Direct Determination of Required Rigidities for Multiple-Bracing Frames

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INTRODUCTION

The first step for sizing the vertical braces of a braced steel structure is to prevent them from exceeding allowable drift. This is a deformation analysis, by which the rigidity of R of any brace (Fig. 1) is determinate with respect to Eq. 1 when the required strength is not taken into consideration:

$$R = \frac{F}{\triangle} = \frac{AE\cos^2\alpha}{\ell} = \frac{AE\cos^2\alpha\ \sin\alpha}{h} = \frac{AEb^2}{\ell^3} \quad (1)$$

In Eq. 1, E is the elasticity modulus of steel; F, the lateral load carried by the brace, $\triangle$, its drift; A its cross-sectional area; α, its slope;

$$\ell = \sqrt{h^2 + b^2}, \text{ brace length} \quad (2)$$

h and b, the vertical and the horizontal projection respectively of the brace length.

In the second step, namely the strength checking of the considered braces, adjustments may be necessary and the result is the repetition of the analysis as long as the drift requirement is not in accordance with the strength requirement.

From the study presented here, which leads to two semi-empirical formulas, one can determine directly the total rigidities required for both directions of elastically designed brace structures and overloading of braces can be prevented. This method can also be used for non-linear designed structures for which the uniformity of the brace yielding is an advantage. Deflections due to vertical displacements of columns are disregarded, but the redistribution of the shear due to horizontal torsional moments is taken into consideration.

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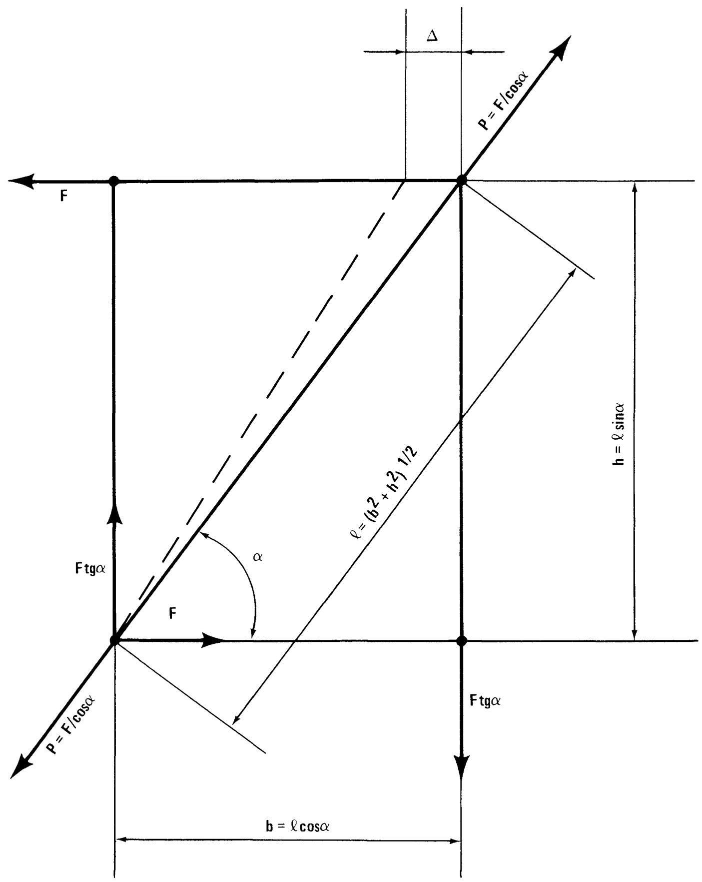

Figure 1

OUTLINE OF METHODOLOGY

Allowable Stresses

The drift of braces, resulting from Eq. 1, shall not exceed the allowable drift $\triangle_a$:

$$\triangle = \frac{Fh}{AE}(\cos^2\alpha\ \sin\alpha)^{-1} \leqslant \triangle_a$$

Replacing:

$$P = \frac{F}{\cos\alpha} = \frac{F\ell}{b} \text{ or } F = P\cos\alpha = \frac{Pb}{\ell}, \tag{3}$$

where P is the axial load carried by the brace, it follows:

$$f_a = \frac{P}{A} \leqslant \left(\frac{\triangle_a}{h}\right) E \cos\alpha \sin\alpha = \frac{\delta Ebh}{\ell^2} = F_a. \tag{4}$$

In Eq. 4, f_a and F_a are the actual and the allowable stresses on the brace; and

$$\delta = \frac{\triangle_a}{h} = \frac{f}{Rh} \tag{5}$$

is the drift index.

Equation 4 indicates it is useless to strive for larger allowable stresses than $\delta Ebh/\ell^2$. However, if the value of this expression is too large with respect to the allowable stress permitted by the AISC Specification for single A36 braces, then solutions with stiffer shapes, double diagonal braces (X braces), or stronger steels should be taken into consideration.

The allowable stress F_a, as determined by Eq. 4, leads to the ideal slenderness ratio $K\ell/r$ in accordance with the AISC Specification. The ideal radius of gyration for the brace with respect to its critical axis may be determined by the equation:

$$r = K\ell/(K\ell/r) \tag{6}$$

where K is the effective length factor of a brace with respect to its critical axis.

Required Rigidities

Consider a system of multiple bracing frames between two adjacent diaphragms of a rectangular structure (Fig. 2). Call d_s its transverse (short) dimension: d_ℓ its longitudinal dimension; h, the distance between the two adjacent diaphragms; R_s, the rigidity of any transverse brace, and a_ℓ its distance to the rigidity center (R.C.); R_ℓ, the rigidity of any longitudinal brace, and a_s its distance to the R.C.; ΣF_s, the resultant of the lateral loads acting in the transverse direction, and e_ℓ its eccentricity with respect to R.C.; ΣF_ℓ, the resultant of the lateral loads acting in the longitudinal direction, and e_s its eccentricity with respect to the R.C.; ΣR_s and ΣR_ℓ, the total rigidities in transverse and longitudinal directions respectively; and I_r, the polar moment of inertia of the brace rigidities.

The lateral loads carried by the transverse and longitudinal braces, taking into account the torsional effects, are:

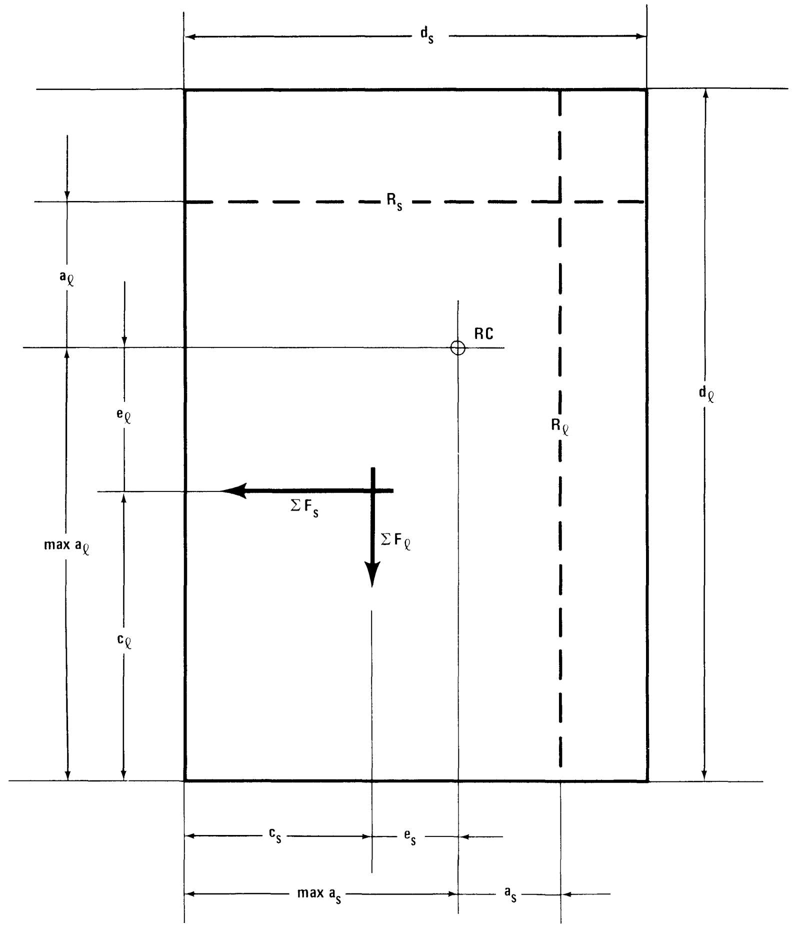

Figure 2

$$F_s = \frac{R_s}{\Sigma R_s} \times \Sigma F_s + \frac{R_s \times a_\ell}{I_r} \times e_\ell \Sigma F_s$$

$$= R_s \Sigma F_s \left(\frac{1}{\Sigma R_s} + \frac{a_\ell e_\ell}{I_r}\right) \tag{7}$$

$$F_\ell = \frac{R_\ell}{\Sigma R_\ell} \times \Sigma F_\ell + \frac{R_\ell \times a_s}{I_r} \times e_s \Sigma F_\ell$$

$$= R_\ell \, \Sigma F_\ell \left(\frac{1}{\Sigma R_\ell} + \frac{a_s e_s}{I_r}\right) \tag{8}$$

The axial load acting on any transversal brace is:

$$P_s = \frac{F_s}{\cos\alpha_s} = \frac{A_s E \cos\alpha_s \sin\alpha_s}{h} \Sigma F_s \left(\frac{1}{\Sigma R_s} + \frac{a_\ell e_\ell}{I_r}\right) \tag{9}$$

The axial load acting on any longitudinal brace is:

$$P_\ell = \frac{F_\ell}{\cos\alpha_\ell} = \frac{A_\ell E \cos\alpha_\ell \sin\alpha_\ell}{h} \Sigma F_\ell \left(\frac{1}{\Sigma R_\ell} + \frac{a_s e_s}{I_r}\right) \tag{10}$$

where

$$I_r = \Sigma R_s\, a_\ell^2 + \Sigma R_\ell\, a_s^2 \qquad (11)$$

The terms $\Sigma R_s\, a_\ell^2$ and $\Sigma R_\ell a_s^2$ are the moments of inertia of the rigidities of the vertical frame system for the two directions of the structure. These terms may be expressed in terms of the radii of gyration as:

$$\Sigma R_s\, a_\ell^2 = r_\ell^2 \Sigma R_s = (\rho_\ell d_\ell)^2\, \Sigma R_s \quad \text{and}$$

$$\Sigma R_\ell a_s^2 = r_s^2\, \Sigma R_\ell = (\rho_s d_s)^2\, \Sigma R_\ell,$$

where r_ℓ and r_s are the gyration radii of the frame rigidities in longitudinal and transverse directions respectively;

$$\rho_\ell = \frac{r_\ell}{d_\ell} \qquad (12)$$

$$\text{and } \rho_s = \frac{r_s}{d_s} \qquad (13)$$

It follows:

$$I_r = (\rho_\ell d_\ell)^2\, \Sigma R_s + (\rho_s d_s)^2\, \Sigma R_\ell$$

Now we want to express I_r with respect to ΣR_s and to ΣR_ℓ. It may be assumed that the ratio of the required rigidities (ΣR_s and ΣR_ℓ) in each direction be in the ratio of the respective total lateral loads (ΣF_s and ΣF_ℓ), adjusted by an unknown reduction m as stated in Eq. 14.

$$\frac{\Sigma R_\ell}{\Sigma R_s} = \frac{\Sigma F_\ell}{\Sigma F_s}\, m \qquad (14)$$

The factor m illustrates: (1) the effect of lateral load eccentricity; and (2) transverse frames are more helpful for carrying longitudinal lateral loads (by torsional effect) than longitudinal frames are for carrying transverse lateral loads. The following results:

$$\Sigma R_\ell = \frac{\Sigma F_\ell}{\Sigma F_s}\, m\, \Sigma R_s$$

$$\Sigma R_s = \frac{\Sigma F_s}{\Sigma F_\ell}\, \frac{\Sigma R_\ell}{m}$$

$$I_R = \Sigma R_s \left[(\rho_\ell d_\ell)^2 + (\rho_s d_s)^2\, \frac{\Sigma F_\ell m}{\Sigma F_s} \right] \qquad (15)$$

$$\text{and } I_R = \Sigma R_\ell \left[(\rho_s d_s)^2 + (\rho_\ell\, d_\ell)^2\, \frac{\Sigma F_s}{\Sigma F_\ell m} \right] \qquad (16)$$

Replace I_R from Eqs. 15 and 16 into Eqs. 9 and 10, and write conditions $max\ P_s \leq A_s\, F_a$ and $max\ P_\ell \leq A_\ell\, F_a$. It follows:

$$\frac{A_s E \cos\alpha_s \sin\alpha_s}{h}\, \Sigma F_s \times$$

$$\left\{ \frac{1}{\Sigma R_s} + \frac{max\ a_\ell\, e_\ell}{\Sigma R_s \left[(\rho_\ell d_\ell)^2 + (\rho_s d_s)^2\, \dfrac{\Sigma F_\ell m}{\Sigma F_s} \right]} \right\} \leq A_s F_a$$

where $A_s F_a = A_s\, \delta E \cos\alpha_s \sin\alpha_s$ and

$$P_\ell = \frac{A_\ell E \cos\alpha_\ell \sin\alpha_\ell}{h}\, \Sigma F_\ell \times$$

$$\left\{ \frac{1}{\Sigma R_\ell} + \frac{max\ a_s e_s}{\Sigma R_\ell \left[(\rho_s d_s)^2 + (\rho_\ell d_\ell)^2\, \dfrac{\Sigma F_s}{\Sigma F_\ell m} \right)} \right\} \leq A_\ell F_a$$

where $A_\ell F_a = A_\ell\, \delta E \cos\alpha_\ell \sin\alpha_\ell$.

After simplification and expressing $max\ a_\ell$ and $max\ a_s$ as fractions of d_ℓ and d_s :

$$max\ a_\ell = \phi_\ell d_\ell \qquad (17)$$

$$\text{and } max\ a_s = \phi_s d_s \qquad (18)$$

it results:

$$\Sigma R_s \geqslant \frac{\Sigma F_s}{\delta h} \times \frac{BL + m}{B + m} \qquad (19)$$

$$\Sigma R_\ell \geqslant \frac{\Sigma F_\ell}{\delta h} \times \frac{B + Sm}{B + m} \qquad (20)$$

$$\text{where } B = \frac{\Sigma F_s}{\Sigma F_\ell} \left(\frac{\rho_\ell d_\ell}{\rho_s d_s} \right)^2 \qquad (21)$$

$$L = 1 + \frac{\phi_\ell e_\ell}{\rho_\ell^2 d_\ell} \qquad (22)$$

$$\text{and } S = 1 + \frac{\phi_s e_s}{\rho_s^2 d_s} \qquad (23)$$

The eccentricities e_ℓ and e_s can be determined by the equations:

$$e_\ell = |\phi_\ell\, d_\ell - c_\ell| \quad \text{and} \tag{24}$$

$$e_s = |\phi_s d_s - c_s| \tag{25}$$

where c_ℓ and c_s are the distances of ΣF_s and ΣF_ℓ, respectively, to the location furthest from the R.C. If the lateral loads are resulting from earthquake forces, both e_ℓ and e_s shall be considered not less than 0.05 d_ℓ.[1]

By substituting Eqs. 19 and 20 into Eq. 14, m can be determined by solving a second degree algebraic equation:

$$m^2 + (BL - S)m - B = 0 \tag{26}$$

The two semi-empirical Eqs. 19 and 20 and the auxiliary Eqs. 4 and 21-26, may be used for determining the total rigidities requirements for both directions of a braced structure between two adjacent diaphragms, so that overloading of braces is prevented.

These formulas are expressed with respect to the data of the problem: E; δ; ΣF_s; ΣF_ℓ; h; d_ℓ; d_s; c_ℓ; c_s; b for all the bays where braces may be provided, as well as the factors ϕ_s, ϕ_ℓ, ρ_s and ρ_ℓ which depend upon the frame rigidity distribution of the structure. These factors can be determined by a simple procedure. The procedure is based on the assumption that, in the final design, brace shapes will be selected from those having a maximum radius of gyration per steel weight group. From Fig. 3, in which these shapes are shown, it follows their cross-sectional areas can be considered as the cube of their minor radius of gyration:

$$A = r_y^3$$

Since it is expected the braces will be uniformly stressed, they will have the same allowable stress for compression, and the same slenderness ratio ℓ/r_y. It follows the cross-sectional area A of the braces be proportional to the cube of their lengths:

$$A = \psi\ell^3 \tag{27}$$

Substituting this expression in Eq. 1, it follows:

$$R = \frac{\psi\ell^3\, Eb^2}{\ell^3} = \psi E b^2 \tag{28}$$

The conclusion is the relative rigidities of braces can be considered as proportional to the square of their horizontal projections. By computing the centroid coordinates, as well as the radii of gyration for these "squares," values for the terms ϕ_s, ϕ_ℓ, ρ_s and ρ_ℓ result.

For a preliminary design, the distribution of the rigidity can be assumed to be that of regular geometric sections (rectangle, triangle, rhombus, circle, parabola, etc.). By this procedure, the above factors result from the properties of the corresponding geometric section.[2]

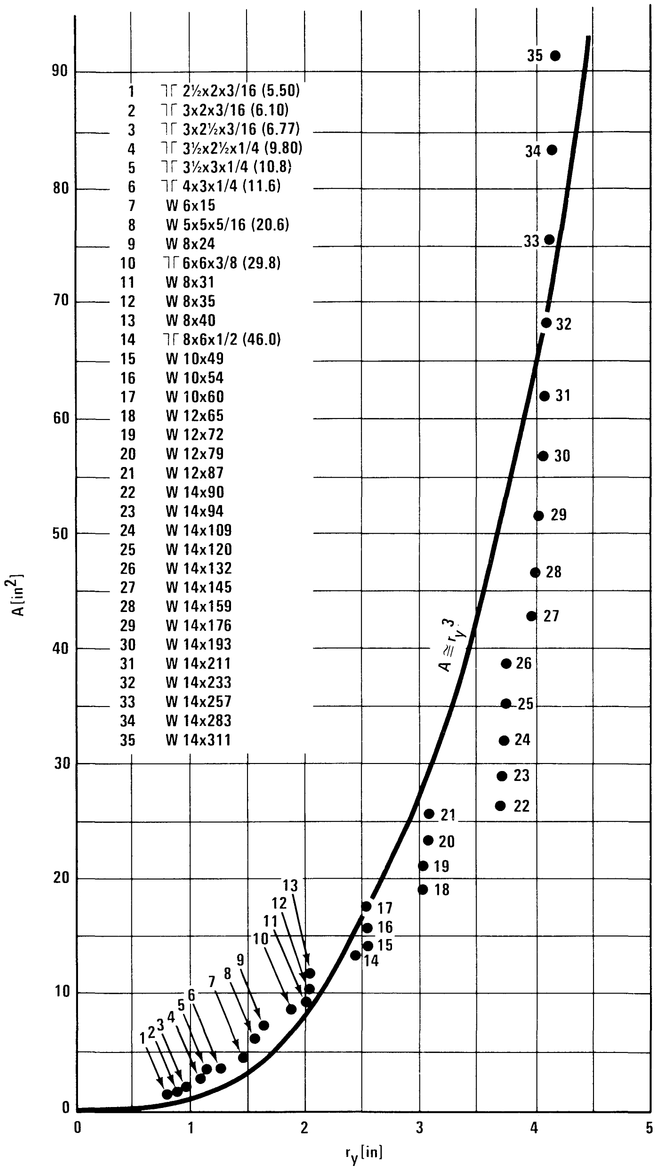

Figure 3

DESIGN PROCEDURE

1. State the data of problem: E; δ; ΣF_s; ΣF_ℓ; h; d_ℓ; d_s; c_ℓ c_s; as well as the horizontal dimensions, b, of all the bays where braces may be provided.
2. Based on Eq. 28, determine the ideal coordinates of the R.C.
3. Based on Eqs. 24 and 25, determine the ideal eccentricities.
4. Based on Eqs. 17 and 18, determine ϕ_ℓ and ϕ_s.
5. Based on Eqs. 12 and 13, determine ρ_ℓ and ρ_s.

6. Based on Eqs. 21-23, determine B, L and S.
7. Based on Eq. 26, determine m.
8. Based on Eqs. 19 and 20, determine the required total rigidities ΣR_s and ΣR_ℓ.
9. Based on Eq. 14, check the correctness of the calculation.
10. Select a detail for brace connections and consequently determine K.
11. Based on Eqs. 2 and 4, compute the brace length ℓ and the allowable stress, F_a for all braces. Then determine the slenderness ratio, $K\ell/r$ (per A36) and the minimum required radius of gyration, r.
12. Distribute ΣR_s and ΣR_ℓ to the potential braces so the slenderness ratio determined from the preceding step is not exceeded. The proportionality expressed in Eq. 27 must also be maintained. In principle, braces shall be provided wherever possible, because columns will be penalized with less up and down supplementary loads and girders will carry smaller axial loads. However, some of the potential braces may be deleted when the multitude of the potential braces lead to understressed cross-sections.
13. Check the structure for the actual sizes of braces.

ILLUSTRATIVE EXAMPLE

Given the structure represented in Fig. 4, in which the bays where braces may be provided are shown with thick lines and marked with Roman numerals I, II and III corresponding to their type. Double diagonal bracing shall be avoided.

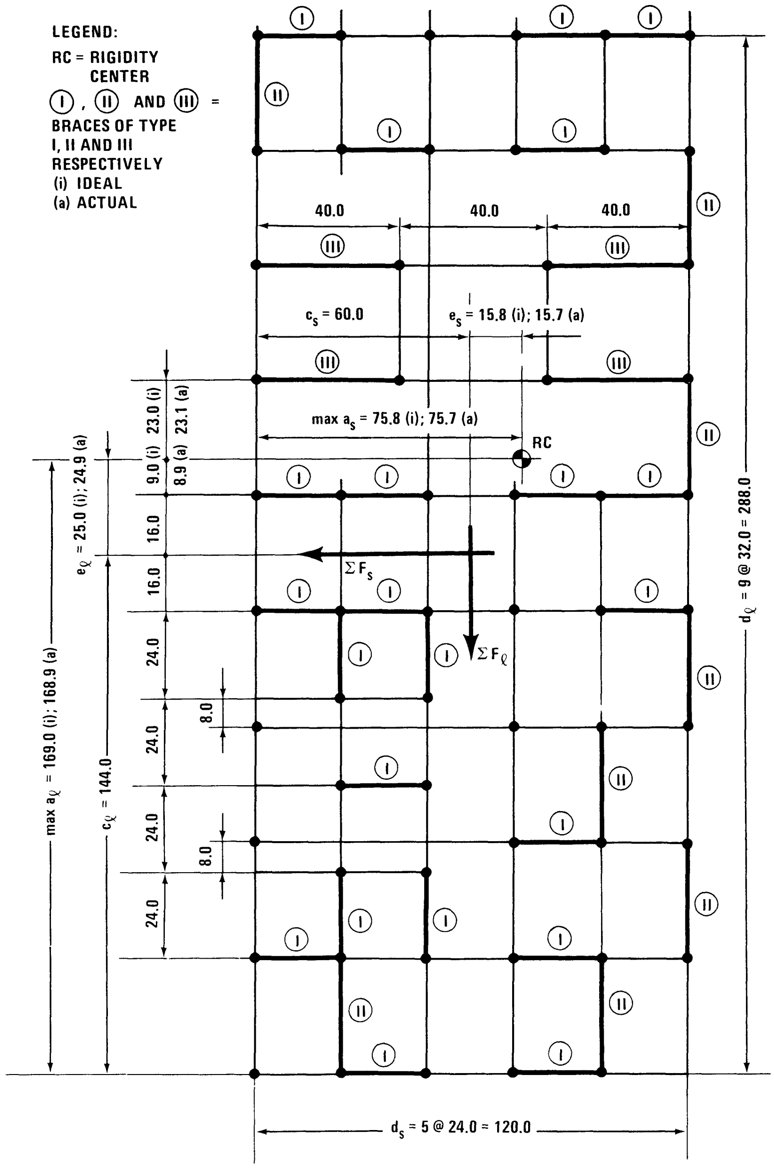

Figure 4

1. $E = 29{,}000$ ksi
   $\delta = 0.001$
   $\Sigma F_s = 5{,}280$ kips
   $\Sigma F_\ell = 4{,}400$ kips
   $h = 24.0$ ft
   $d_\ell = 288.0$ ft
   $d_s = 120.0$ ft
   $c_\ell = 144.0$ ft
   $c_s = 60.0$ ft
   $b_{\text{I}} = 24.0$ ft
   $b_{\text{II}} = 32.0$ ft
   $b_{\text{III}} = 40.0$ ft

2. $max\ a_\ell = [(2 \times 32 + 64 + 80 + 3 \times 128 + 4 \times 160 + 2 \times 256 + 3 \times 288) \times 576 + 4 \times 208 \times 1{,}600]/(18 \times 576 + 4 \times 1{,}600)$
   $= (2{,}608 \times 576 + 832 \times 1{,}600)/16{,}768$
   $= 168.977 \approx 169.0$ ft
   $max\ a_s = [(4 \times 120 + 2 \times 96 + 24) \times 1{,}024 + 4 \times 36 \times 576]/(8 \times 1{,}024 + 4 \times 576)$
   $= (696 \times 1{,}024 + 144 \times 576)/10{,}496$
   $= 75.8$ ft

3. $e_\ell = 169.0 - 144.0 = 25.0$ ft
   $e_s = 75.8 - 60.0 = 15.8$ ft

4. $\phi_\ell = \dfrac{169.0}{288.0} = 0.587$
   $\phi_s = \dfrac{75.8}{120.0} = 0.632$

5. $\rho_\ell = \{[(2 \times 169^2 + 2 \times 137^2 + 105^2 + 89^2 + 3 \times 41^2 + 4 \times 9^2 + 2 \times 87^2 + 3 \times 119^2) \times 576 + (2 \times 23^2 + 2 \times 55^2) \times 1{,}600]/16{,}768\}^{1/2}/288.0$
   $= 0.285$
   $\rho_s = \{[(75.8^2 + 51.8^2 + 2 \times 20.2^2 + 4 \times 44.2^2) \times 1{,}024 + (2 \times 51.8^2 + 2 \times 27.8^2) \times 576]/10{,}496\}^{1/2}/120.0$
   $= 0.377$

6. $B = \dfrac{5{,}280}{4{,}400}\left(\dfrac{0.285 \times 288}{0.377 \times 120}\right)^2$
$= 3.95$

$L = 1 + \dfrac{0.587 \times 25.0}{0.285^2 \times 288.0}$
$= 1.63$

$S = 1 + \dfrac{0.632 \times 15.8}{0.377^2 \times 120.0}$
$= 1.59$

7. $m^2 + (3.95 \times 1.63 - 1.59)\, m - 3.95 = 0$
$m^2 + 4.85\, m - 3.95 = 0$
$m = [-4.85 + (4.85^2 + 4 \times 3.95)^{1/2}]/2$
$= 0.71$

8. $\Sigma R_s \geq \dfrac{5{,}280}{0.001 \times 24} \times \dfrac{3.95 \times 1.63 + .71}{3.95 + .71}$
$\geq 337{,}482.8$ kip/ft

$\Sigma R_\ell \geq \dfrac{4{,}400}{0.001 \times 24} \times \dfrac{3.95 + 1.59 \times .71}{3.95 + .71}$
$\geq 199{,}813.7$ kip/ft

9. $337{,}482.8/199{,}813.7 = 1.69$
$5{,}280/(4{,}400 \times .71) = 1.69$ **o.k.**

10. $K = 0.75$

11. *Braces Type I:*
$\ell = (24.0^2 + 24.0^2)^{1/2}$
$= 33.94$ ft
$F_a = 0.001 \times 29{,}000 \times 24.0 \times 24.0/33.94^2$
$= 14.50$ ksi
$K\ell/r_y = 87.50$ (per A36)
min r_y (per $K = 0.75$)
$r_y = 0.75 \times 33.94 \times 12/87.50$
$= 3.49$ in.

Braces Type II:
$\ell = (24.0^2 + 30.0^2)^{1/2}$
$= 40.0$ ft
$F_a = 0.001 \times 29{,}000 \times 24.0 \times 30.0/40.0^2$
$= 13.92$ ksi
$K\ell/r_y = 92.38$ (per A36)
min r_y (per $K = 0.75$)
$r_y = 0.75 \times 40.0 \times 12/92.38$
$= 3.90$ in.

Braces Type III:
$\ell = (24.0^2 + 40.0^2)^{1/2}$
$= 46.65$ ft
$F_a = 0.001 \times 29{,}000 \times 24.0 \times 40.0/46.65^2$
$= 12.79$ ksi
$K\ell/r_y = 101.46$ (per A36)
min r_y (per $K = 0.75$)
$= 0.75 \times 46.65 \times 12/101.46$
$= 4.14$ in.

12. *Braces Type I:*
W14x90
$r_y = 3.70$ in.
$A = 26.5$ in.^{2}
$R = 26.5 \times 29{,}000 \times 24.0^2/33.94^3$
$= 11{,}322.2$ kip/ft

Braces Type II:
W14x145
$r_y = 3.98$ in.
$A = 42.7$ in.^{2}
$R = 42.7 \times 29{,}000 \times 24.0^2/40.00^3$
$= 19{,}812.8$ kip/ft

Braces Type III:
W14x233
$r_y = 4.10$ in.^{2}
$A = 68.5$ in.^{2}
$R = 68.5 \times 29{,}000 \times 24.0^2/46.65^3$
$= 31{,}307.9$ kip/ft

13. $\Sigma R_s = 18 \times 11{,}322.2 + 4 \times 31{,}307.9$
$= 329{,}031.2 \approx 337{,}482.8$ kip/ft **o.k.**
$\Sigma R_\ell = 8 \times 19{,}812.8 + 4 \times 11{,}322.2$
$= 203{,}791.2 > 199{,}813.7$ kip/ft **o.k.**
max $a_\ell = (2{,}608 \times 11{,}322.2 \times 832 \times 31{,}307.9)/329{,}031.2$
$= 168.9$ ft
max $a_s = (696 \times 19{,}812.8 + 144 \times 11{,}322.2)/203{,}791.2$
$= 75.7$ ft
$e_\ell = 168.9 - 144.0$
$= 24.9$ ft
$e_s = 75.7 - 60.0$
$= 15.7$ ft
$I_r = (2 \times 168.9^2 + 2 \times 136.9^2 + 104.9^2 + 88.9^2 + 3 \times 40.9^2 + 4 \times 8.9^2 + 2 \times 87.1^2 + 3 \times 119.1^2 + 2 \times 51.7^2 + 2 \times 27.7^2) \times 11{,}322.2 + (75.7^2 + 51.7^2 + 2 \times 20.3^2 + 4 \times 44.3^2) \times 19{,}812.8 + (2 \times 23.1^2 + 2 \times 55.1^2) \times 31{,}307.9$
$= 2{,}638{,}220{,}770$ kip/ft

Braces Type I:
$K\ell/r_y = 0.75 \times 33.94 \times 12/3.70$
$= 82.56 \therefore$
$F_a = 15.07$ ksi (per A36)
max $a = 168.9$ ft
$F = 11{,}322.2 \times 5{,}280\,(1/329{,}031.2 + 24.9 \times 168.9/2{,}638{,}220{,}770)$
$= 277.0$ kips
$\delta = 277.0/11{,}322.2 \times 24.0$
$= 0.00102 \approx .001$ **o.k.**
$P = 277.0 \times 33.94/24.0$
$= 391.7$ ksi
$f_a = 391.7/26.5$
$= 14.78 < 15.07$ ksi **o.k.**

Braces Type II:

$K\ell/r = 0.75 \times 40.0 \times 12/3.98$
$= 90.45 \therefore$

$F_a = 14.04$ ksi (per A36)

$max\ a = 75.7$ ft

$F = 19{,}812.8 \times 4{,}400\ (1/203{,}791.2 + 15.7 \times 75.7/2{,}638{,}220{,}770)$
$= 467.0$ kips

$\delta = 467.0/19{,}812.8 \times 24.0$
$= .00098 < 0.001$ **o.k.**

$P = 467.0 \times 40.0/32.0$
$= 583.8$ kips

$f_a = 583.8/42.7$
$= 13.67 < 14.04$ ksi **o.k.**

Braces Type III:

$K\ell/r_y = 0.75 \times 46.65 \times 12/4.10$
$= 102.40 \therefore$

$F_a = 12.67$ ksi (per A36)

$max\ a = 55.1$ ft

$F = 31{,}307.9 \times 5{,}280\ (1/329{,}031.2 + 24.9 \times 55.1/2{,}638{,}220{,}770)$
$= 588.4$ kips

$\delta = 588.4/31{,}307.9 \times 24.0$
$= 0.0078 < 0.001$ **o.k.**

$P = 588.4 \times 46.65/40.0$
$= 686.2$ kips

$f_a = 686.2/68.5$
$= 10.02 < 12.67$ ksi **o.k.**

CONCLUSIONS

The analysis of vertical braces of a braced steel structure requires concurrent consideration of drift and strength requirements. Done by hand calculations, or even by digital computers, this kind of analysis is time-consuming and expensive. The method presented in this paper avoids this tedious work, even though only the simplest arithmetic calculations are used. The simplicity of the described algorithm permits implementation on a pocket calculator, which makes it an important aid for the design of braced steel structures. The guidelines presented here allow the direct determination of the required rigidities for both directions of a braced steel structure so that overloading of braces can be prevented.

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NOMENCLATURE

a = distance of a brace to R.C.
b = horizontal dimensions of a bay or horizontal projection of a brace length ($b = \ell \cos\alpha$)
c = distance of resultant ΣF of lateral loads to that extremity of structure which is farthest with respect to R.C.
d = overall dimension of structure
e = eccentricity of ΣF with respect to R.C.
f_a = actual axis stress on a brace
h = distance between two adjacent diaphragms or vertical projection of a brace length ($h = \ell \sin\alpha$)
ℓ = length of a brace
m = a factor defined by Eq. 14
r = gyration radius of frame rigidities or of a brace
r_y = gyration radius of a brace with respect to its minor axis
A = cross-sectional area of a brace
B = expression defined by Eq. 21
E = elasticity modulus of steel = 29,000 ksi
F = lateral load carried by a brace
F_a = allowable stress for sizing braces in accordance with the AISC Specification, but no larger than that defined by Eq. 4
I_r = polar moment of inertia of frame rigidities
K = effective length factor of a brace with respect to its critical axis
L = expression defined by Eq. 22
P = axial load carried by a brace
R = rigidity of a brace
R.C. = rigidity center
S = expression defined by Eq. 23
α = slope of a brace
δ = drift index ($\delta = \triangle_a/h$)
ϕ = ratio of the maximum brace distance from R.C. to overall structure dimension, in same direction = $(max\ a)/d$
ρ = ratio of gyration radius to overall structure dimension, in same direction ($\rho = r/d$)
ψ = factor defined by Eq. 27
$\triangle$ = displacement of top of loaded brace from its position when brace is not loaded (drift)
$\triangle_a$ = allowable drift
Σ = summation

Subscripts

s = short, refers to transverse dimension of a structure
ℓ = long, refers to longitudinal dimension of a structure
I,II,III = refers to the braces Type I, Type II and Type III respectively shown in Fig. 4

REFERENCES

1. Uniform Building Code *1982, Sec. 2312(e) 4, p. 135.*
2. *American Institute of Steel Construction, Inc.* Manual of Steel Construction *8th Ed., pp. 6–19 and 21–24.*