A General Solution for the Governing Bending Equation

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The allowable bending stress F_b for most wide-flange sections is governed only by AISC Eq. 1.5-7, and Eq. 1.5-6 never need to be checked for these sections. Those wide-flange sections for which F_b is determined by Eq. 1.5-6, and the unbraced lengths for which these forms govern, are easily identified by simultaneous solution of the bending equations for the common roots of length for any given yield stress F_y and bending coefficient C_b. Tables of these sections and a general solution for determining the governing bending equation are presented in this paper.

The tables are useful supplements to Stockwell's simplification of the bending equations[1] and to Burgett and Tide's extension of the use of the beam charts to designs with bending coefficients greater than unity.[2] The tables and their use are presented in the body of this paper; simultaneous solutions of the bending equations for common roots of length are given in the appendix.

An example of an allowable bending stress curve for a shape with buckling resistance governed by both torsional and warping resistance is shown in Fig. 1. However, for A36 steel with a C_b of 1, the curves do not intersect for 141 wide-flange shapes and are similar to Fig. 2. Accordingly, there is no need to check these shapes for Eqs. 1.5-6a and 1.5-6b.

The wide-flange sections for which Eqs. 1.5-6a and 1.5-6b may govern bending for $F_y = 36$ ksi and $C_b = 1$ are listed in Table 1A. Tables 1B, 1C and 1D extend the tables to $C_b = 1.4$, 1.8 and 2.3. Similar lists for Gr. 50 steel are in Tables 2A, B, C and D.

The left hand column in these tables, captioned "WID FL," lists the wide-flange sections.

The second column, captioned "L_u," lists the maximum unbraced length for which $F_b = 0.6\ F_y$.

The third column, captioned "Ll," lists the length where Eq. 1.5-6a begins to govern.

The fourth column, captioned "TRANS," lists the transition length for which bending begins to be determined by Eq. 1.5-6b. If N/A (not applicable) is in this column, Eq. 1.5-6a governs between L1 and L2.

The fifth column, captioned "L2," lists the length beyond which Eq. 1.5-7 again governs.

The following general statements may be made about the tables:

1. If a wide-flange section is not listed in the table for $C_b = 1$ and given F_y, Eq. 1.5-6 will not govern for any C_b for that F_y.
2. As C_b increases, L1 and L2 increase and fewer wide-flange sections are governed by Eq. 1.5-6a and 1.5-6b.
3. As F_y increases, more sections are governed by Eq. 1.5-6.

While tables could be prepared for C_b ranging from 1 to 2.3 in small increments, one finds tables for $C_b = 1$ (Tables 1A and 2A) are the only ones used in practice. Intelligent extrapolation from either Table 1A or 2A will preclude needless calculation or shuffling through tables. If such extrapolation does not identify the correct equation for maximum bending stress, the error will be small and conservative.

The tables are a useful supplement to Stockwell's simplification[1] for solutions requiring an exact determination of F_b, such as the interaction equations. If the section under investigation does not appear on the table for $C_b = 1$, F_b is a function of Eq. 1.5-7 only and Stockwell's common variable Q need never be calculated. If the section does appear, the likelihood of Eq. 1.5-6 governing may be easily estimated.

When using the beam charts and Burgett and Tide's equivalent length L_e for C_b greater than 1.0, a section selected using an L_e of L/C_b will require no additional checking if it is not listed in the appropriate table. For beams listed, the user may estimate the likelihood that the warping equations will govern and proceed accordingly.

Note that on Fig. 1 the plot of Eq. 1.5-7 is always concave, whereas the plot of 1.5-6a is always convex, a quality which can be useful when using equivalent lengths in the beam charts. For example, on the lower

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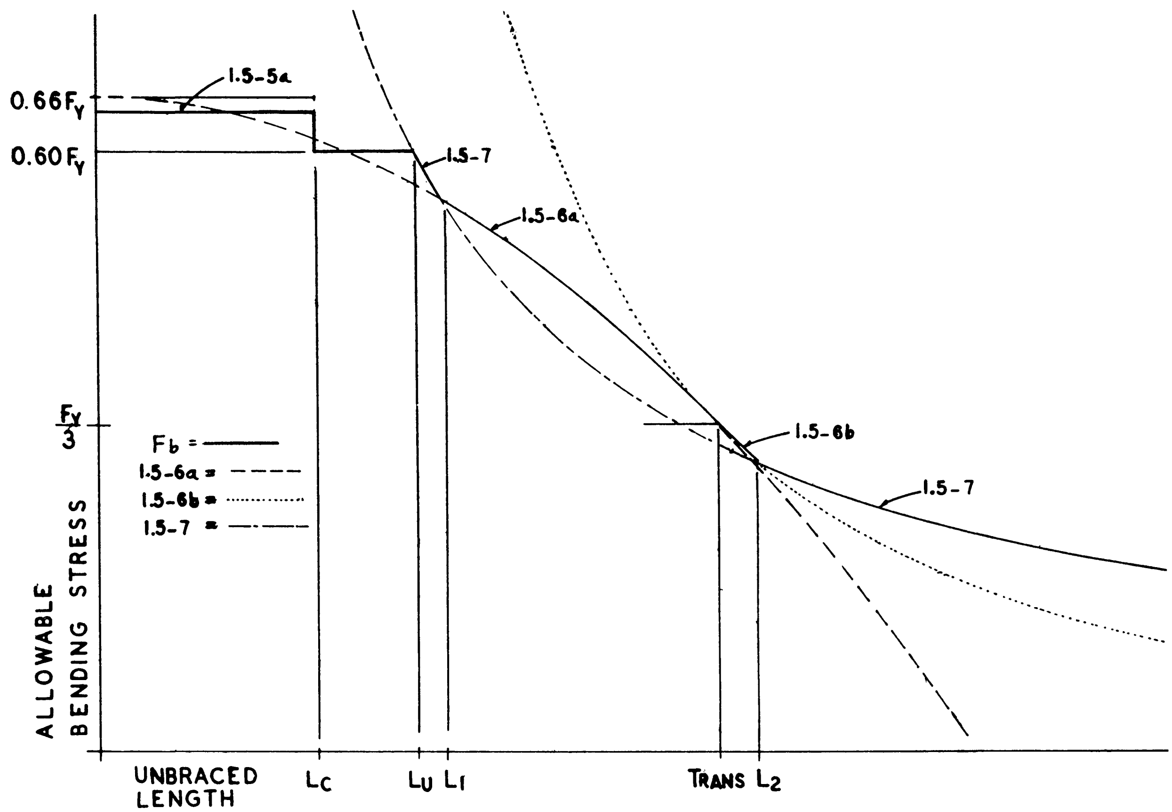

Figure 1

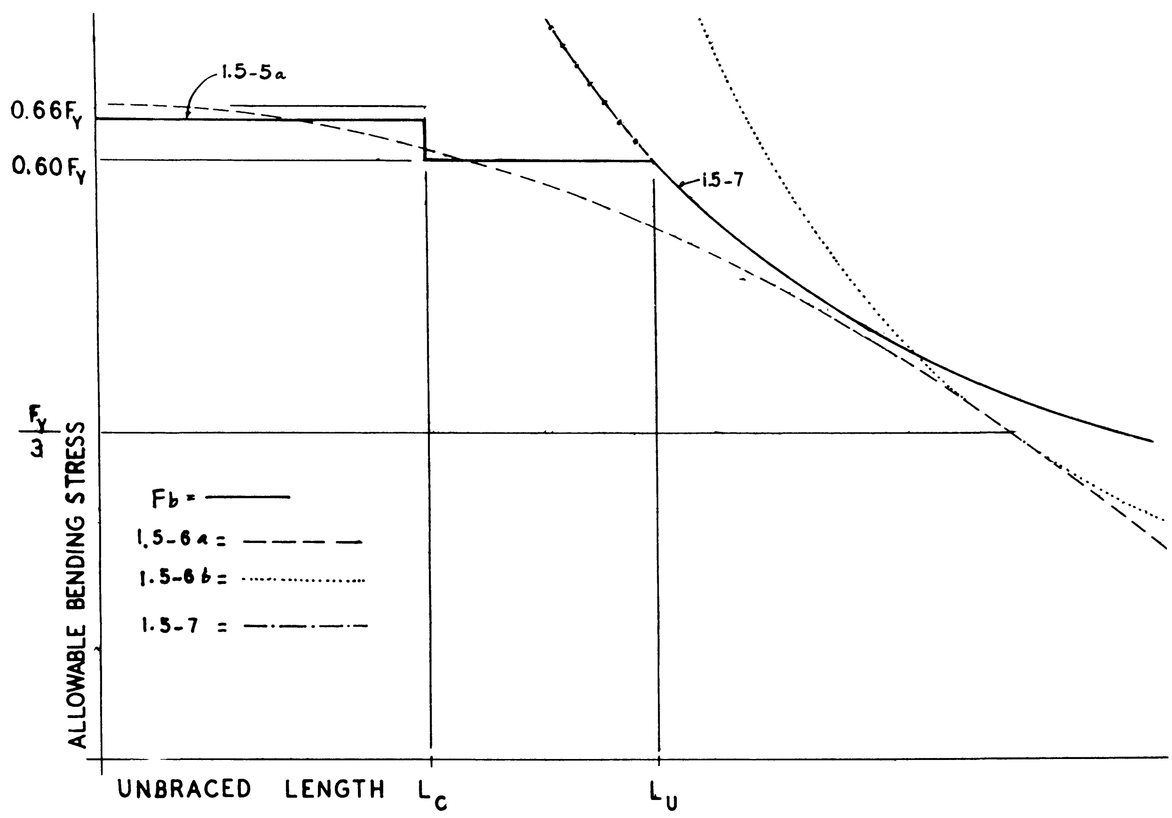

Figure 2

half of the beam chart on p. 2-57 in the AISC *Manual*, the curves for the W27 × 84 through 102 are convex and therefore are governed by Eq. 1.5-6a, which can be verified by Table 1A. Moreover, the intersection of 1.5-7 and 1.5-6a may be observed to occur at lengths of approximately 11 ft, 13.5 ft and 16 ft for the W27 group listed above, which may again be verified from Table 1A. The scale of the charts often prevents the identification of this quality of the curves, but when perceivable it is a useful assistance to Burgett and Tide's equivalent length procedure.

DEVELOPMENT OF TABLES

Find l for Four Conditions

1. $l \leqq L_u$

$F_b = 0.6F_y$

$l = L_u =$ Maximum of

$$r_T \sqrt{\frac{102{,}000C_b}{F_y}} \text{ or } \frac{12{,}000C_b}{F_y(d/A_f)}$$

2. $L_u < l \leqq r_T \sqrt{\dfrac{510{,}000C_b}{F_y}}$

Set F_b from Eq. 1.5-6a equal to F_b from Eq. 1.5-7 and solve for l.

$$\left[\frac{2}{3} - \frac{F_y(l/r_T)^2}{1{,}530 \times 10^3\ C_b}\right] F_y = \frac{12{,}000C_b}{l(d/A_f)}$$

$$l\left[\frac{2F_y}{3}\right] - l^3\left[\frac{F_y^2}{1{,}530 \times 10^3\ C_b r_T^2}\right] - \frac{12{,}000C_b}{d/A_f} = 0$$

$$l^3 + l\left[\frac{-1.02 \times 10^6 r_T^2 C_b}{F_y}\right] + \left[\frac{1.836 \times 10^{10} r_T^2 C_b^2}{(d/A_f)F_y^2}\right] = 0$$

Following Burington's "Handbook of Mathematical Formulas," the equation is in the normal cubic form:

$$l^3 + Al + B = 0$$

Where

$$A = \frac{-1.02 \times 10^6 \times r_T^2 \times C_b}{F_y} \text{ and}$$

$$B = \frac{1.836 \times 10^{10} \times (r_T C_b / F_y)^2}{(d/A_f)}$$

Let $A_o = (A/3)^3$ and $B_o = (B/2)^3$

Note: A and A_o *always* negative

B and B_o *always* positive

If $A_o + B_o < 0$ *there are 3 real and unequal roots*

$\cos\phi = -\sqrt{B_o / -A_o}$ so ϕ is a 2nd or 3rd quadrant angle

$$\phi = \pi - \cos^{-1} -\sqrt{B_o / -A_o}$$

$$l_1 = -2\sqrt{\frac{-A}{3}}\cos\left(\frac{\phi}{3}\right)$$

$$l_2 = -2\sqrt{\frac{-A}{3}}\cos\left(\frac{\phi}{3} - \frac{2\pi}{3}\right) = -2\sqrt{\frac{-A}{3}}\cos\left(\frac{\phi + 2\pi}{3}\right)$$

$$l_3 = -2\sqrt{\frac{-A}{3}}\cos\left(\frac{\phi}{3} - \frac{4\pi}{3}\right) = -2\sqrt{\frac{-A}{3}}\cos\left(\frac{\phi + 4\pi}{3}\right)$$

Solutions for $A_o + B_o < 0$ yield two positive roots (l_1 and l_3 above) and one negative, and therefore meaningless, root (l_2).

l_3 is the smaller meaningful length and is the entry length where Eq. 1.5-6a begins to govern; this length is call "L1" in the tables and is so marked on Fig. 1a. When l_3 is less than L_u, "L1" is printed as L_u in the tables.

l_1 is the larger of the two roots and is the exit length where Eq. 1.5-6a ceases to govern; this length is called "L2" in the tables and is so marked on Fig. 1. When l_1 is greater than the transition length to 1.5-6b (where $F_b = F_y/3$), the exit length for Eq. 1.5-6b will be printed as "L2." (See Sections 3 and 4.)

If $A_o + B_o = 0$, there are three real roots of which at least two are equal, and the general solution is the same as the previous solution for $A_o + B_o < 0$. If this situation occurs it will be meaningless, because the two positive roots will necessarily be the equal roots and will represent a single point intersection between 1.5-6a and 1.5-7.

If $A_o + B_o < 0$, there are one real and two imaginary roots.

$$A_1 = \sqrt[3]{-\frac{B}{2} + \sqrt{A_o + B_o}}$$

$$B_1 = \sqrt[3]{-\frac{B}{2} - \sqrt{A_o + B_o}}$$

$$l_1 = A_1 + B_1$$

The real root for this condition will be negative and therefore meaningless.

3. $l = r_T \sqrt{\dfrac{510{,}000C_b}{F_y}}$

$$F_b = \frac{F_y}{3}$$

l is the transition length between Eqs. 1.5-6a and 1.5-6b and is printed under "Trans" in the Tables. If the exit root (l_1) for Eq. 1.5-6a is smaller than this transition length, "N/A" (for not applicable) is printed under "Trans" and the exit root for 1.5-6a is listed under L2.

4. $l > r_T \sqrt{\frac{510{,}000C_b}{F_y}}$

Set F_b from Eq. (1.5-6b) equal to F_b from Eq. 1.5-7 and solve for l.

$$\frac{170 \times 10^3 C_b}{(l/r_T)^2} = \frac{12{,}000C_b}{l(d/A_f)}$$

$$l = \frac{170r_T^2\ (d/A_f)}{12} = \frac{85r_T^2\ (d/A_f)}{6}$$

l is the exit length for Eq. 1.5-6b and is listed under L2 whenever a length is listed under "Trans" (See Section 3).

REFERENCES

1. *Stockwell, Frank W., Jr.* Simplified Approach to AISC Bending Formulas *AISC Engineering Journal, 3rd Qtr., 1974, Chicago, Ill.*
2. *Burgett, Lewis B. and Raymond H.R. Tide* Fast Design of Beams with C_b Greater than 1.0 *AISC Engineering Journal, 3rd Qtr., 1980, Chicago, Ill.*
3. *American Institute of Steel Construction, Inc.* Manual of Steel Construction *8th Ed., 1980, Chicago, Ill.*

Table 1B

F_y = 36 ksi WID FL	L_u (ft)	L1 (ft)	TRANS (ft)	C_b = 1.4 L2 (ft)
W 8×10	6.64	7.59	N/A	11.26
W10×12	5.46	5.61	11.27	12.91
W10×15	7.01	9.07	N/A	9.90
W12×14	4.99	4.99	11.15	14.21
W12×16	5.72	5.99	11.27	12.34
W14×22	7.90	8.58	14.67	15.13
W14×30	12.13	14.84	N/A	18.44
W16×26	7.84	8.10	15.96	18.06
W16×36	12.27	14.50	N/A	19.67
W18×35	9.34	10.08	17.49	18.19
W21×44	9.18	9.54	18.43	20.55
W21×50	10.87	12.66	N/A	17.85
W24×55	9.73	10.07	19.72	22.20
W24×62	11.34	12.83	N/A	19.70
W24×68	14.32	15.58	26.52	27.28
W27×84	15.47	16.60	29.22	30.67
W27×94	17.92	23.32	N/A	25.16
W30×99	15.31	16.05	30.16	33.02
W30×108	17.30	19.55	N/A	30.10
W33×118	16.76	17.49	33.33	36.83
W33×130	19.28	22.02	N/A	32.82
W36×135	17.21	17.93	34.39	38.17
W36×150	20.35	23.77	N/A	33.27

Table 1A

F_y = 36 ksi WID FL	L_u (ft)	L1 (ft)	TRANS (ft)	C_b = 1.0 L2 (ft)
W 8×10	4.74	4.86	9.82	11.30
W 8×13	5.91	7.11	N/A	9.20
W10×12	4.26	4.26	9.52	12.91
W10×15	5.01	5.26	9.82	10.70
W12×14	4.21	4.21	9.42	14.21
W12×16	4.26	4.26	9.52	12.34
W12×19	5.34	5.79	9.92	10.24
W12×26	9.34	10.27	17.06	17.31
W14×22	5.64	5.67	12.40	15.13
W14×26	7.02	7.79	12.70	12.75
W14×30	8.67	9.04	17.26	19.09
W14×34	10.16	11.94	N/A	16.45
W16×26	6.03	6.03	13.49	18.06
W16×31	7.09	7.48	13.79	14.90
W16×36	8.77	9.08	17.75	19.97
W16×40	10.21	11.57	N/A	17.70
W18×35	6.67	6.69	14.78	18.19
W18×40	8.17	8.90	15.08	15.46
W18×50	10.99	12.59	N/A	18.64
W21×44	6.96	6.96	15.57	20.55
W21×50	7.76	8.01	15.87	18.02
W21×57	9.37	10.83	N/A	15.59
W21×62	11.18	12.10	20.83	21.56
W21×68	12.41	14.95	N/A	19.30
W24×55	7.45	7.45	16.66	22.20
W24×62	8.10	8.27	16.96	19.73
W24×68	10.23	10.29	22.42	27.28
W24×76	11.83	12.58	22.71	24.22
W24×84	13.34	15.68	N/A	21.58
W24×104	18.40	20.41	33.23	33.33
W27×84	11.05	11.05	24.70	30.67
W27×94	12.80	13.45	25.09	27.33
W27×102	14.21	15.91	N/A	25.21
W30×99	11.40	11.40	25.49	33.02
W30×108	12.36	12.62	25.89	30.13
W30×116	13.76	14.70	26.19	27.68
W30×124	15.01	17.09	N/A	25.70
W33×118	12.60	12.60	28.17	36.83
W33×130	13.77	14.11	28.57	32.93
W33×141	15.40	16.57	28.96	30.27
W33×152	16.87	19.62	N/A	27.76
W36×135	13.00	13.00	29.06	38.17
W36×150	14.54	15.00	29.66	33.61
W36×160	15.74	16.81	29.95	31.68
W36×170	16.94	19.04	N/A	29.81
W36×182	18.16	22.92	N/A	26.44

Table 1C

F_y = 36 ksi C_b = 1.8

WID FL	L_u (ft)	L1 (ft)	TRANS (ft)	L2 (ft)
W10×12	7.02	7.74	12.77	12.91
W12×14	6.25	6.47	12.64	14.21
W12×16	7.35	8.49	N/A	12.26
W16×26	10.08	11.24	N/A	18.05
W21×44	11.80	13.33	N/A	20.53
W24×55	12.51	14.00	N/A	22.20
W27×84	19.89	25.10	N/A	28.96
W30×99	19.68	22.73	N/A	32.81
W33×118	21.54	24.60	N/A	36.71
W36×135	22.13	25.15	N/A	38.08

Table 1D

F_y = 36 ksi C_b = 2.3

WID FL	L_u (ft)	L1 (ft)	TRANS (ft)	L2 (ft)
W12×14	7.99	8.93	N/A	14.20

Table 2A

F_y = 50 ksi C_b = 1.0

WID FL	L_u (ft)	L1 (ft)	TRANS (ft)	L2 (ft)
W 6×09	4.79	5.30	8.67	8.72
W 8×10	3.73	3.73	8.33	11.30
W 8×13	4.26	4.43	8.50	9.43
W 8×15	5.20	6.54	N/A	7.61
W10×12	3.61	3.61	8.08	12.91
W10×15	3.73	3.73	8.33	10.70
W10×17	4.36	4.60	8.50	9.20
W10×19	5.17	6.38	N/A	7.75
W10×22	6.78	7.32	12.71	13.22
W12×14	3.58	3.58	8.00	14.21
W12×16	3.61	3.61	8.08	12.34
W12×19	3.84	3.87	8.42	10.24
W12×22	4.64	5.05	8.58	8.83
W12×26	6.73	6.81	14.48	17.31
W12×30	7.75	8.35	14.56	15.20
W14×22	4.70	4.70	10.52	15.13
W14×26	5.06	5.14	10.77	12.75
W14×30	6.55	6.55	14.64	19.09
W14×34	7.32	7.58	14.81	16.66
W14×38	8.24	9.14	14.90	14.96
W14×43	10.34	12.06	N/A	16.94
W16×26	5.12	5.12	11.45	18.06
W16×31	5.23	5.23	11.70	14.90
W16×36	6.74	6.74	15.07	19.97
W16×40	7.35	7.53	15.32	17.72
W16×45	8.21	8.86	15.40	16.04
W16×50	9.13	10.97	N/A	14.25
W16×67	13.89	17.54	N/A	20.22
W18×35	5.61	5.61	12.54	18.19
W18×40	5.88	5.93	12.79	15.46
W18×46	6.77	7.20	12.96	13.79
W18×50	7.92	8.13	16.33	18.71
W18×55	8.73	9.40	16.41	17.14
W18×60	9.60	11.26	N/A	15.56

Table 2A

F_y = 36 ksi C_b = 1.0

WID FL	L_u (ft)	L1 (ft)	TRANS (ft)	L2 (ft)
W18×76	13.74	15.22	24.83	24.93
W21×44	5.91	5.91	13.21	20.55
W21×50	6.02	6.02	13.47	18.02
W21×57	6.74	6.95	13.80	15.69
W21×62	8.05	8.09	17.67	21.56
W21×68	8.94	9.31	17.84	19.79
W21×73	9.63	10.44	17.93	18.53
W21×83	10.85	13.65	N/A	15.87
W21×101	15.34	17.12	N/A	27.42
W24×55	6.32	6.32	14.14	22.20
W24×62	6.44	6.44	14.39	19.73
W24×68	8.51	8.51	19.02	27.28
W24×76	8.62	8.62	19.27	24.22
W24×84	9.61	9.95	19.44	21.86
W24×94	10.88	12.08	19.61	19.64
W24×104	13.25	13.46	28.19	33.33
W24×117	14.95	16.00	28.36	29.90
W24×131	16.80	20.03	N/A	26.54
W27×84	9.37	9.37	20.96	30.67
W27×94	9.52	9.52	21.29	27.33
W27×102	10.23	10.43	21.55	25.21
W27×114	11.44	12.24	21.71	22.90
W27×146	16.58	17.92	30.97	32.15
W27×161	18.29	21.82	N/A	28.87
W30×99	9.67	9.67	21.63	33.02
W30×108	9.82	9.82	21.97	30.13
W30×116	9.94	9.94	22.22	27.68
W30×124	10.80	11.08	22.39	25.77
W30×132	11.60	12.24	22.56	24.37
W30×173	17.48	18.70	33.16	34.96
W30×191	19.36	22.59	N/A	31.72
W33×118	10.69	10.69	23.90	36.83
W33×130	10.84	10.84	24.24	32.93
W33×141	11.08	11.11	24.58	30.27
W33×152	12.14	12.54	24.74	28.01
W33×201	17.92	18.96	34.67	37.27
W33×221	19.80	22.46	N/A	34.17
W36×135	11.03	11.03	24.66	38.17
W36×150	11.25	11.25	25.16	33.61
W36×160	11.37	11.37	25.42	31.68
W36×170	12.20	12.45	25.59	29.82
W36×182	13.07	13.73	25.67	28.00
W36×194	13.94	15.16	25.84	26.60
W36×210	15.05	17.52	N/A	24.74
W36×230	19.27	20.75	36.19	37.76
W36×245	20.59	23.34	N/A	35.60
W36×260	21.91	27.58	N/A	32.02

Table 2B

F_y = 50 ksi WID FL	L_u (ft)	L1 (ft)	TRANS (ft)	C_b = 1.4 L2 (ft)
W 8×10	4.78	4.91	9.86	11.30
W 8×13	5.96	7.23	N/A	9.16
W10×12	4.28	4.28	9.56	12.91
W10×15	5.05	5.31	9.86	10.70
W12×14	4.23	4.23	9.46	14.21
W12×16	4.28	4.28	9.56	12.34
W12×19	5.38	5.85	9.96	10.24
W12×26	9.42	10.39	17.13	17.31
W14×22	5.69	5.73	12.45	15.13
W14×26	7.08	7.88	12.75	12.75
W14×30	8.74	9.13	17.33	19.09
W14×34	10.24	12.11	N/A	16.40
W16×26	6.06	6.06	13.54	18.06
W16×31	7.14	7.55	13.84	14.90
W16×36	8.84	9.17	17.83	19.97
W16×40	10.30	11.71	N/A	17.68
W18×35	6.72	6.75	14.84	18.19
W18×40	8.23	9.00	15.14	15.46
W18×50	11.08	12.75	N/A	18.61
W21×44	6.99	6.99	15.63	20.55
W21×50	7.83	8.09	15.93	18.02
W21×57	9.44	10.98	N/A	15.56
W21×62	11.27	12.23	20.91	21.56
W21×68	12.51	15.20	N/A	19.20
W24×55	7.48	7.48	16.73	22.20
W24×62	8.16	8.35	17.03	19.73
W24×68	10.31	10.39	22.51	27.28
W24×76	11.93	12.71	22.80	24.22
W24×84	13.45	15.91	N/A	21.52
W24×104	18.55	20.65	N/A	33.33
W27×84	11.14	11.15	24.80	30.67
W27×94	12.90	13.59	25.19	27.33
W27×102	14.32	16.10	N/A	25.20
W30×99	11.45	11.45	25.59	33.02
W30×108	12.45	12.74	25.99	30.13
W30×116	13.87	14.86	26.29	27.68
W30×124	15.13	17.31	N/A	25.68
W33×118	12.65	12.65	28.28	36.83
W33×130	13.88	14.25	28.68	32.93
W33×141	15.52	16.74	29.08	30.27
W33×152	17.00	19.89	N/A	27.70
W36×135	13.05	13.05	29.18	38.17
W36×150	14.65	15.15	29.78	33.61
W36×160	15.86	16.98	30.07	31.68
W36×170	17.07	19.27	N/A	29.79
W36×182	18.30	23.56	N/A	26.01

Table 2C

F_y = 50 ksi WID FL	L_u (ft)	L1 (ft)	TRANS (ft)	C_b = 1.8 L2 (ft)
W 8×10	6.14	6.77	11.18	11.30
W10×12	5.06	5.12	10.84	12.91
W10×15	6.49	7.58	N/A	10.59
W12×14	4.80	4.80	10.73	14.21
W12×16	5.29	5.45	10.84	12.34
W14×22	7.31	7.75	14.11	15.13
W14×30	11.23	12.87	N/A	19.02
W16×26	7.26	7.38	15.36	18.06
W16×31	9.19	10.88	N/A	14.66
W16×36	11.36	12.78	N/A	19.96
W18×35	8.64	9.12	16.82	18.19
W21×44	8.49	8.69	17.73	20.55
W21×50	10.06	11.22	N/A	18.02
W24×55	9.01	9.18	18.97	22.20
W24×62	10.50	11.47	19.31	19.73
W24×68	13.26	14.08	25.52	27.28
W24×76	15.33	18.66	N/A	23.48
W27×84	14.32	15.04	28.12	30.67
W27×94	16.59	19.41	N/A	27.03
W30×99	14.17	14.60	29.02	33.02
W30×108	16.01	17.49	29.47	30.13
W30×116	17.84	22.25	N/A	26.37
W33×118	15.51	15.92	32.07	36.83
W33×130	17.84	19.64	32.52	32.93
W36×135	15.93	16.33	33.08	38.17
W36×150	18.84	21.04	N/A	33.61
W36×160	20.39	25.41	N/A	30.21

Table 2D

F_y = 50 ksi WID FL	L_u (ft)	L1 (ft)	TRANS (ft)	C_b = 2.3 L2 (ft)
W10×12	6.46	6.91	12.25	12.91
W12×14	5.75	5.86	12.13	14.21
W12×16	6.76	7.48	12.25	12.34
W14×22	9.35	11.09	N/A	14.87
W16×26	9.27	10.01	17.36	18.06
W18×35	11.05	12.93	N/A	17.99
W21×44	10.85	11.83	20.04	20.55
W24×55	11.51	12.46	21.44	22.20
W24×68	16.94	20.21	N/A	26.75
W27×84	18.30	21.15	N/A	30.47
W30×99	18.10	20.02	32.80	33.02
W33×118	19.82	21.76	36.25	36.83
W36×135	20.36	22.27	37.40	38.17