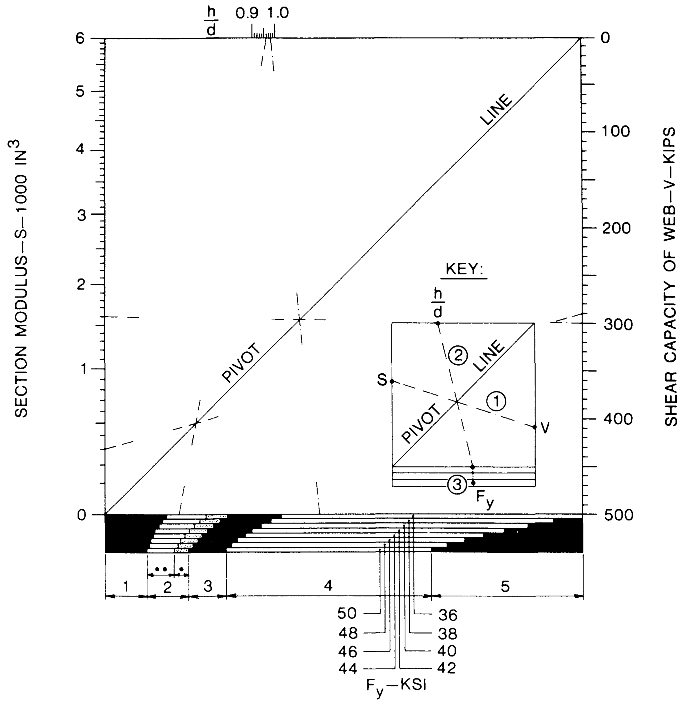
Graph 4



GENERAL CONDITION

RANGES 3, 4, 5: $C_V < 0.8$ (ELASTIC WEB BUCKLING)

Graph 5



OPTIMIZED CONDITION

RANGE 1: $F_V = 0.4\ F_y$
RANGE 2••: $C_V \geqslant 0.8$ (COMPACT WEB—INELASTIC BUCKLING)
RANGE 2•: $C_V \geqslant 0.8$ (NO OPTIMUM POSSIBLE)
RANGE 3: $C_V < 0.8$ (COMPACT WEB—ELASTIC BUCKLING)
RANGE 4: $C_V < 0.8$ (NONCOMPACT WEB—ELASTIC BUCKLING)
RANGE 5: $C_V < 0.8$ (NONCOMPACT—REDUCTION IN ALLOW COMPR. FLG. STRESS)

Optimized Condition (inelastic):

1. Find the range from Graph 5
2. Find $t \geqq$ (t from Graph 3)
3. Find $d =$ (d^* from Graph 6)
4. Continue at Step 5 of Optimized Design Procedure — inelastic

The nomographic solutions for Ex. 1-3 are marked on the pertinent graphs in the following manner:

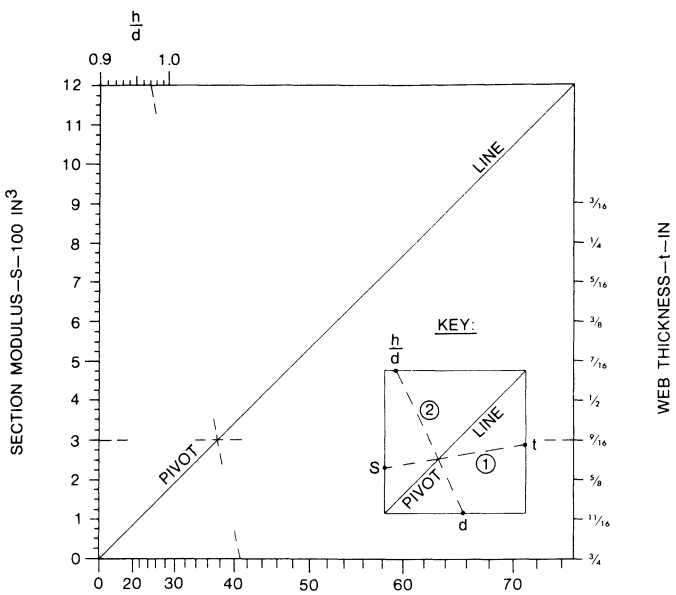
Example 1: . . . .
Example 2: —·—·—·—·—·—·—·
Example 3: -------------------

For the Optimized Condition, Graphs 8 and 9 will yield a quick check on expected optimized girder weights for both inelastic and elastic conditions without going through any design effort.

SUMMARY

The techniques presented allow the designer to obtain material sizes for plate girders and weld-fabricated beams directly, either mathematically or nomographically. Trial designs are not necessary to establish web-buckling ranges. The developed design approaches can be used for general as well as for material-optimized design situations.

Graph 6

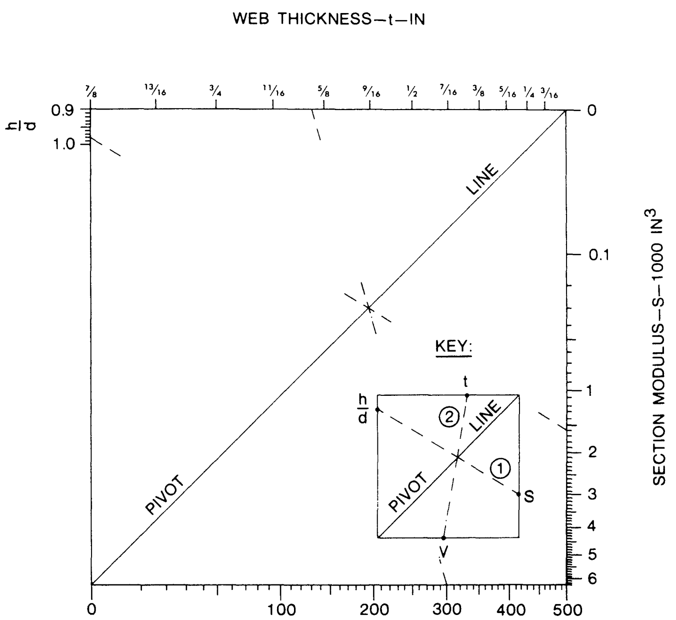


GIRDER DEPTH—d—IN

OPTIMIZED CONDITION

RANGE 2 •• : $C_V \geq 0.8$ (COMPACT WEB—INELASTIC BUCKLING)

Graph 7

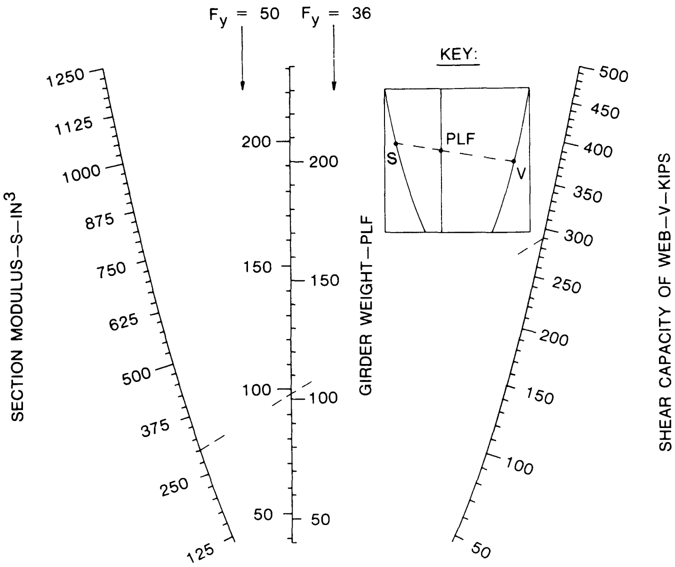


SHEAR CAPACITY OF WEB—V—KIPS

OPTIMIZED CONDITION

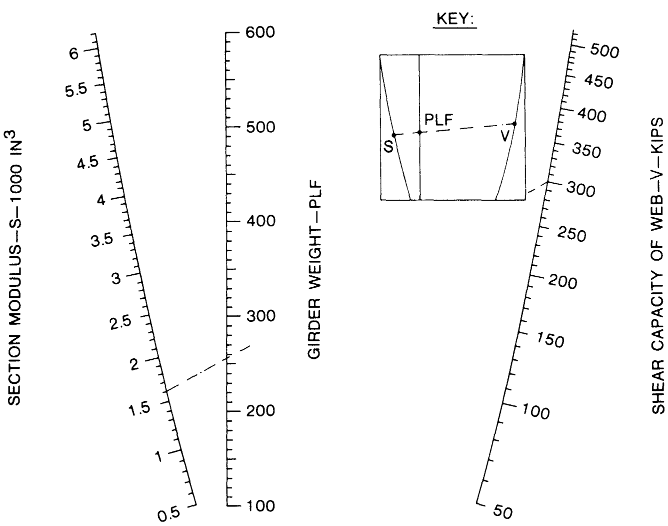
RANGES 3, 4, 5: $C_V < 0.8$ (ELASTIC WEB BUCKLING)

Graph 8



WEIGHT OF GIRDER FOR RANGE 2 ••
OPTIMIZED CONDITION—$C_V \geq 0.8$

Graph 9



WEIGHT OF GIRDER FOR RANGES 3 & 4
OPTIMIZED CONDITION—$C_V < 0.8$

REFERENCES

1. *American Institute of Steel Construction, Inc.* Specification for the Design, Fabrication and Erection of Structural Steel for Buildings *Chicago, Ill., Nov. 1, 1978.*

NOMENCLATURE (unless given in text)

A = cross-sectional total area of girder—chosen, in.^{2}
A^* = cross-sectional total area of girder—material optimized, in.^{2}
A_f = cross-sectional area of one flange as shown in Fig. 1, in.^{2}
A_w = cross-sectional area of web as shown in Fig. 1, in.^{2}
F_b = allowable bending stress in flange, ksi
F_v = allowable shear stress in web, ksi
F_y = yield stress, ksi
I = moment of inertia of girder, in.^{4}
S = section modulus of girder—furnished, in.^{3}
V = shear force capacity of web—furnished, kips
b = full nominal width of flange, in.
d = depth of girder—furnished, or chosen, in.
d^* = depth of girder—material optimized, in.
f_v = shear stress in web, ksi
h = clear distance between top and bottom flanges, in.
t = web thickness—chosen, in.
t^* = web thickness—material optimized, in.
t_f = flange thickness, in.
w = net weight of girder (= 3.4 A), plf
ϕ = h/d, furnished

ACKNOWLEDGMENTS

Editorial review by Roger H. Wildt, director of construction marketing; art work by Leonard M. Gentilcore, Consulting Engineering Division; transcribing supervision by Donna E. Savitske, Central Transcribing; all of Bethlehem Steel Corporation, Bethlehem, Pa. Verification of the manuscript by Dr. Spiro S. Thomaides, Consulting Engineer, Bethlehem, Pa.